8.4 Systems of Equations

Three Strategies to Solve a System of Linear Eqns

① **Graphing**: this method is useful to provide a visual, but it doesn't produce very precise answers.

② **Substitution** method

Vocab

System of eqns: Two or more eqns w/ two or more variables (that we can solve simultaneously)

Solution: set of values for the variables that makes all the eqns true.
(the intersection point of all the curves represented by the eqns)

③ **Elimination** method (note: this book calls this Addition or Linear Combinations method).

8.4 (cont)

Ex 1 Solve.

(a) $y = x^2 - 4x$
$y = x^2 - 4x + 8$
} solve by graphing.

(b) $\frac{x}{3} - y = 7$
$x + \frac{y}{2} = 7$
} Solve by substitution

8.4 (cont)

Ex 2 Solve.

(a) $3u+2v=5$
$4v=10-6u$ } Solve by elimination (or linear combinations) method

(b) $100x-y=0$
$50x+y=300$

8.4 (cont)

Ex3 Solve $3x + 4y^2 = 19$
$x^2 + y^2 = 5$

Ex4 Find the lengths of the legs of a right Δ whose area is 60 ft^2 and whose hypotenuse is 17 ft.

8.5 Matrix Solution of a System of Equations

Vocab/Notation

matrix (plural is matrices): an ordered array of numbers

characterized by # of rows by # of columns as its size, and typically denoted by capital letters

ex

$$\begin{aligned} a_{11}x + a_{12}y &= b_1 \\ a_{21}x + a_{22}y &= b_2 \end{aligned} \iff \overbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}^{\text{matrix eqn}}$$

$$\iff AX = B$$

where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

if # rows = # columns, A is **square**

Size (a.k.a. order or dimension) of A is $m \times n$ where m = # rows, n = # columns (read "m by n")

augmented matrix: in above example, it would be $\left[\begin{array}{cc|c} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \end{array}\right]$

(the coefficient matrix augmented w/ constant matrix)

8.5 (cont)

Ex 1 Write augmented matrix for

$$x+y+z=10$$
$$3x+2y+z=7$$
$$x \quad +5z=0$$

what size is this matrix?

Ex 2 Perform elementary row operations to get 1 in a_{11} place.

$$\left[\begin{array}{ccc|c} 5 & 20 & 15 & 6 \\ 7 & -5 & 3 & 2 \\ 12 & 0 & 1 & 4 \end{array}\right]$$

Elementary Row Operations

① Multiply any row by a nonzero constant

② Exchange order of two rows

③ Temporarily multiply one row by a nonzero constant and add to another row, replacing on of those rows by the sum. (book calls one of these rows a pivot row)

☆ using these is called Gauss-Jordan Elimination

8.5 (cont)

Ex3 Solve using Gauss-Jordan method

(a) $5x + z = 1$
$x - 5z = 21$
$x + y - z = 0$

Row-Echelon Form

$$\left[\begin{array}{ccc|c} 1 & a_{12} & a_{13} & b_1 \\ 0 & 1 & a_{23} & b_2 \\ 0 & 0 & 1 & b_3 \end{array}\right]$$

Reduced Row Echelon Form

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & b_1 \\ 0 & 1 & 0 & b_2 \\ 0 & 0 & 1 & b_3 \end{array}\right]$$

☆ use elementary row ops to change augmented matrix into these forms

8.5 (cont)

Ex 3 (b)

$$x+2y-z=3$$
$$z-3x-6y=-9$$
$$4y+2x-z=6$$

(c)

$$x+2y=9$$
$$x+y=5$$
$$3x-y=26$$

8.5 (cont)

Ex 3 (d) $2x+3y-z=5$
$x+5y-4y=2$

Ex 4 Find eqn of parabola of the form $y=ax^2+bx+c$ that passes through $(1,-2)$, $(-2,-14)$ and $(3,-4)$

8.6 Inverse Matrices

Arithmetic w/ Matrices

① $A = B$ iff every entry in A = every corresponding entry in B

i.e. $a_{ij} = b_{ij} \quad \forall\, i,j$

② we can add or subtract matrices only if they are the same size (add or subtract corresponding entries)

③ multiplication: two types

(a) scalar multiplication ⇒ cA, means multiply each entry of A by $\mathbb{R}$ # c.

(b) matrix multiplication ⇒ AB, can only do this if # cols in A = # rows in B; if A is $m \times n$ and B is $n \times p$, then AB is $m \times p$ (but BA is not doable).

☆ note: There is NO matrix division.

Properties	Addition	Multiplication
Commutativity	$A + B = B + A$	$(AB \neq BA)$
Associativity	$(A+B)+C = A+(B+C)$	$A(BC) = (AB)C$
Identity	$A + O = O + A$	$AI = IA = A$
Inverse	$A + -A = -A + A = O$	$A \cdot A^{-1} = A^{-1} \cdot A = I$
Distributivity	$M(N+P) = MN + MP$ $(N+P)M = NM + PM$	

④ O is the zero matrix (filled w/ zeros; can be any size)

⑤ I is the identity matrix: a square $(n \times n)$ matrix w/ zeros on the off diagonal entries and 1's on the diagonal

§6 (cont)

ex $\mathbf{0} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

ex $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

inverse matrix: A^{-1} (read "A inverse") is the square matrix such that

$$A^{-1}A = AA^{-1} = I$$

(where A is also square)

(note: $A^{-1} \neq \frac{1}{A}$)

- if A^{-1} exists, we say A is nonsingular

B^T = "B transpose"

if $B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, $B^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$

basically exchange rows w/ columns

Ex 1 For $A = \begin{bmatrix} 1 & 2 \\ 4 & 0 \\ -1 & 3 \\ 7 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 1 & 3 & 2 \\ 1 & 0 & -2 & 9 \end{bmatrix}$

(a) Find $B^T + A$

(b) Find $-3A$

8.6 (cont)

Ex1 (cont)

(c) Find AB.

(d) Find BA.

8.6 (cont)

Ex 2 Show that A and B are inverses.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} -16 & -2 & 7 \\ 7 & 1 & -3 \\ -3 & 0 & 1 \end{bmatrix}$$

Ex 3 Find inverse of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

8.6 (cont)

Ex 4 Find inverse of $A = \begin{bmatrix} 6 & 1 & 20 \\ 1 & -1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Ex 5 (a) Set up $\begin{aligned} 4x - 7y &= -65 \\ -x + 2y &= 18 \end{aligned}$ as $AX = B$, (b) find A^{-1},
(c) Solve for X.

MOOO

8.6 (cont)

Ex 6 Use the inverse from Ex 4 to solve

$$\begin{aligned} 6x + y + 20z &= 5 \\ x - y &= 2 \\ y + 3z &= -1 \end{aligned}$$

Ex 7 If M & N are nonsingular, then MN is nonsingular. Show that the inverse of MN is

$N^{-1} M^{-1}$!

8.7 System of Inequalities

Ex 1 Graph solution set for

$$x+y \le 8$$
$$y \le 4$$
$$x \le 6$$

Shaded solution region is called feasible region

Ex 2 Graph solution set for this system of inequalities.

$$2x+3y \le 30 \quad , \quad x \ge 0$$
$$3x+2y \ge 20 \quad , \quad y \ge 0$$

8.7 (cont)

Ex 3 Graph solution set $y \geq \frac{1}{2}(x-3)^2$
$y \leq x+1$

Ex 4 Graph solution set for $x^2+y^2 \leq 16$
$x^2+y^2-6x \geq -16$