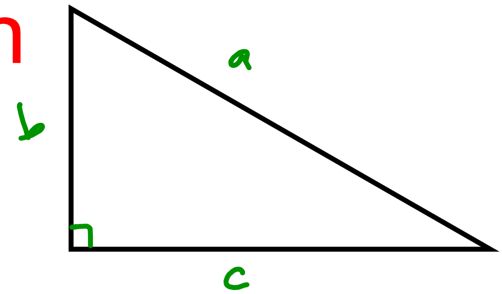
14.2 Pythagorean Theorem

The Pythagorean Theorem

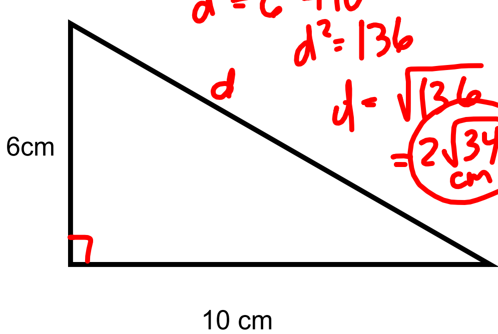


What does it say? What does it really mean?

For a right triangle, the sum of squares of the leg lengths is the square of the hypotenuse length. $b^2+c^2=a^2$

How do we use it?

Find the missing side:

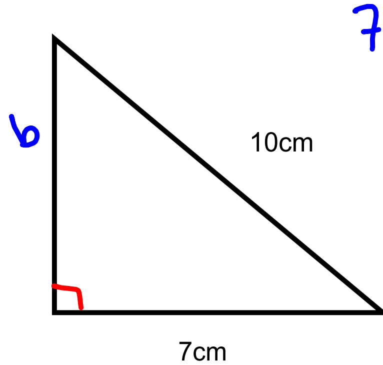


$d^2=6^2+10^2$

$d^2=136$

$d=\sqrt{136}$

$=2\sqrt{34}$ cm

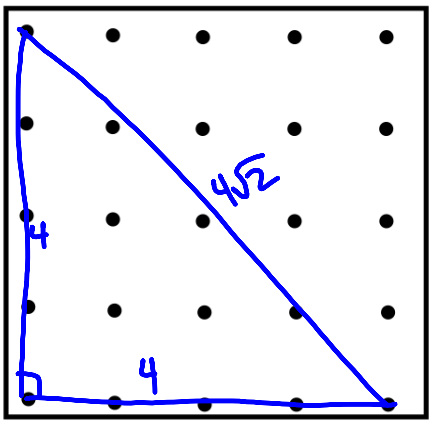


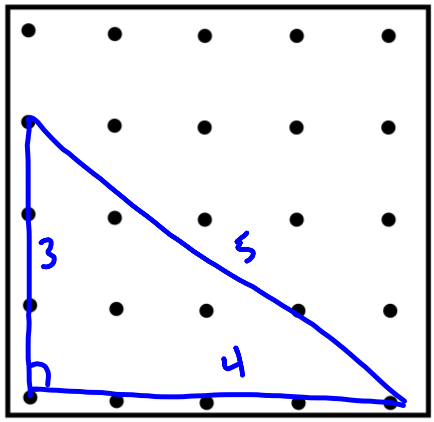
$7^2+b^2=10^2$

$b^2=100-49$

$b^2=51$

$b=\sqrt{51}$ cm





What is the longest segment you can get on a geoboard? $4\sqrt{2}$

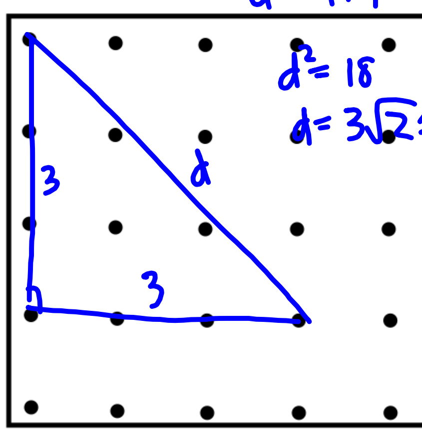
$4^2+4^2=d^2$

$d^2=32 \Rightarrow d=\sqrt{32}=4\sqrt{2}\simeq 5.65$

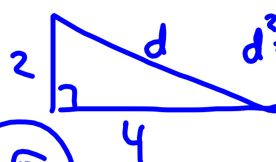
$d^2=9+9$

$d^2=18$

$d=3\sqrt{2}\simeq 4.24$



What is the second longest? 5



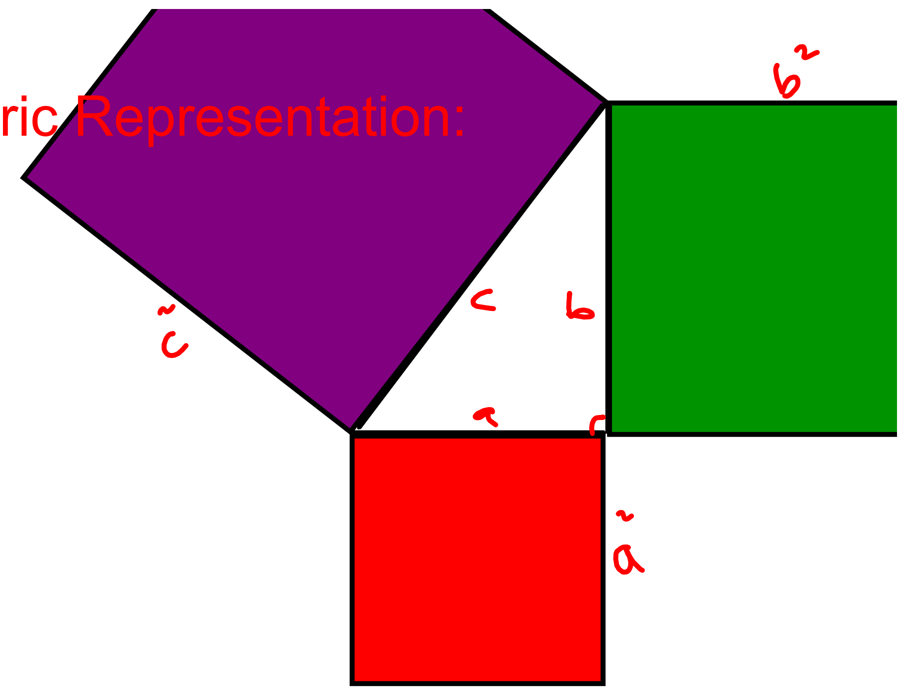
$d^2=4+16$

$d^2=20$

$d=2\sqrt{5}\simeq 4.47$

Third? $2\sqrt{5}$

A Geometric Representation:

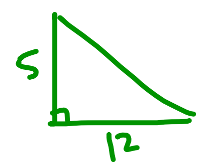


Pythagorean Triples

You will need a sheet of graph paper.

On your graph paper, draw a vertical segment the number of units in length that is designated in the table.

Draw a horizontal segment using the table below and then measure or compute the hypotenuse.

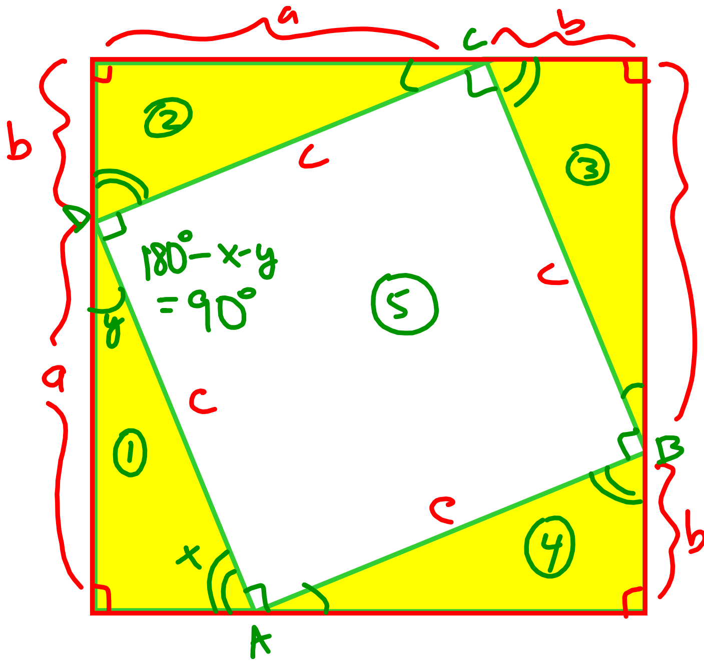


Vertical	Horizontal	Hypotenuse
3	4	5
5	12	13
6	8	10
8	15	17
12	16	20
15	20	25

x4

x5

Carpet proof



We start w/ a square w/ side lengths $a+b$

① $A = (a+b)^2$

② $A =$ add area of pieces

We proved that quadrilateral ABCD is a square.

claim:

http://www.youtube.com/watch?v=pVo6szYE13Y&feature=related

all 4 yellow triangles are congruent. by SAS (all have sides of lengths a and b w/ included rt angle)

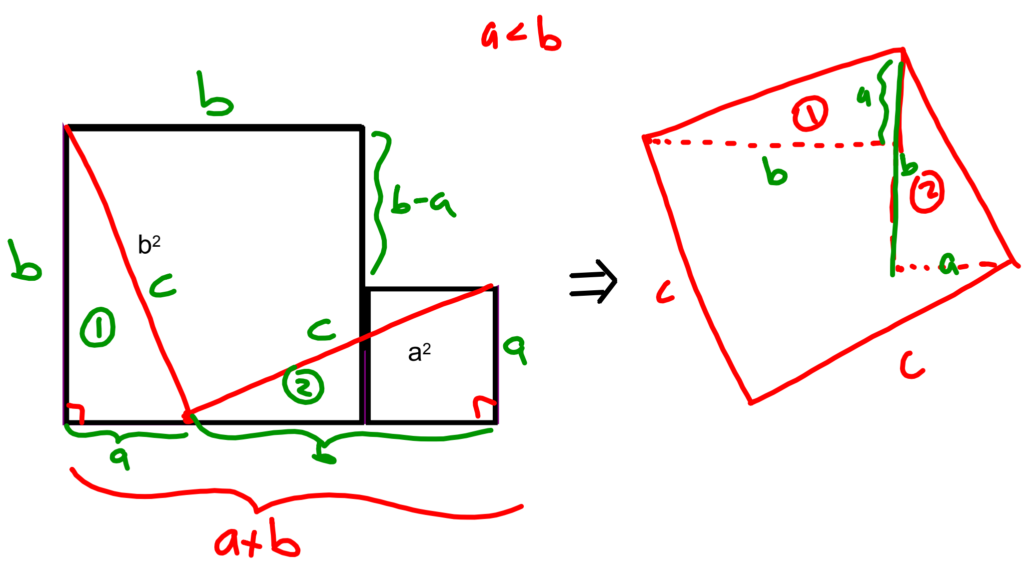
② $A = 4\left(\frac{1}{2}ab\right) + c^2$

Now equate ① and ②. $(a+b)^2 = 4\left(\frac{1}{2}ab\right) + c^2$

$$a^2 + 2ab + b^2 = 2ab + c^2$$
$$-2ab \qquad\qquad -2ab$$

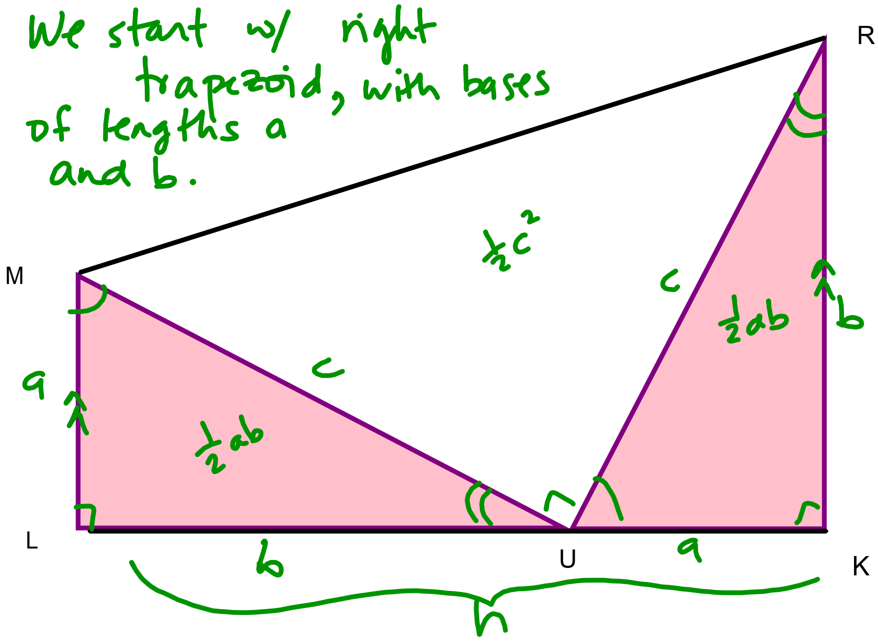
$$a^2 + b^2 = c^2 \ /\!/$$

An Indian Proof



Garfield's Proof

We start w/ right trapezoid, with bases of lengths a and b.



base lengths: a, b
height: a+b

a = short leg   b = long leg   c = hypotenuse

MLKR is a right trapezoid

Δ MUR is a right triangle; why?

area of MLKR = $\frac{1}{2}(a+b)(a+b)$

Area of the three triangles = $\frac{1}{2}ab$ + $\frac{1}{2}ab$ + $\frac{1}{2}c^2$

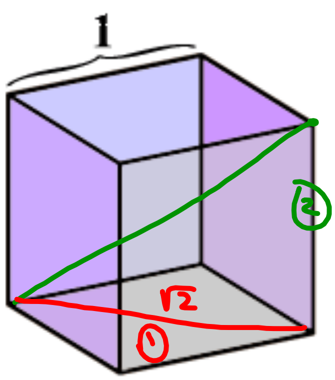
$$\frac{1}{2}(a+b)(a+b) = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$$

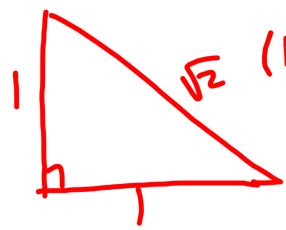
$$\frac{1}{2}(a^2+2ab+b^2) = \frac{1}{2}(2ab) + \frac{1}{2}c^2$$

$$a^2+2ab+b^2 = 2ab + c^2$$

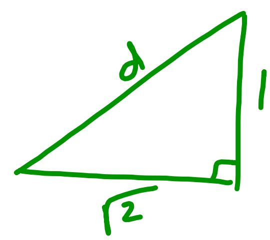
$$a^2+b^2=c^2 \; \#$$

Can we use Pythagorean Theorem to find the diagonal length of a unit cube?



① 

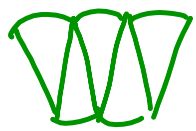
(base of cube)

② 

$(\sqrt{2})^2 + 1^2 = d^2$

$3 = d^2 \Rightarrow d = \sqrt{3}$ unit

Circle Worksheet



tell how this convinces you of area formula for circle.

14.1
A 16 B)

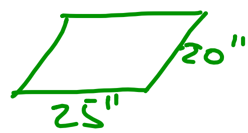
Sector of circle w/ $r = 4$ cm

circle $A = \pi r^2$

sector $A = \pi r^2\left(\frac{60^\circ}{360^\circ}\right)$

$A = \frac{1}{6}\pi(4^2) = \frac{8}{3}\pi$ cm^2

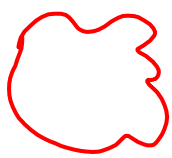
14.1 MC #16)



Jimmy says $A = 20(25) = 500$ in^2

not correct. need height (makes rt angle with base)

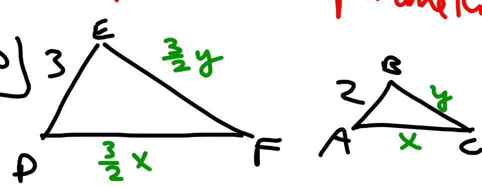
MC #12)



idea
measure perimeter, then form square w/ same perimeter & compute area

no relationship between perimeter and area

14.1 B #10)

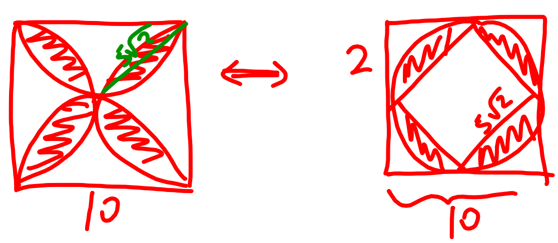


$\triangle ABC \sim \triangle DEF$

$\frac{3}{2}h_{ABC} = h_{DEF}$

$\frac{3}{2}P_{ABC} = P_{DEF}$ $\quad\frac{9}{4}A_{ABC} = A_{DEF}$

14.1 A #20d)



$\sqrt{50}$
$= \sqrt{25(2)} = \sqrt{25}\sqrt{2}$
$= 5\sqrt{2}$

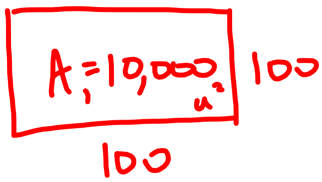
$A = 2\left(A_{\text{circle of } r=5} - A_{\text{square side length } 5\sqrt{2}}\right)$

$A = 2\left(\pi(25) - (5\sqrt{2})^2\right) = 2(25\pi - 50)$

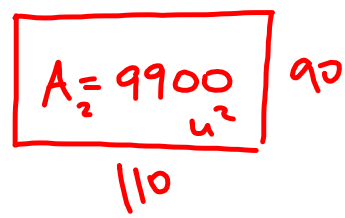
$= 50\pi - 100$ cm^2

14.1 MC #6

original



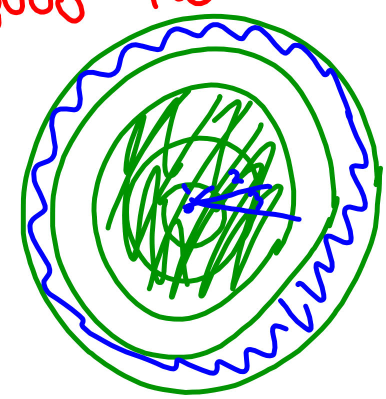
change



A_2 = what percent of A_1?

$$\frac{A_2}{A_1} = \frac{9900}{10000} = \frac{99}{100} = 99\%$$

14.1B #21



$A = \pi(5^2) - \pi(4^2)$

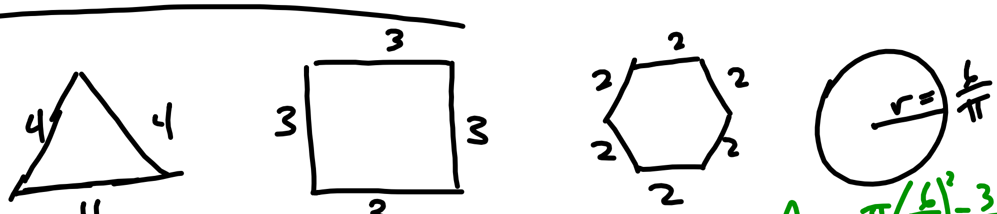
$= 9\pi$ in^2

$A = 9\pi$ in^2

Area/Perimeter Worksheet

#2)

$P = 12$ in



$A_\triangle = \frac{\sqrt{3}}{4}(4^2) = 4\sqrt{3} \approx 6.8$

$A_\square = 9$

$A_{\text{hex}} = \frac{3\sqrt{3}}{2}(2^2) = 6\sqrt{3} \approx 10.2$

$A_\circ = \pi\left(\frac{6}{\pi}\right)^2 = \frac{36}{\pi} \approx 11.4$

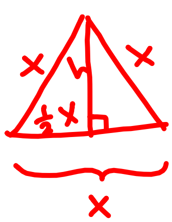
same perimeter

$P_\circ = 2\pi r = 12$

$r = \frac{12}{2\pi}$

$r = \frac{6}{\pi}$

A of equilateral $\triangle$:



$h^2 + \left(\frac{1}{2}x\right)^2 = x^2$

$h^2 = x^2 - \frac{1}{4}x^2$

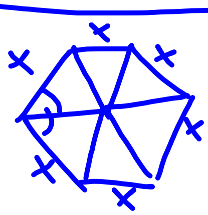
$h^2 = \frac{3}{4}x^2$

$\boxed{h = \frac{\sqrt{3}}{2}x}$

$\Rightarrow A_\triangle = \frac{1}{2}\left(\frac{\sqrt{3}}{2}x\right)x$

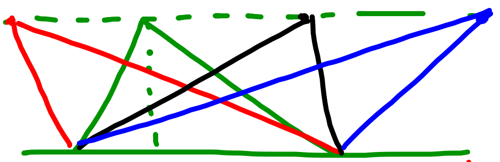
$\boxed{A_{\triangle\,\text{equil}} = \frac{\sqrt{3}}{4}x^2}$

area of regular hexagon:



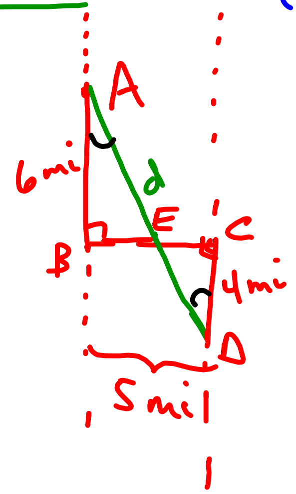
$A = 6A_{\text{equil}\,\triangle} = 6\left(\frac{\sqrt{3}}{4}x^2\right) = \boxed{\frac{3\sqrt{3}}{2}x^2}$ area regular hexagon

Graph Paper Book



all triangles have same area (because b and h are same)

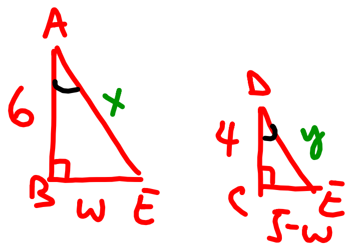
14.2 A #9)



$d = ?$

2 black angles are $\cong$ because they are alt. int. for 2 parallel lines

by AA, $\triangle ABE \sim \triangle DCE$



$d = x + y$

$x^2 = 6^2 + 3^2$

$x^2 = 36 + 9$

$x^2 = 45$

$x = \sqrt{45} = \sqrt{9(5)}$

$= 3\sqrt{5}$

$y^2 = 4^2 + 2^2$

$y^2 = 20 \Rightarrow y = \sqrt{20}$

$y = 2\sqrt{5}$

$\frac{w}{5-w} = \frac{6}{4} \Leftrightarrow 4w = 6(5-w)$

$4w = 30 - 6w$

$+6w \quad +6w$

$10w = 30$

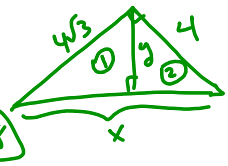
$w = 3$

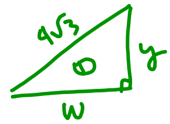
$\Rightarrow d = x + y = 3\sqrt{5} + 2\sqrt{5}$

$= 5\sqrt{5}$ mi

14.2 A #14

(a)





$(4\sqrt{3})^2 = 4^2\sqrt{3}^2 = 16\cdot 3 = 48$

① $w^2 + y^2 = (4\sqrt{3})^2$

$w^2 + y^2 = 48$

③ (big triangle)

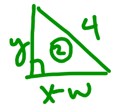
$x^2 = 4^2 + (4\sqrt{3})^2$

$x^2 = 16 + 48$

$x^2 = 64$

$x = 8$

② $(x-w)^2 + y^2 = 4^2$



$x^2 - 2xw + \underbrace{w^2 + y^2}_{} = 16$

$x^2 - 2xw + 48 = 16$

$64 - 2(8)w + 48 = 16$

$-112 \qquad -112$

$-16w = -96$

$w = 6$

$\Rightarrow$ ① $w^2 + y^2 = 48$

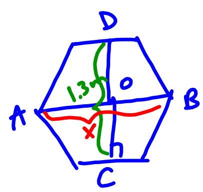
$6^2 + y^2 = 48$

$y^2 = 48 - 36$

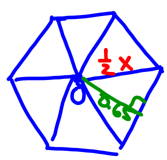
$y^2 = 12$

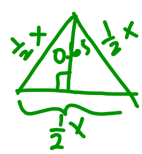
$y = \sqrt{12} = 2\sqrt{3}$

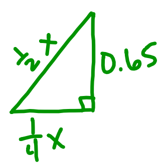
14.2 B #14



$x = ?$







$\left(\frac{1}{4}x\right)^2 + (0.65)^2 = \left(\frac{1}{2}x\right)^2$

$\frac{1}{16}x^2 + 0.65^2 = \frac{1}{4}x^2$

$-\frac{1}{16}x^2 \qquad -\frac{1}{16}x^2$

$\left(\frac{16}{3}\right) 0.65^2 = \frac{3}{16}x^2 \quad \left(\frac{16}{3}\right)$

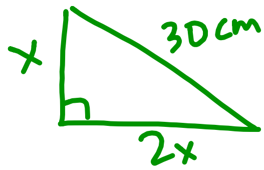
$\sqrt{\frac{16(0.65)^2}{3}} = \sqrt{x^2}$

$\frac{4(0.65)}{\sqrt{3}} = x$

$\frac{2.60}{\sqrt{3}} = x$

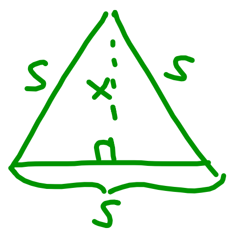
$x = \frac{2.6}{\sqrt{3}}$ m

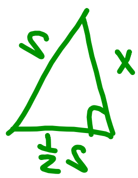
14.2A #6)



$$x^2 + (2x)^2 = 30^2$$
$$x^2 + 4x^2 = 30^2$$
$$5x^2 = 30(30)$$
$$x^2 = 6(30)$$
$$x^2 = 6^2 \cdot 5$$
$$x = \sqrt{6^2 \cdot 5}$$
$$x = 6\sqrt{5} \text{ cm}$$

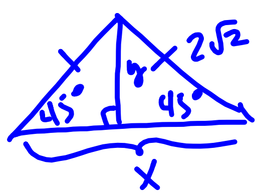
14.2A 2d)

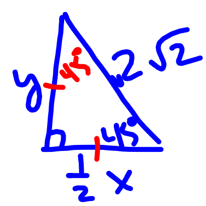




$$x^2 + \left(\tfrac{1}{2}s\right)^2 = s^2$$
$$x^2 + \frac{s^2}{4} = s^2$$
$$x^2 = \frac{3}{4}s^2 \Longrightarrow x = \frac{\sqrt{3}}{2}s$$

14.2A 14b)



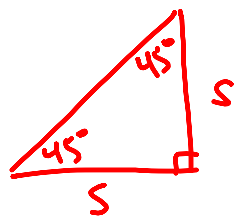


$$y = \tfrac{1}{2}x$$

$$x = 2y = 4$$

$$y^2 + y^2 = (2\sqrt{2})^2$$
$$2y^2 = 4 \cdot 2$$
$$y^2 = 4 \Rightarrow y = 2$$

14.2B #12) (b)



$A = \frac{1}{2}s^2$