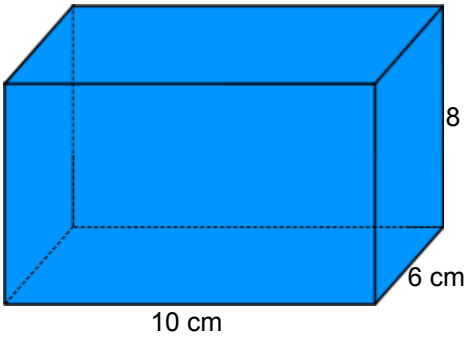
14.5 Volume/Temperature

How do we find the volume of a solid figure?



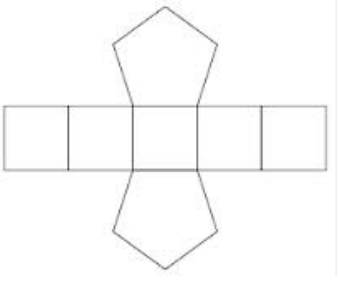
$V = 10(6)(8) = 480 \text{ cm}^3$

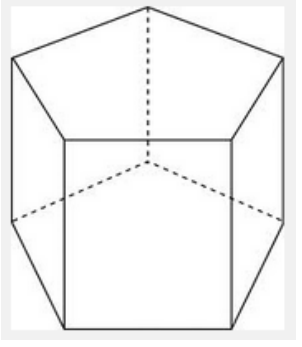
Let A = area of base

P = perimeter of base

h = height of solid

Right Prism



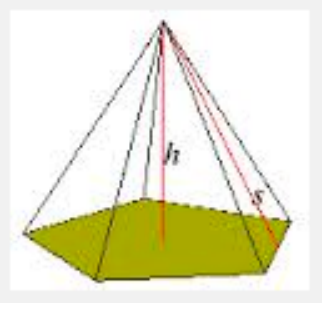


$V = Ah$

$V = Ah$

Let s = slant height

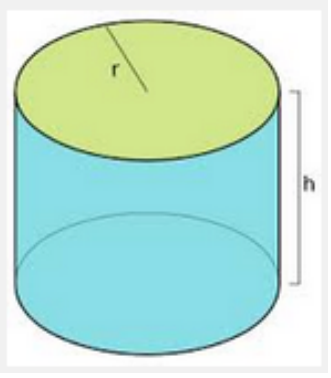
Right Pyramid



$V = \frac{Ah}{3}$ or $\frac{1}{3}Ah$

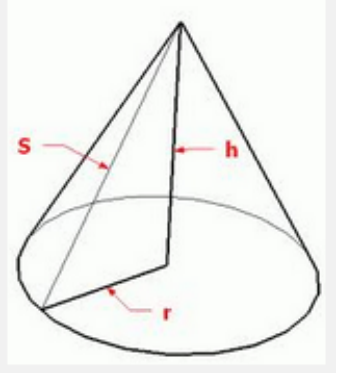
V = (Ah)/3

Right Circular Cylinder



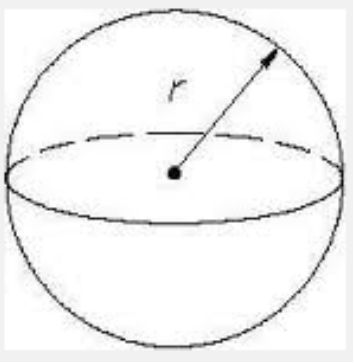
$V = Ah = \pi r^2 h$

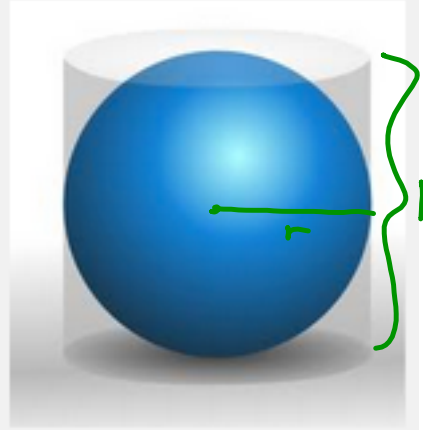
Right Circular Cone



$V = \frac{1}{3}Ah = \frac{1}{3}\pi r^2 h$

Sphere





The ratio of volume of the sphere to the volume of the smallest cylinder containing the sphere is still 2/3 !!!

$$\frac{2}{3} = \frac{V_{sphere}}{V_{cylinder}}$$

$$V_{cylinder} = \pi r^2 h = \pi r^2 (2r) = 2\pi r^3$$

$$\frac{2}{3} = \frac{V_{sphere}}{2\pi r^3}$$

$$V_{sphere} = \frac{2}{3}(2\pi r^3) = \boxed{\frac{4}{3}\pi r^3}$$

Math4020

Scaling Worksheet

Cube

Side Length	Surface Area	Volume
1 m	6 m^2	1 m^3
2 m	24 m^2	8 m^3
3 m	54 m^2	27 m^3
5 m		
7 m		
10 m		

Sphere

Radius	Surface Area	Volume
1 ft		
2 ft		
3 ft		
5 ft		
6 ft		
10 ft		

Rt. Circular Cylinder

Radius	Height	Surface Area	Volume
1 in	3 in		
2 in	6 in		
3 in			
5 in			
8 in			
10 in			

Scaling Relationship:

If we double the lengths in a solid,
We multiply the surface area by $2^2 = 4$
We multiply the volume by $2^3 = 8$

If we triple the lengths in a solid,
We multiply the surface area by $3^2 = 9$
We multiply the volume by $3^3 = 27$

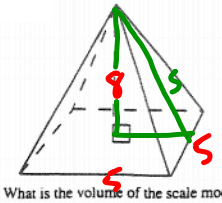
Rt. Circular Cone

Radius	Height	Surface Area	Volume
1 unit	2 units		
2 units	4 units		
3 units			
5 units			
9 units			
10 units			

If we multiply the lengths in a solid by 5,
We multiply the surface area by $5^2 = 25$
We multiply the volume by $5^3 = 125$

If we multiply the lengths in a solid by n,
We multiply the surface area by n^2
We multiply the volume by n^3

The square pyramid drawn below is a scale model of the package for a new candy bar.



scale model
height = 8 in
apex located over center of square base
side of square base = 5 in

a) What is the volume of the scale model? Volume:_____

$h = 8$ in

right square pyramid

$V = \frac{1}{3}Ah$

$A = s(s) = 25$ in^2

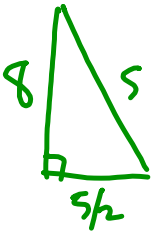
$V = \frac{1}{3}(25)(8) = \boxed{\frac{200}{3} \text{ in}^3}$

b) What is the surface area of the scale model? Surface Area: _____

$SA = A + \frac{1}{2}Ps = 25 + \frac{1}{2}(20)s$

$= 25 + 10\left(\frac{\sqrt{281}}{2}\right)$

$= \boxed{25 + 5\sqrt{281} \text{ in}^2}$



$8^2 + \left(\frac{5}{2}\right)^2 = s^2$

$s^2 = 64 + \frac{25}{4}$

$s^2 = \frac{256+25}{4}$

$s^2 = \frac{281}{4}$

$s = \frac{\sqrt{281}}{2}$

c) Suppose the actual package will have a height 32 inches. What will the surface area and volume be for the actual package? Surface Area:_____

Volume:_____

length scaling factor $= \frac{32}{8} = 4$

$SA = \left(25 + 5\sqrt{281}\right)4^2 \text{ in}^2$

$V = \left(\frac{200}{3}\right)4^3 = \frac{200(64)}{3} \text{ in}^3$

The two commonly used systems of temperature, Celcius and Fahrenheit, are not as simply related.

The problem is this: A basic principle of Celcius is that the freezing point of water is zero. In Fahrenheit, that freezing point is 32°. To make things even harder, the Fahrenheit and Celcius degrees are not the same size -- they represent a different amount of temperature change.

The boiling point of water is set at 100° C. In Fahrenheit, the boiling point of water is 212° F. Then a Celcius degree is defined as 1/100 of the change from freezing to boiling, and Fahrenheit degrees are defined similarly.

1. Which degree represents a larger change in temperature, a Celcius degree or a Fahrenheit degree?

celsius

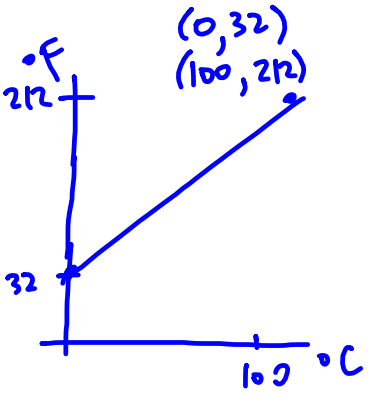
2. How would you convert from "Celcius degrees above freezing" to "Fahrenheit degrees above freezing".

$F = \frac{9}{5}C + 32$

3. Turn your answer to #2 into a formula that takes the temperature C in Celcius, and returns the temperature F in Fahrenheit.

$F - 32 = \frac{9}{5}C \Rightarrow \frac{5}{9}(F-32) = C$

4. Using any reasoning you like, produce a formula that takes the temperature F in Fahrenheit and returns the temperature C in Celcius.



$m = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$

pt (0,32)

$F = \frac{9}{5}C + 32$

5. There is one temperature that is the same in both systems. Use one of your formulas to find it.

$F = \frac{9}{5}C + 32$

$C = \frac{9}{5}C + 32$

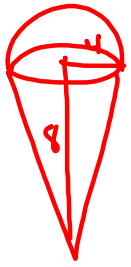
$\left(-\frac{5}{4}\right)-\frac{4}{5}C = 32\left(-\frac{5}{4}\right)$

$C = 8(-5) = -40°$

$C - \frac{9}{5}C = \frac{5}{5}C - \frac{9}{5}C$

$= -\frac{4}{5}C$

14.5 A #4a) (a)



$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) + \frac{1}{3}\pi r^2 h$

$= \frac{2}{3}\pi(4^3) + \frac{1}{3}\pi(4^2)(8)$

$= \frac{128\pi}{3} + \frac{128\pi}{3} = \boxed{\frac{256\pi}{3}\ cm^3}$

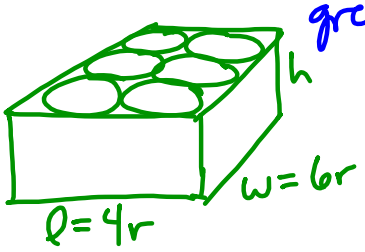
14.5 A #16) length is increased by 30%

$\Rightarrow$ scale factor (length) $= 1.3$

$\Rightarrow$ new volume is $(1.3)^3$ times bigger than original volume

$1.3^3 = 2.197$ $\Rightarrow$ volume grew by 119.7%

A #18)



$w = 6r$

$V_{box} = 4r(6r)h = 24r^2h$

$V_{cans} = 6(\pi r^2 h)$

$= 6\pi r^2 h$

$V_{\text{empty space}} = 24r^2h - 6\pi r^2 h = r^2h(24-6\pi)$

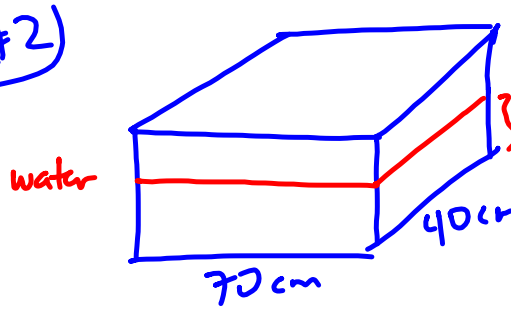
$\frac{r^2h(24-6\pi)}{\text{part}}$ is $\underline{\quad x \quad}$% of $\frac{24r^2h}{\text{whole}}$

$r^2h(24-6\pi) = x(24r^2h) \Rightarrow x = \frac{r^2h(24-6\pi)}{24r^2h}$

$= \frac{24-6\pi}{24} = \left(1 - \frac{\pi}{4}\right)$

$\simeq 0.21$ or 21%

14.5 B #2)



$1\ cm^3$ of liquid $= 1\ ml$

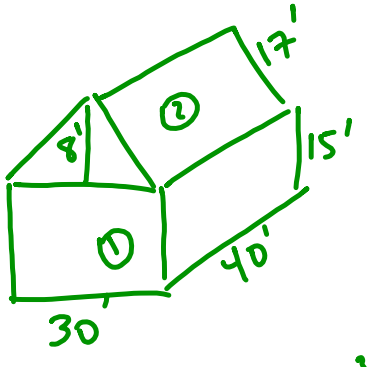
$V_{rock} = 70(40)(2)$

$= 5600\ cm^3$

$= 5600\ ml$

$= 5.6\ l$

B #4 d)



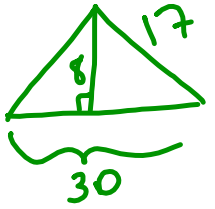
$V = V_1 + V_2$

① rt. rectangular prism

② rt. triangular prism

$V_1 = 30(40)(15) = 18000\ ft^3$

$V_2 = Ah = A(40)$



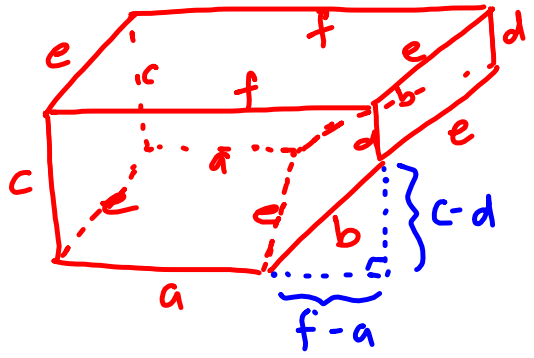
$A = \frac{1}{2}(8)(30)$

$= 120\ ft^2$

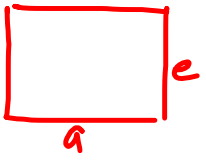
$V_2 = 120(40) = 4800 ft^3$

$\Rightarrow V = 18000 + 4800 = 22800\ ft^3$

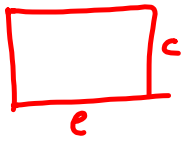
a = 3m
b =
c = 6m
d = 1m
e = 20m
f = 25m



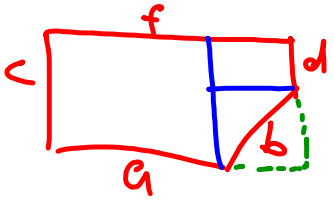
bottom



left side



front/back sides:



right side:

slopy: