9.1 Introduction to Probability

Experiment-->The act of making an observation.

Outcome-->One of the possible things that can occur.

Sample Space-->The set of all possible outcomes (usually denoted as S). *(The sample space is called uniform if all outcomes in S are equally likely.)*

Event-->A subset of the sample space (usually denoted as E or A).

Probability-->The relative frequency we expect an event to occur
(can be written as a fraction, decimal, ratio or percent)

$P(E)=\frac{n(E)}{n(S)}$ That is, the probability of an event E is the number of ways the event E can happen divided by the number of outcomes of S.

Since $\varnothing \subseteq E \subseteq S$, and we know that the number of elements in the empty set is 0, i.e. $n(\varnothing)=0$, then

$0 \le n(E) \le n(S)$. And dividing by $n(S)$, we get

$\frac{0}{n(S)} \le \frac{n(E)}{n(S)} \le \frac{n(S)}{n(S)}$. Finally, we have

$0 \le P(E) \le 1$. In other words, the probability of any event is always a number between 0 and 1 (inclusive).

and $P(\varnothing)=0$ and $P(S)=1$

<u>Experimental Probability</u>-->The probability calculated from an actual experiment. The probability for the same event may vary from observation to observation.

<u>Theoretical Probability</u>-->This is based on ideal occurrences (usually what we mean when we say "probability").

<u>Law of Large Numbers (Bernoulli's Theorem)</u>:
If an experiment is repeated a large number of times, the experimental or empirical, probability of a particular outcome approaches a fixed number as the number of repetitions increases.
(As it turns out, the experimental mean approaches the theoretical mean.)

The probability of A ***OR*** B is equal to the probability of A plus the probability of B minus the probability of A ***AND*** B (that got counted twice).

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are *mutually exclusive* (meaning that they have nothing in common), then

$$P(A \cup B) = P(A) + P(B) \quad \text{since} \quad A \cap B = \emptyset$$

The probability of the the event not happening equals 1 minus the probability of the event happening.

$$P(\bar{E}) = 1 - P(E)$$

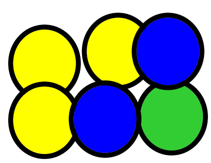
Ex 1 Roll a die.

(a) What is the sample space, S?

(b) Let E = event that you roll an even number. What is P(E)?

(c) P(rolling a six) = ?

Ex 2 Balls P(2 of same color)



Ex 3 For a deck of cards

P(red or face card)

Ex 4

Coins P(2H)

Ex 5

Dice P(sum 6)

P(doubles)

Ex 6 We have 5 skittles in a bag: 1 red, 1 purple, 1 green, 1 yellow and 1 orange.

Draw two skittles out of the bag at once.

Let A = event that you get one yellow skittle

B = event that both skittles are the same color

C = event that neither skittle is orange

D = event that one skittle is green and the other is red

(a) List the sample space, and each of these events as a set, in list form.

(b) What is the probability of each event A-D?

Ex 7

There are 10 numbered chips in a bag, numbered 0 through 9.

You pull one chip out of the bag at random.

Let A = the event that the chip is even

B = the event that the chip is a multiple of 3

C = the event that the chip is a nonzero multiple of 5

D = the event that the chip is a multiple of 4

(a) Write out the sample space and each event in set (list) format.

(b) Use a Venn Diagram (for A and B) to help you determine P(A), P(B), P(not A), P(not B), P(A or B), and P(A and B).

(c) Follow the instructions for (b), using events C and D instead of A and B.