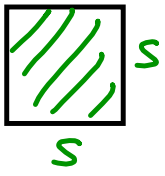
14.1 Area

Area--2-d measurement of space inside 2-d shapes (or on the outside of a 3-d shape)

(a) square

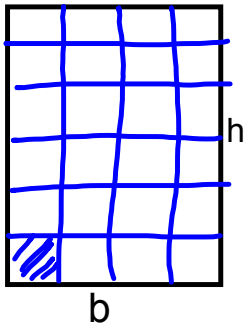


s^2 = area

Units of measure

ft^2, yd^2, m^2, cm^2, in^2, ...

(b) rectangle

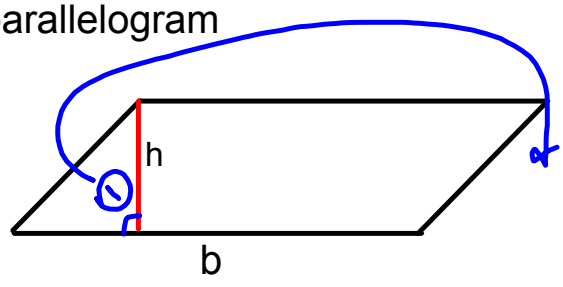


(break into squares)

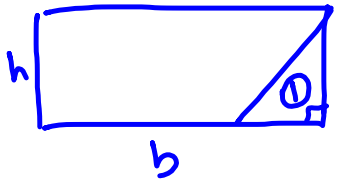
$A = bh$

1 unit square

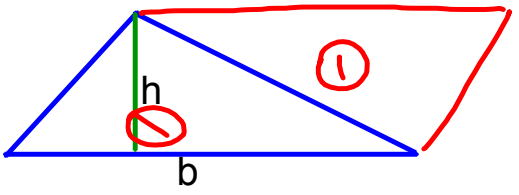
(c) parallelogram



$A = bh$ $\Rightarrow$

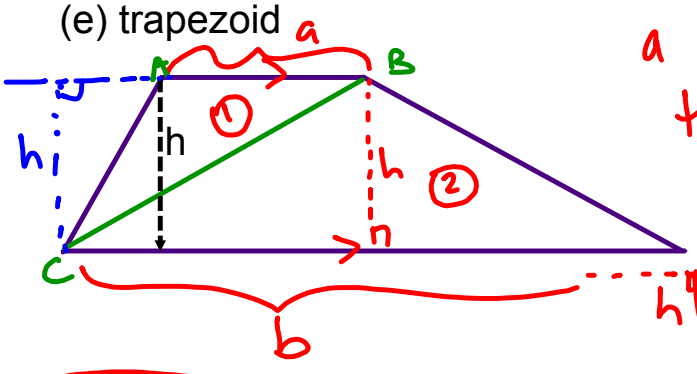


(d) triangle



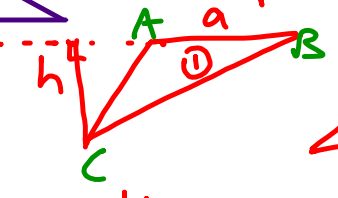
$A = \frac{1}{2}bh = \frac{bh}{2}$

(e) trapezoid



a and b are lengths of the two bases of the trapezoid

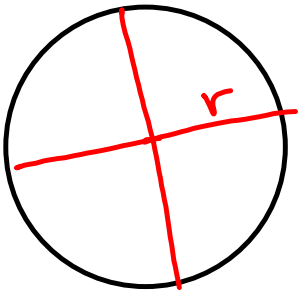
$A = \frac{1}{2}ah + \frac{1}{2}bh = \frac{1}{2}h(a+b)$



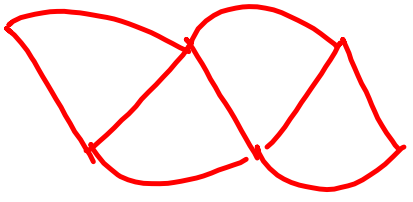
$A_1 = \frac{1}{2}ha$

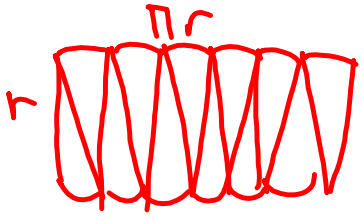
$A_2 = \frac{1}{2}bh$

(f) circle

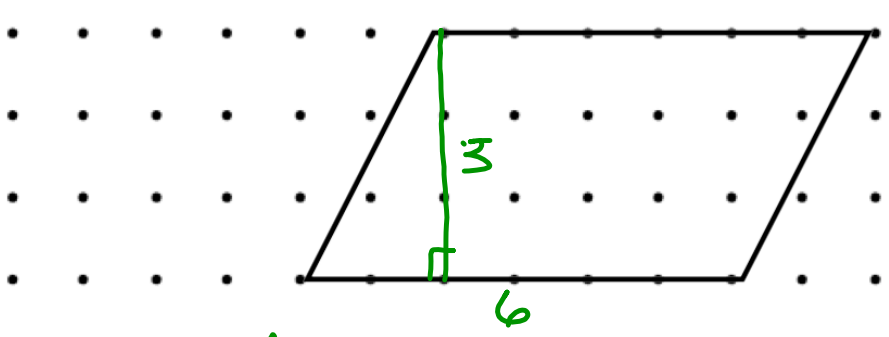


$A = \pi r^2$





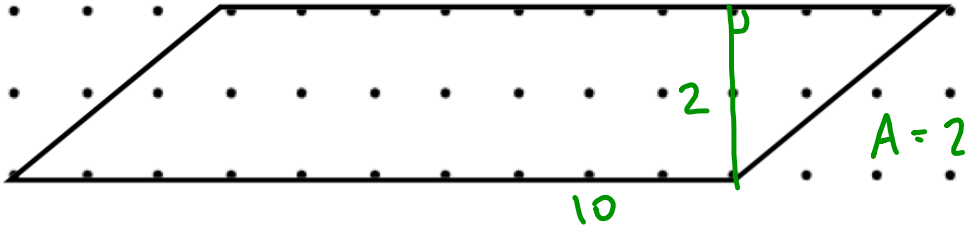
$C = 2\pi r$



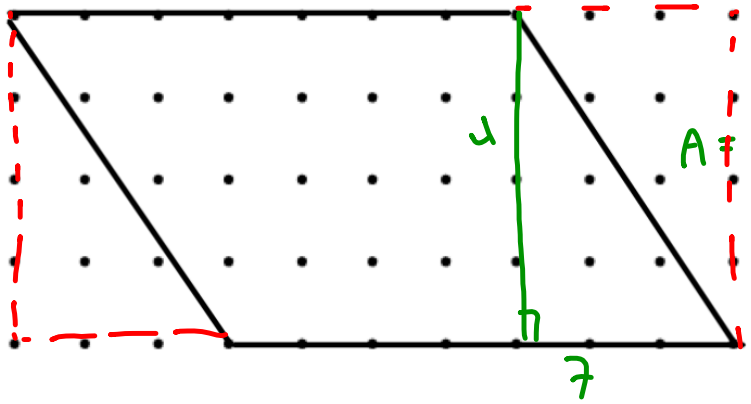
Find Area.

1 unit² of area

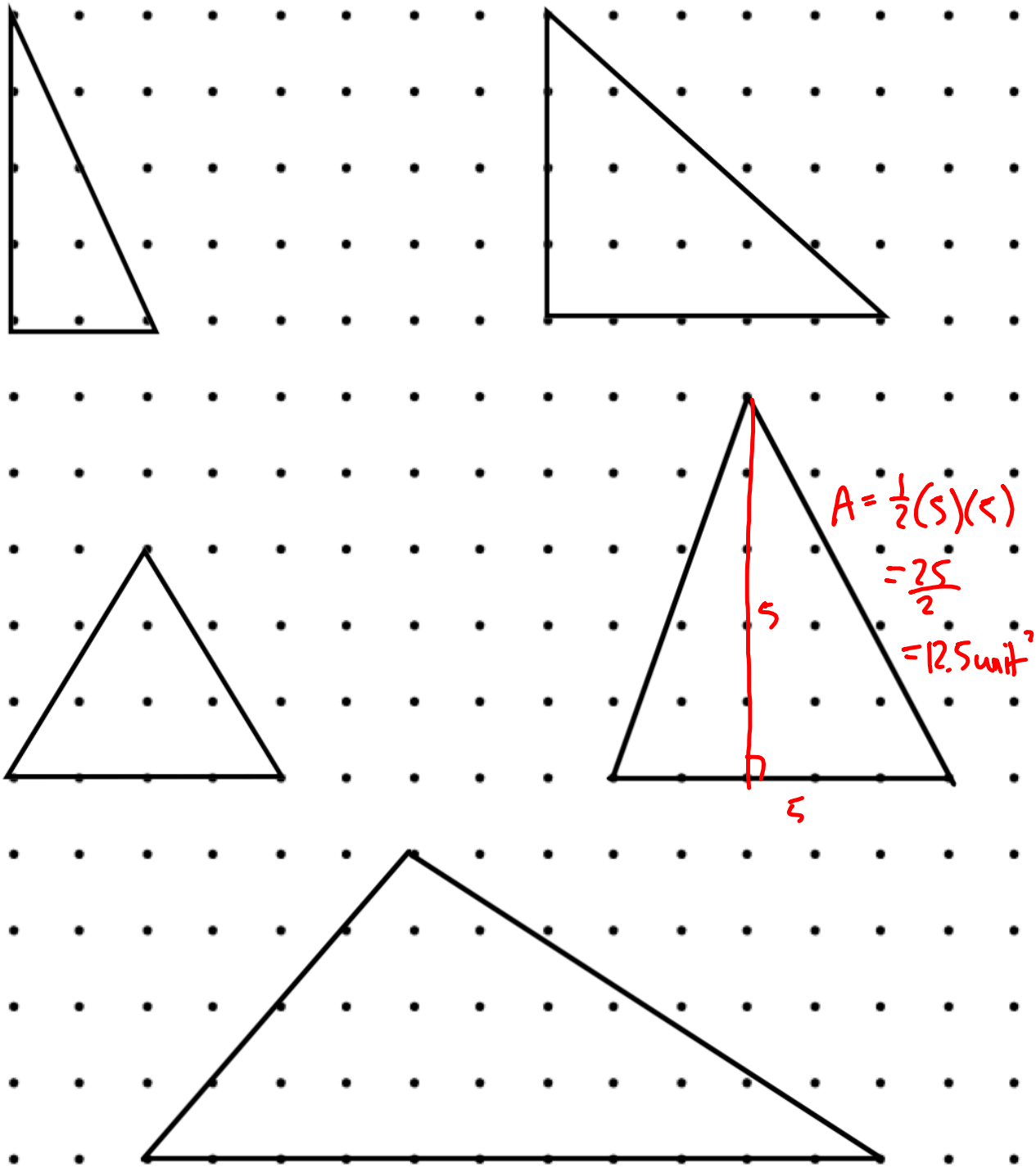
$A = 6(3) = 18$ units²

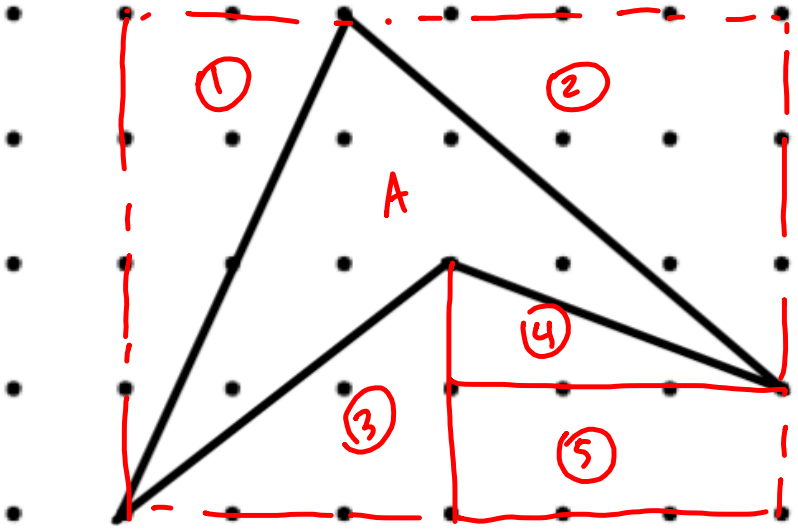


$A = 20$ units²



$A = 28$ in²

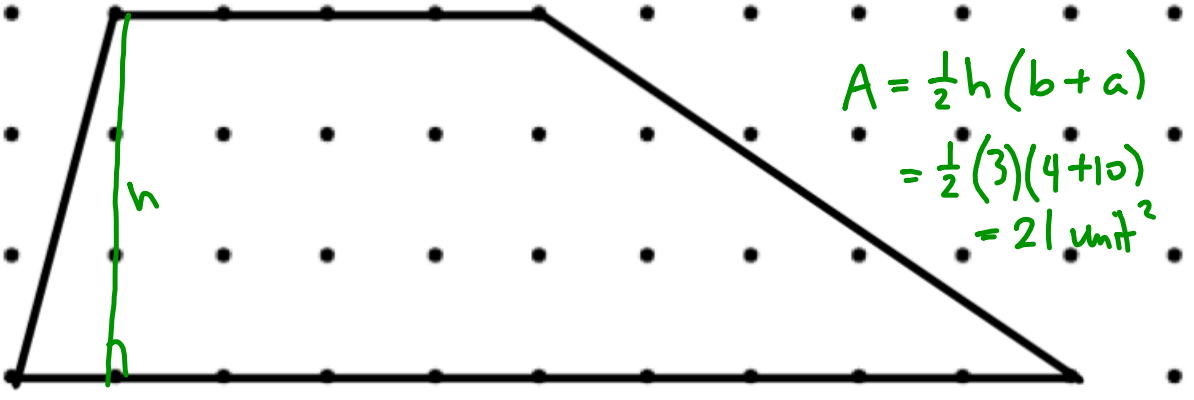

A = $\frac{1}{2}$(5)(5)
= $\frac{25}{2}$
= 12.5 unit^2
5
5



$A = \underset{①}{4(6)} - \underset{②}{\tfrac{1}{2}(4)(2)} - \underset{③}{\tfrac{1}{2}(4)(3)} - \underset{④}{\tfrac{1}{2}(1)(3)} - \underset{⑤}{1(3)}$

$= 24 - 4 - 6 - 3 - \tfrac{3}{2} - 3$

$= 6\tfrac{1}{2}\ \text{unit}^2$



$A = \tfrac{1}{2}h(b+a)$

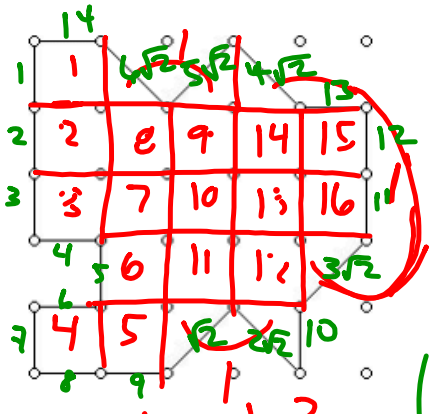
$= \tfrac{1}{2}(3)(4+10)$

$= 21\ \text{unit}^2$

Area and Perimeter

Find the area and perimeter of the lattice polygons:

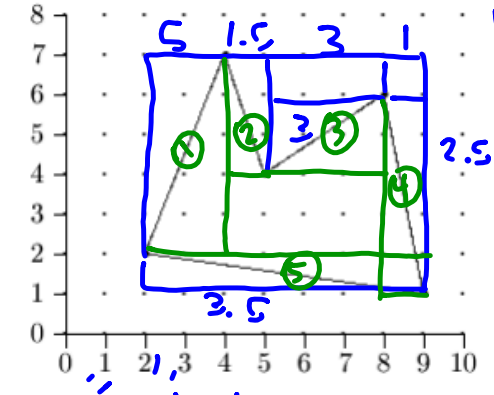
①



$a = 19 \text{ units}^2$

$P = 14 + 6\sqrt{2}$ units

②



$42 - 19.5 = 22.5$ units^2

42.0
$-\ 19.5$
22.5

diagonal ①

$2^2 + 5^2 = d^2$

$29 = d^2 \Rightarrow d = \sqrt{29}$

diagonal ②

$d^2 = 1^2 + 3^2$

$d^2 = 10$

$d = \sqrt{10}$

$P = (\sqrt{10} + \sqrt{29} + 5\sqrt{2} + \sqrt{13} + \sqrt{26})$ units

measurement units of area

1. How many square inches in a square foot?

$$1\ \text{ft}^2\left(\frac{12\ \text{in}}{1\ \text{ft}}\right)\left(\frac{12\ \text{in}}{1\ \text{ft}}\right) = 144\ \text{in}^2$$

2. Compute the following:

(a) 450 square yards = 583,200 sq. inches

$$450\ \text{yd}^2\left(\frac{3\ \text{ft}}{1\ \text{yd}}\right)\left(\frac{3\ \text{ft}}{1\ \text{yd}}\right)\left(\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right) = 450(9)(144)\ \text{in}^2 = 583{,}200\ \text{in}^2$$

(b) 12 sq. miles = 7680 acres

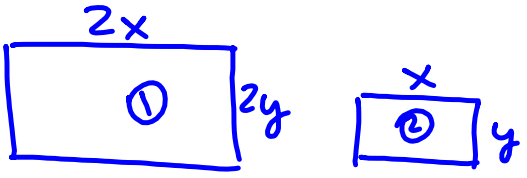
(note: 1 acre = 0.0015625 sq miles)

$$12\ \text{mi}^2\left(\frac{1\ \text{acre}}{0.0015625\ \text{mi}^2}\right) = 7680\ \text{acres}$$

(c) 800 sq. cm = 0.08 sq. m

$$800\ \text{cm}^2\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right) = \frac{800}{10000}\ \text{m}^2 = 0.08\ \text{m}^2$$

3. If one rectangle's sides are twice as big as another similar rectangle, what is the scaling factor of their areas? (In other words, if the length scaling factor is 2, what is the area scaling factor?)



$A_1 = 2x(2y) = 4xy$

$A_2 = xy$

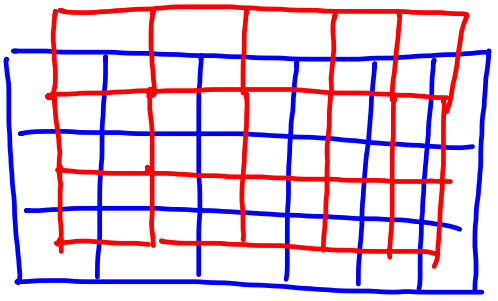
$A_1 = 4A_2$

$\Rightarrow$ area scaling factor is 4

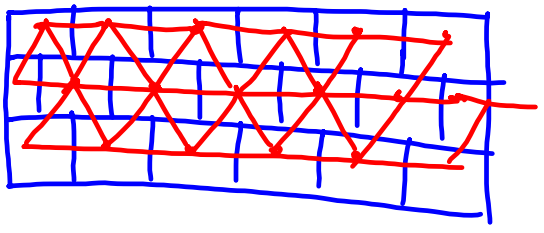
(length/perimeter scaling factor is 2)

HW ?'s

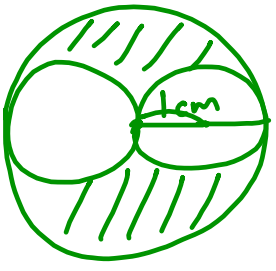
13.4A #7 (a)



(b)



14.1A #20) (a)



area of circle $= \pi r^2$

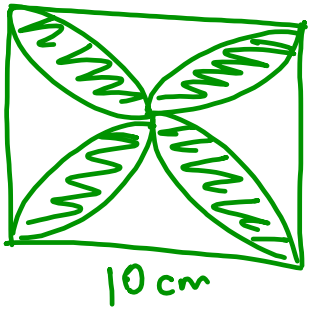
Area of shaded region

$= A = A_{\text{big circle}} - 2A_{\text{smaller circle}}$

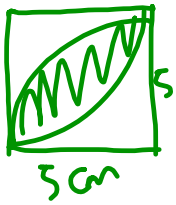
$A = \pi(2^2) - 2(\pi(1^2)) =$

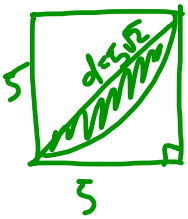
$4\pi - 2\pi = 2\pi \text{ cm}^2$

(d)



$A = 4A_{\text{one piece}} = 4(2A_{\text{half petal}})$

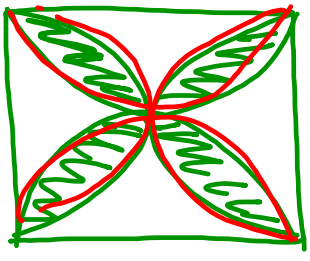


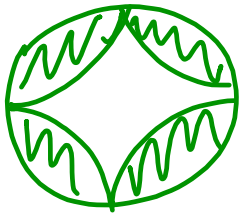


$d^2 = 5^2 + 5^2$

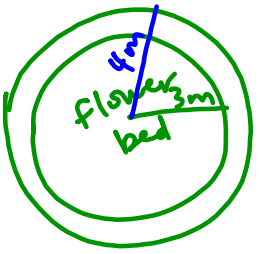
$d^2 = 50$

$d = \sqrt{50} = 5\sqrt{2}$





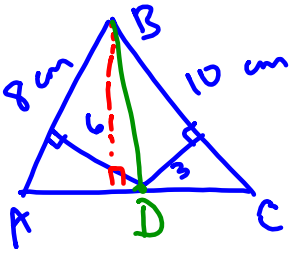
14.1A #21



Area of sidewalk

$= \pi(4^2) - \pi(3^2) = 7\pi \text{ m}^2$

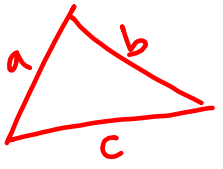
14.1B #8 (b

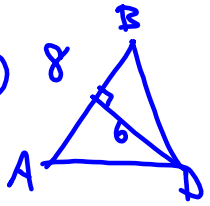


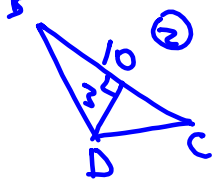
Heron's Formula:

$A_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{a+b+c}{2}$



① 

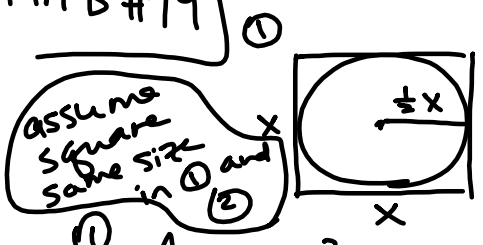
② 

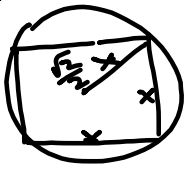
$A_1 = \frac{1}{2}8(6)$

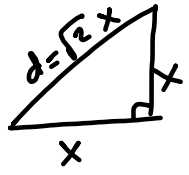
$A_2 = \frac{1}{2}(10)(3)$

$A = 24 + 15 = 39 \text{ cm}^2$

14.1 B #19

① 

② 



$d^2 = x^2 + x^2$

$d^2 = 2x^2$

$d = \sqrt{2}\,x$

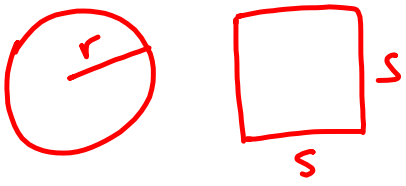
① $\frac{A_\square}{A_\circ} = \frac{x^2}{\pi(\frac{x}{2})^2} = \frac{x^2}{\frac{\pi}{4}x^2} = \frac{1}{\pi/4} = \frac{4}{\pi} \simeq 1.273$

② $\frac{A_\square}{A_\circ} = \frac{x^2}{\pi(\frac{\sqrt{2}}{2}x)^2} = \frac{x^2}{\pi(\frac{1}{2})x^2} = \frac{1}{\pi/2} = \frac{2}{\pi} \simeq 0.6366$

① $A_\square - A_\circ = x^2 - \frac{\pi}{4}x^2 = (1-\frac{\pi}{4})x^2$ "wasted space" $\simeq 0.2146\,x^2$

② $A_\circ - A_\square = \frac{\pi}{2}x^2 - x^2 = x^2(\frac{\pi}{2} - 1) \simeq 0.5707\,x^2$

14.1A #19b)

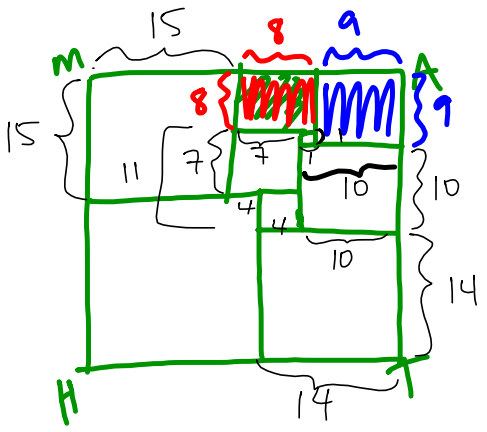


$\pi r^2 = s^2$

$r^2 = \frac{s^2}{\pi}$

$r = \frac{\sqrt{s^2}}{\sqrt{\pi}} = \frac{s}{\sqrt{\pi}}$

14.1B #24)



$A = (15+8+9)(9+10+14)$

$= 32(33)$

$= 1056 \text{ units}^2$