What does it mean to measure?

- decide on units to use for measuring.
  observe how many units complete a whole
- "sizing up"
- sort of assessment
- quantifier

What attributes can we measure?

- distance
- area
- height
- volume
- weight
- ~~amount~~ value of money
- time

What are some of the units we use to measure length?

- inches
- cm
- km
- mm
- miles
- feet
- yards

- The earliest recorded unit for measuring length was *cubit:* the distance from the point of the elbow to the tip of the middle finger.

- The Roman writer Plutarch wrote that the *foot* was based on the actual length of Hercules' foot.

- In the 10th century King Edgar I declared that the *yard* would be the distance from the tip of his nose to the tip of the finger of his outstretched arm

- The Romans divided foot into 12 units (unicae – inch)

- The Roman *pace* consisted of 5 feet and their mile was 1000 paces long.

- When Alexander the Great conquered the world he had professional pacers hired so that the maps would be accurate. The pacers followed the army and measured the distance they went: and were very precise.

There are about 6 billion people on Earth. If they all lined up and held hands, how long would the line be?

① 12,000,000,000 ft (2 ft wide per person)

9,000,000,000 ft ($1\frac{1}{2}$ ft wide per person)

① 2,727,272.27 miles

Circumference of Earth = 24901 miles

91.27 times around Earth.

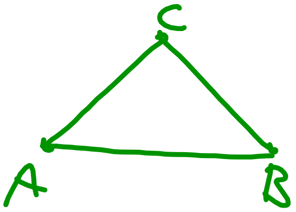
Properties of Distance between points A and B

note

$\overline{AB}$ is line seg from A to B

AB = length of $\overline{AB}$

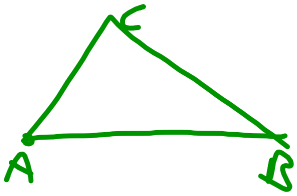
1. AB >= 0
2. AB = BA
3. For any three points, A, B and C, AB + BC >= AC (the Triangle Inequality).



Example:

Can the following be lengths of the sides of a triangle? 24 cm, 45 cm, 70 cm

$24+45 = 69 \overset{?}{>} 70$ no ⇒ these cannot be lengths of a Δ.



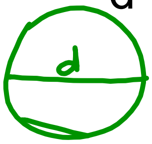
we have something

like this

Circumference of a Circle

(1-d measurement of a 2-d circle)

$C = \pi d$ d = diameter of the circle



$d = 2r$

Ex 2 What happens to the circumference of a circle if the length of the radius is tripled?

$\overset{\text{old}}{r} \longrightarrow \overset{\text{new}}{3r}$

$C_o = 2\pi r \longrightarrow C_n = 2\pi(3r) = 3(2\pi r) = 3C_o$

new circumfrence is 3 times bigger than original circumference

Ex 3 Find a formula that will give the length of the circle that has a central angle of θ.

The measures of length in the English system are:

1 inch
1 foot = 12 inches
1 yard = 3 feet
1 mile = 5280 feet

1. How many yards are in one mile?

$$1\,\cancel{mi}\left(\frac{5280\,\cancel{ft}}{1\,\cancel{mi}}\right)\left(\frac{1\,yd}{3\,\cancel{ft}}\right) = 1760\,yd$$

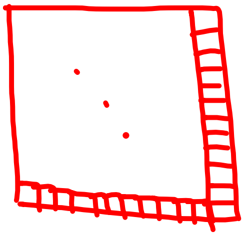
2. Convert 10 ft/s to miles/hr.

$$\frac{10\,\cancel{ft}}{\cancel{sec}}\left(\frac{1\,mi}{5280\,\cancel{ft}}\right)\left(\frac{60\,\cancel{sec}}{1\,\cancel{min}}\right)\left(\frac{60\,\cancel{min}}{1\,hr}\right) = \frac{36000}{5280}\,\frac{mi}{hr} \approx 6.81\,\frac{mi}{hr}$$

3. A snail travels at a pace of one inch per second. How long does it take the snail to travel one mile? $\boxed{1056\text{ min}}$

$$\frac{1\,\cancel{in}}{\cancel{sec}}\left(\frac{1\,\cancel{ft}}{12\,\cancel{in}}\right)\left(\frac{1\,mi}{5280\,\cancel{ft}}\right)\left(\frac{60\,\cancel{sec}}{1\,min}\right) = \frac{60\,mi}{12(5280)\,min} = \frac{5\,mi}{5280\,min} = \frac{1\,mi}{1056\,min}$$

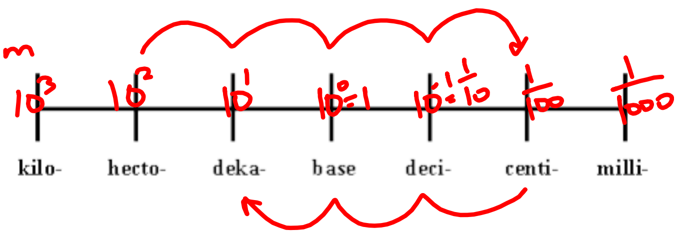
4. I have square tiles one inch on a side. How many does it take to tile a square one foot on a side?



12(12) tiles = 144 tiles

5. I have some cubes one foot on a side. How many does it take to fill a large cubical box one yard on a side?

1 km = 1000m



The "ruler" above is called a *metric converter*. All conversions within the metric system involve multiplying or dividing by powers of ten. In other words, they are all accomplished by moving the decimal point (and adding zeros, if necessary, after the move). When you move to the right from one prefix to another, the decimal point moves to the right. For example, 1.23 kg in dekagrams is 123 dkg; the decimal point moves two to the right.

A mnemonic device for remembering the order that is often used is: "King Henry Doesn't (Usually) Drink Chocolate Milk". The "Usually" stands for "Unit", since the metric prefixes may be applied to any metric unit.

Some conversions:

1. What is 236 centimeters in dekameters?

236 cm = 0.236 dkm

2. What is 0.456 hg in cg?

0.456 hg = 4560 cg

3. Convert 20 km/hr to m/s.

$$20\frac{\cancel{km}}{\cancel{hr}}\left(\frac{1\ \cancel{hr}}{60\ \cancel{min}}\right)\left(\frac{1\ \cancel{min}}{60\ sec}\right)\left(\frac{10^3\ m}{1\ \cancel{km}}\right)=\frac{2(10^4)}{3600}\ \frac{m}{sec}$$

$$=\frac{100}{18}\ \frac{m}{sec}$$

$$=\frac{50}{9}\ \frac{m}{sec}$$

$$=5.\overline{5}\ m/sec$$

4. How much money is a kilocent?

kilocent = 1000 ¢ = $10

5. What is a centidollar?

$$\$\frac{1}{100}=\$0.01=1\ ¢$$

6. A snail moves 1 cm per minute. How many kilometers has the snail covered in one week?

$$\frac{1\ cm}{min}\cdot 1\ wk=\frac{1\ cm}{\cancel{min}}(\cancel{wk})\left(\frac{7\ \cancel{days}}{1\ \cancel{wk}}\right)\left(\frac{24\ \cancel{hr}}{1\ \cancel{day}}\right)\left(\frac{60\ \cancel{min}}{1\ \cancel{hr}}\right)$$

$$=\frac{7(24)(60)\ \cancel{cm}}{1}\left(\frac{1\ km}{10^5\ \cancel{cm}}\right)$$

$$=\frac{7(24)(60)}{10^5}\ km$$

Since two countries in the world still use the English system, we need to be able to convert between the metric and English systems.

The basic fact for length is: 1 in = 2.54 cm (exactly).

1. How many kilometers are in a mile?

$$1\,\cancel{\text{mi}}\left(\frac{5280\,\cancel{\text{ft}}}{1\,\cancel{\text{mi}}}\right)\left(\frac{12\,\cancel{\text{in}}}{1\,\cancel{\text{ft}}}\right)\left(\frac{2.54\,\cancel{\text{cm}}}{1\,\cancel{\text{in}}}\right)\left(\frac{1\,\text{km}}{10^5\,\cancel{\text{cm}}}\right)=\frac{5280(12)(2.54)}{10^5}\,\text{km}$$

2. How many inches are in a meter?

$$1\,\cancel{\text{m}}\left(\frac{100\,\cancel{\text{cm}}}{1\,\cancel{\text{m}}}\right)\left(\frac{1\,\text{in}}{2.54\,\cancel{\text{cm}}}\right)=\frac{100}{2.54}\,\text{in}$$

3. A typical highway speed limit in Canada is 100 km/hr. How fast is this in miles/hr?

$$100\,\frac{\cancel{\text{km}}}{\text{hr}}\left(\frac{10^5\,\cancel{\text{cm}}}{1\,\cancel{\text{km}}}\right)\left(\frac{1\,\cancel{\text{in}}}{2.54\,\cancel{\text{cm}}}\right)\left(\frac{1\,\cancel{\text{ft}}}{12\,\cancel{\text{in}}}\right)\left(\frac{1\,\text{mi}}{5280\,\cancel{\text{ft}}}\right)=\frac{10^7\,\text{mi}}{2.54(12)5280\,\text{hr}}$$

4. Now that snail is moving 1 meter per hour. How many miles does the snail travel in the month of January?

$$\frac{1\,\text{m}}{\cancel{\text{hr}}}\cdot 31\,\cancel{\text{days}}\left(\frac{24\,\cancel{\text{hr}}}{1\,\cancel{\text{day}}}\right)=31(24)\,\cancel{\text{m}}\left(\frac{100\,\cancel{\text{cm}}}{1\,\cancel{\text{m}}}\right)\left(\frac{1\,\cancel{\text{in}}}{2.54\,\cancel{\text{cm}}}\right)$$

$$\cdot\left(\frac{1\,\cancel{\text{ft}}}{12\,\cancel{\text{in}}}\right)\left(\frac{1\,\text{mi}}{5280\,\cancel{\text{ft}}}\right)$$

$$=\frac{31(24)(100)}{2.54(12)(5280)}\,\text{mi}$$

Practicum Discussion

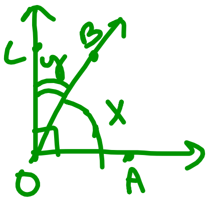
** Due date: Monday, 11/25 (Do not turn in late!!)

*Observations should be done by 10/7

* You can turn in rough draft for review on 11/11

Homework Questions (9/4)

11.1A #9 (b)



$m(\angle AOB) = 3m(\angle BOC) - 35°$

① $x = 3y - 35°$

② $x + y = 90°$

② $3y - 35 + y = 90$

$4y = 125$

$\boxed{y = 31.25°}$

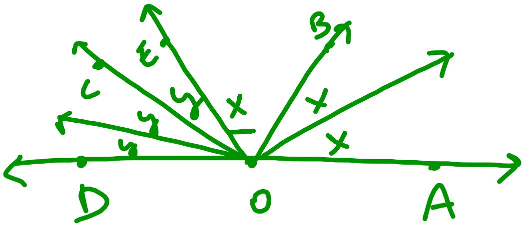
$\Longleftarrow$

① $x = 3(31.25) - 35$

$= 93.75 - 35$

$\boxed{x = 58.75°}$

(c)



m(∠BOC) = ?

why can x and y not be determined?

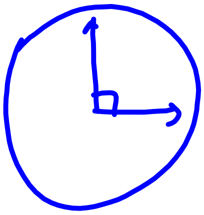
not enough info

① $3x + 3y = 180°$

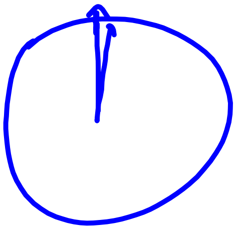
$\Rightarrow x + y = 60°$

$m(\angle BOC) = x + y = \boxed{60°}$

11.1A #8) (a) 90°



(b)



at 12:25

$\theta = ?$

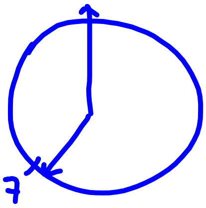
by 1:00 hour hand is
30° away from 12

30° in 60 minutes

x° in 25 minutes

$$\frac{x}{25} = \frac{30}{60} \Rightarrow x = \boxed{12.5^\circ}$$

(a)(iii) at 6:50 p.m.

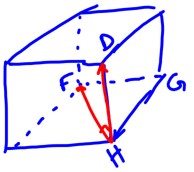


at 6:00 p.m. angle is 180°

$$\frac{x}{50} = \frac{30}{60} \Rightarrow x = 25^\circ \text{ after } 6{:}00$$

at 6:50 $\theta = 180 + 25 = \boxed{205^\circ}$

11.1B #2 (d)



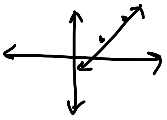
90°

11.1B #10 (a) # of planes defined by 3 noncollinear pts

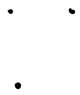
1-d

1 way

2-d



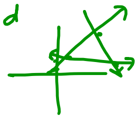
3-d



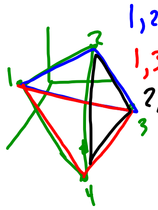
(b) 4 pts, no 3 of which are coplanar

1-d

2-d



3-d



1,2,4
1,2,3
1,3,4
2,3,4

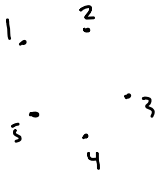
(c) # of planes defined by n pts, no 4 of which are coplanar? (in 3-d)

# pts	# planes
3	1
4	4
5	10
6	20
7	35
⋮	
n	?

+3 +6 +10

+3 +4

$\boxed{{}_nC_3} = \frac{n!}{(n-3)!\,3!}$



1,2,3
1,2,4
1,2,5
1,3,4
1,3,5
1,4,5

2,3,4
2,3,5
2,4,5
3,4,5

6:

1,2,3
1,2,4
1,2,5
1,2,6

1,3,4
1,3,5
1,3,6
1,4,5
1,4,6

1,5,6
2,3,4
2,3,5
2,3,6
2,4,5
2,4,6

2,5,6
3,4,5
3,4,6
3,5,6
4,5,6

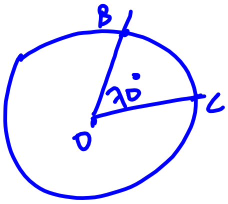
Pascal's Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

${}_3C_3$ ${}_4C_3$ ${}_5C_3$ ${}_6C_3$ ${}_7C_3$

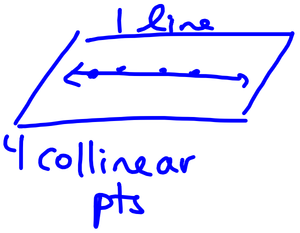
1
4
10
20
35

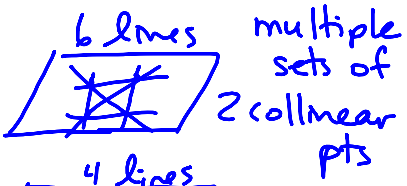
11.1 A) 15(a)

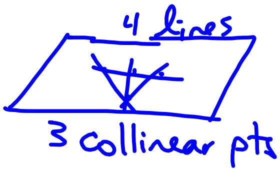


m∠BOC = 70°

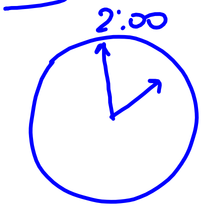
11.1 MC) #2

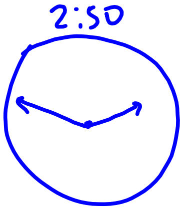


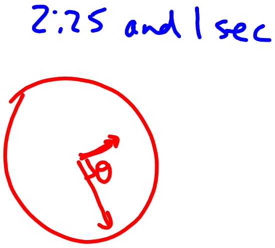




11.1 MC) #4)

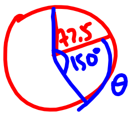






at 2:25

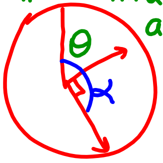
hour hand goes 30° in 60 minutes ⇒ at 2:25, hour hand at 72.5°



and minute hand at 150°

⇒ $\theta = 150 - 77.5 = 77.5°$

t = # of minutes after 2:00



$\theta = \frac{1}{2}t + 60°$ hour hand

$\alpha = 6t$ minute hand

goal: $t = ?$

when $\alpha - \theta = 90°$

$$6t - \left(\frac{1}{2}t + 60\right) = 90$$

$$\frac{11}{2}t = 150$$

$$t = 150\left(\frac{2}{11}\right) = \frac{300}{11} \simeq 27.\overline{27}$$

at about 2:27