2.5 Piecewise Functions

Vocab

- piecewise function ⇒ a fn that's defined in pieces (on separate domain intervals)
- greatest integer fn ⇒ $f(x) = [[x]] = n$ if $n \le x < n+1$, $n \in \mathbb{Z}$
  (it's sometimes called a step fn) "rounds down"

Ex 1 For

$$f(x) = \begin{cases} 10x & \text{if } x \le 0 \\ x^3 & \text{if } 0 < x < 3 \\ 0 & \text{if } x \ge 3 \end{cases}$$

find

(a) $f(-1)$

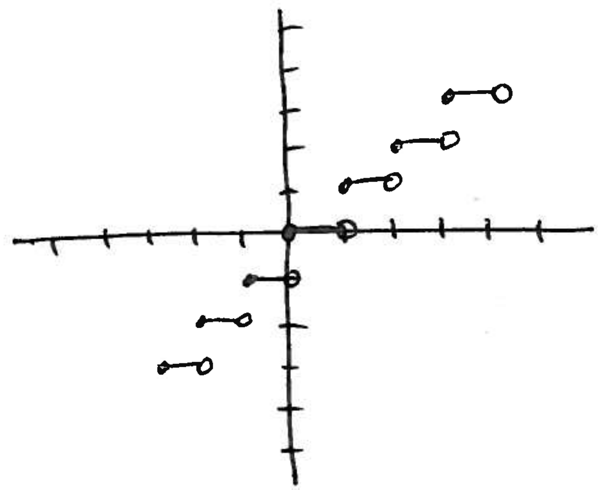
(e) $f(3)$

(b) $f(0)$

(c) $f(2)$

(d) $f(10)$

Graph of $f(x) = [[x]]$



2.5 (cont)

Ex 2 Graph $f(x) = \begin{cases} 10x & \text{if } x \le 0 \\ x^3 & \text{if } 0 < x < 3 \\ 0 & \text{if } x \ge 3 \end{cases}$

Ex 3 Graph $f(x) = 3 - |x|$

Ex 4 The charge for a certain telephone call is 75¢ for the first 3 minutes & 25¢ for each additional minute or fraction thereof. Write a function that gives cost of a call lasting x minutes.

Ex 5 Graph $g(x) = [[x+3]]$

2.6 Composition and Operations of Functions

Functional Operations

f has domain D_f

g " " D_g

then these fns have domain $D_f \cap D_g$

(1) $(f+g)(x) = f(x) + g(x)$

(2) $(f-g)(x) = f(x) - g(x)$

(3) $(f \cdot g)(x) = f(x)\,g(x)$

(4) $(f \div g)(x) = (f/g)(x) = \dfrac{f(x)}{g(x)}$

Composition Fn

$(f \circ g)(x) = f(g(x))$

(read "f composed with g of x" is f of g of x")

where x is in the domain of g and $g(x)$ is in domain of f

Ex1 For $f(x) = \dfrac{x-2}{x+1}$ and $g(x) = x^2 - x - 2$, find

(a) $(f+g)(2)$

(b) $(f-g)(5)$

(c) $(f \circ g)(1)$

(d) $(g \circ f)(1)$

2.6 (cont)

Ex 1 (cont) $f(x) = \frac{x-2}{x+1}$ $g(x) = x^2 - x - 2$

(e) $(f/g)(99)$

(f) $(fg)(102)$

(g) state domain of (i) $f+g$

(ii) $f-g$

(iii) fg

(iv) $\frac{f}{g}$

2.6 (cont)

Ex 2 Given $f(x) = x^4$ and $(f \circ g)(x) = (2x+\sqrt{5})^4$,

find $g(x)$.

Ex 3 Express $f(x)$ as a composition of 2 fns u and g so that $f(x) = g[u(x)]$.

(a) $f(x) = (2x^2 - x + 1)^3$

(b) $f(x) = |x+1|^2 + 6$

2.6 (cont)

Ex 4 If $f(x)=x^2$, $g(x)=3x-2$ and $h(x)=x^2+1$,

find (a) $(f \circ g) \circ h$

(b) $f \circ (g \circ h)$

2.7 Inverse Functions

Vocab/Defns

<u>inverse of f</u>: let f be a fn w/ domain D and range R. Then f^{-1} with domain R and range D is the <u>inverse of f</u> if

$$f^{-1}(f(a)) = a \quad \forall a \in D$$
$$f(f^{-1}(b)) = b \quad \forall b \in R$$

(f inverse basically undoes the function f(x); so if (a, b) is pt on graph $y = f(x)$, then (b, a) is pt on graph of $y = f^{-1}(x)$.)

Graphically, $f^{-1}(x) = y$ is the reflection across $y=x$ line of f(x).

A fn only has an inverse if it's 1-1.

<u>To find $f^{-1}(x)$</u>: first check if f(x) is 1-1.

1. exchange x & y variables in eqn/fn.
2. solve for y.

The result is $f^{-1}(x)$.

2.7 (cont)

Ex 1 Determine if these fns are inverses.

(a) $f(x) = \frac{2}{3}x + 2, \quad g(x) = \frac{3}{2}x - 3$

(b) $f(x) = \frac{1}{x+1}, \quad g(x) = \frac{1}{x-1}$

Ex 2 Find $f^{-1}(x)$ for $\quad f(x) = \frac{3x+1}{2x-5}$

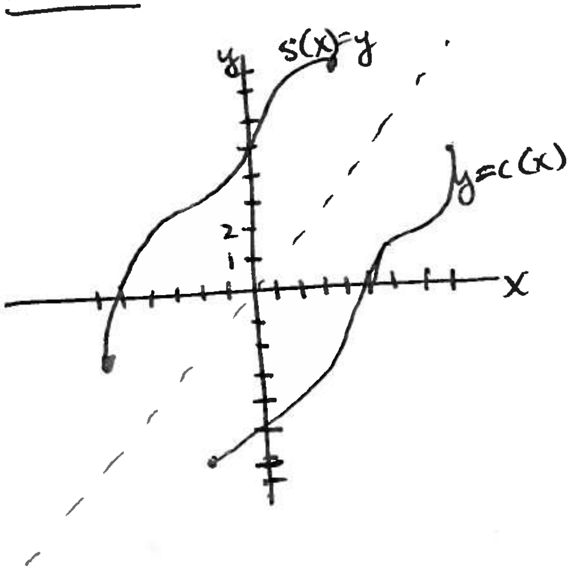
2.7 (cont)

Ex 3 Find $f^{-1}(x)$, if it exists, for these fns.

(a) $g(x) = x^5 - 1$

(b) $h(x) = \frac{1}{\sqrt[3]{x}} + 5$

Ex 4 Use this graph to answer the qns.



(a) $s(1)$

(b) $s^{-1}(1)$

(c) $c(0)$

(d) $c^{-1}(0)$

(e) domain/range for $s(x)$ and $c(x)$

Ex 5 For $y=f(x)=x^2$, does it have an inverse? If not, can we restrict the domain so it has inverse? If so, find $f^{-1}(x)$.

1.6/1.9 Inequalities

(Review)

Defn

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Ex 1 Solve

(a) $|2x+4| = 9$

(b) $|3x-1| = -8$

(c) $|x-5| = |x+2|$

Ex 2 Solve.

(a) $|2-5x| < 1$

(b) $|5-4x| \geq 9$

(c) $x(x+1) < 0$

1.6/1.9 (cont)

Ex 3 Solve.

(a) $x^2+3x+2>0$

(b) $\dfrac{(x-2)}{x(x+1)(5-x)} \leq 0$