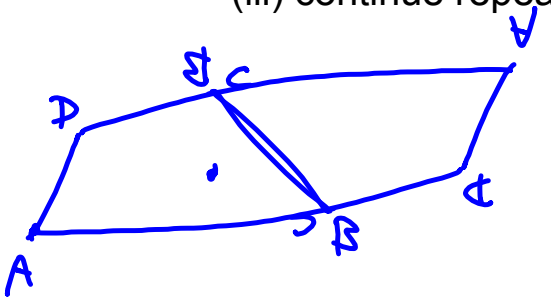
13.4 Tesselations

(a.k.a. tiling)

Tesselation--an arrangement of non-overlapping polygons (having only sides in common) that completely covers the plane

Which polygons tesselate the plane?

1. any triangle
2. any quadrilateral
   - (i) trace quadrilateral
   - (ii) rotate 180 degrees around the midpoint of any side and trace the image
   - (iii) continue repeating step (ii)



Regular Tesselation--a tesselation made of only one regular polygon

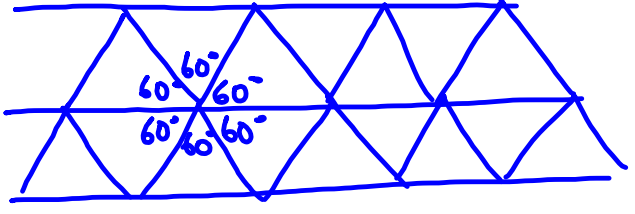
Semiregular Tesselation--a tesselation made of two or more regular polygons

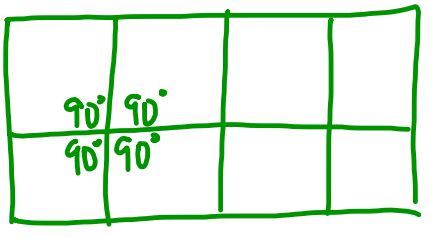
Which regular polygons tesselate the plane?

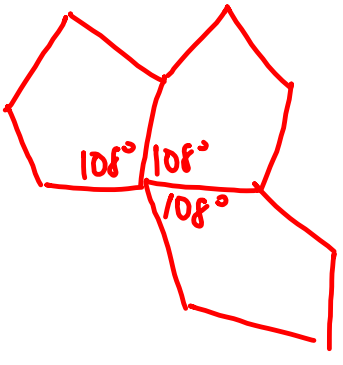
1. equilateral triangle
2. square
3. regular hexagon

$\frac{180(n-2)}{n}$ (interior)

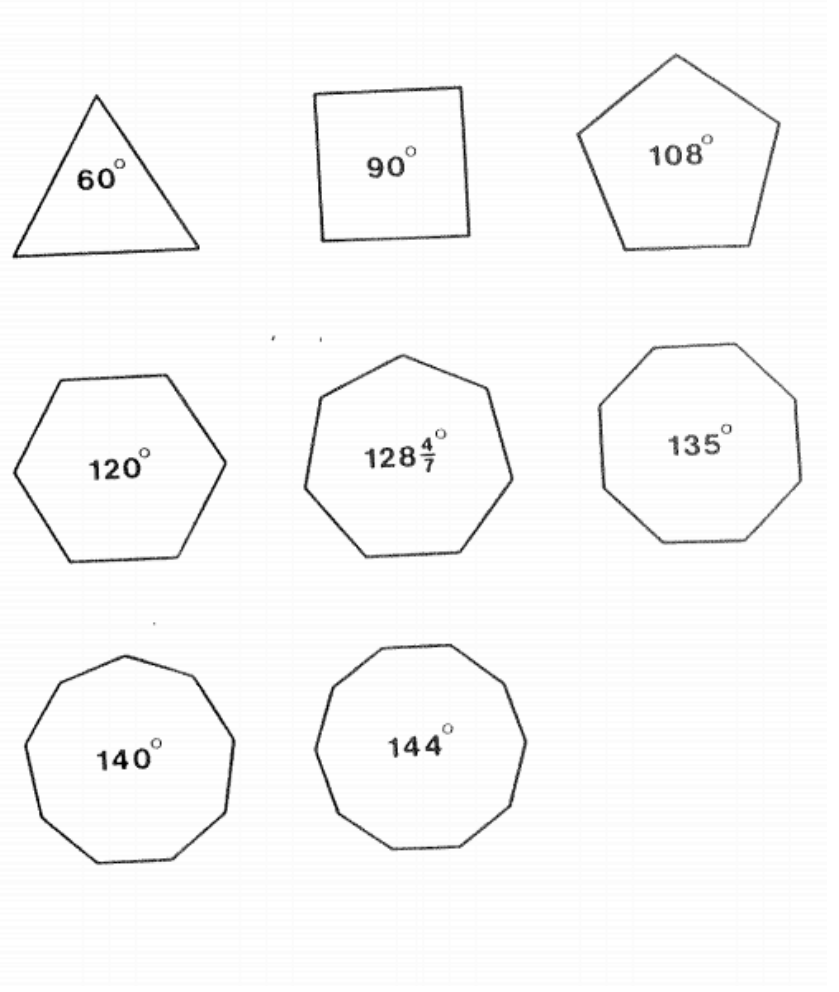
n	vertex angle of regular n-gon	tesselate a plane?
3	60°	yes
4	90°	yes
5	108°	no
6	120°	yes
7	$128\frac{4}{7}$°	

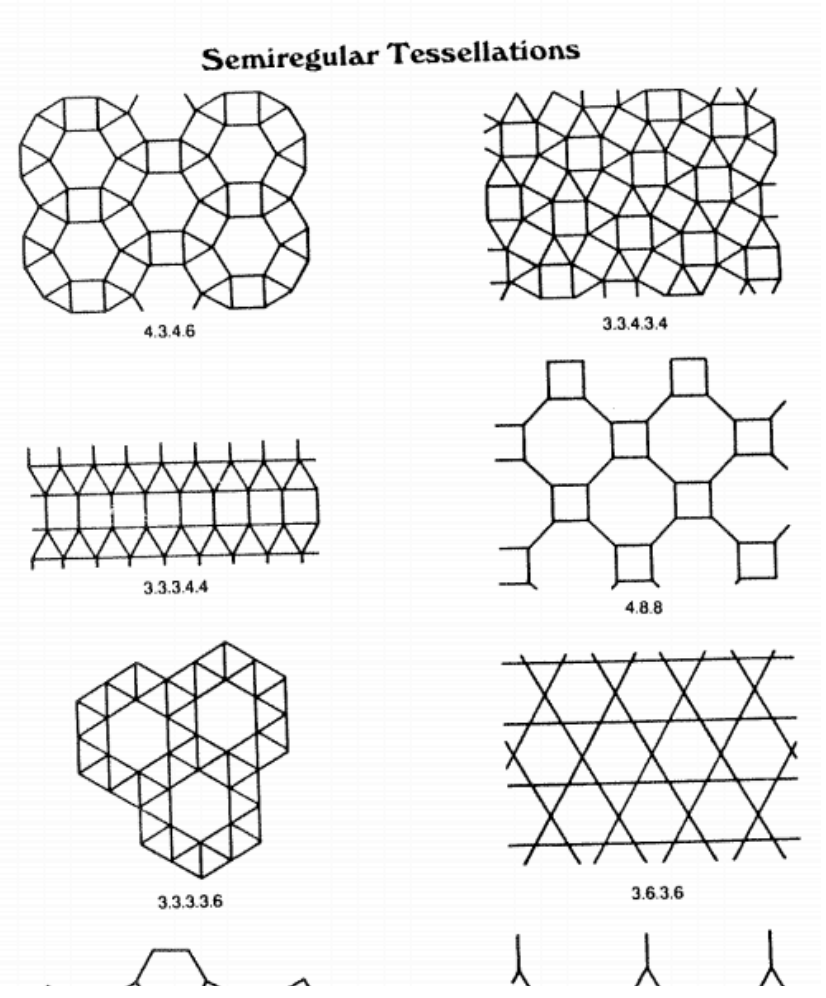






factors of 360:
1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360


60°
90°
108°
120°
$128\frac{4}{7}°$
135°
140°
144°

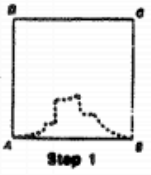

Semiregular Tessellations
4.3.4.6
3.3.4.3.4
3.3.3.4.4
4.8.8
3.3.3.3.6
3.6.3.6

Leap Frog, Robert Caness, Geometry student

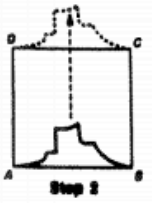
Step 1: Start with one square from a tessellation of squares (although any parallelogram will work with this method). Connect one side $\overline{AB}$ of the square with a curve, call it $\overset{\frown}{AB}$ (curve *AB*).

Step 2: Place tracing paper or clear plastic over $\overset{\frown}{AB}$ and copy it with a felt tip pen onto the tracing paper or clear plastic. Place the copy beneath the original and slide it so that the endpoints of $\overset{\frown}{AB}$ line up with the endpoint of $\overline{CD}$. Retrace the curve on the original so that it now connects with the endpoint of $\overline{CD}$.

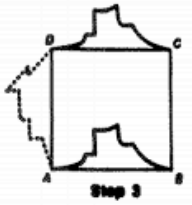
Step 3: Repeat this process with a curve connecting points *A* and *D*. That is, connect one side $\overline{AD}$ of the square with a curve, call it $\overset{\frown}{AD}$.



Step 1



Step 2



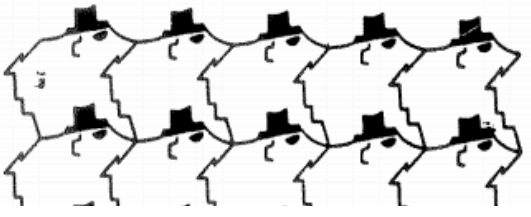
Step 3

Step 4: Copy $\overset{\frown}{AD}$ onto tracing paper or clear plastic and transfer it across to the opposite side $\overline{BC}$.

Step 5: When completed, trace the entire figure onto the tracing paper or clear plastic and move it to the next square. Trace the entire figure onto the next square. Fill the grid of squares with your figure. You have created a non-polygonal translation tessellation.

Step 4

Step 5



M.C. Escher artist (google his work)

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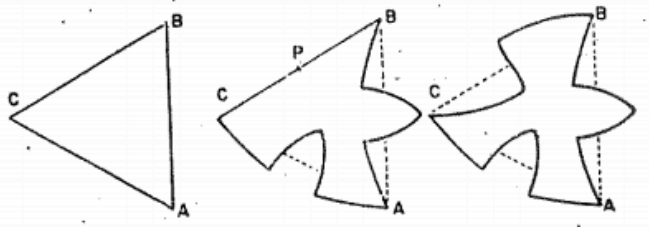
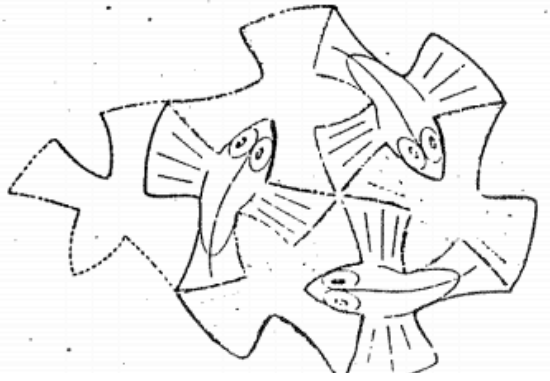
TESSELLATING WITH HEXAGONS

The artist M. C. Escher(1898–1972) is well known for his use of tessellations. By skillfully altering a basic polygon, such as a triangle or hexagon, he was able to produce intricate, artistic tessellations. The figure used here is based on one of Escher's drawings.

Step 1: Start with equilateral triangle ABC. Mark off the same curve on both sides AB and AC as shown. Mark off another curve on side BC that is symmetric about the midpoint P. If you choose the curves carefully, as did Escher, an interesting figure suitable for tessellating will be formed.



Step 2: Six of these figures accurately fit together about a point forming a hexagonal array. Trace and cut out one of the basic figures and show how it can be used to continue the tessellation over the entire sheet.



Mid 1

#3) (a) ext. angle (for regular n-gon)

$$= \frac{360^\circ}{n} = 24^\circ$$

$$\frac{360}{24} = \frac{24n}{24}$$

$$n = 15$$

(b) $\frac{180(n-2)}{n}$ = int. angle of reg. n-gon

$$\frac{180(13)}{15} = 156^\circ$$

(c) # diagonals $= \frac{n(n-3)}{2} = \frac{15(12)}{2} = \boxed{90}$

9) $\overline{AR}$ line segment from A to R

AR the length of line segment $\overline{AR}$

$\overline{AR} \cong \overline{CD}$ $AR = CD$ reasonable

$\overline{AR} = \overline{CD}$ $AR \cong CD$ incorrect