9.4 Odds, Conditional Probability and Expected Value

Odds-->The odds in favor of an event E are given by n(E) : n(not E). That is, the odds in favor of event E is the ratio of the number of ways E can occur to the number of ways E does not occur. The odds against event E are given by n(not E) : n(E).

Example 1: When drawing a card from a deck of cards, what are the odds in favor of
(a) getting a black card.
(b) getting a face card.
(c) getting a 2.
(d) getting the 5 of hearts.

(a) 26 : 26 = 1 : 1

(b) 12 : 40 = 3 : 10

(c) 4 : 48 = 1 : 12

(d) 1 : 51

Example 2: In the game of craps, one wins on the first roll of the pair of dice if a 7 or 11 is thrown (i.e. the sum of the dice is 7 or 11). What are the odds of winning on the first roll?

ways to get 7 or 11:
(1, 6) (6, 1) (5, 6)
(2, 5) (5, 2) (6, 5)
(3, 4) (4, 3)

Odd
8 : 28 = 2 : 7

Given the probability of event E, P(E) , determine the odds in favor of E and the odds against E.

We know the odds in favor of E are given by

$$\frac{n(E)}{n(\bar{E})}=\frac{n(E)}{n(\bar{E})}\cdot\frac{1/n(S)}{1/n(S)}$$
$$=\frac{P(E)}{P(\bar{E})}$$
$$=\frac{P(E)}{1-P(E)}$$

or $\boxed{P(E):1-P(E)}$

$$\frac{n(E)}{n(\bar{E})}\left(\frac{\cancel{1/n(S)}}{\cancel{1/n(S)}}\right)=\frac{\cancel{n(E)/n(S)}}{\cancel{n(\bar{E})/n(S)}}$$

$$=\frac{P(E)}{P(\bar{E})}$$

Likewise, the odds against E are given by

$$1-P(E):P(E)$$

Thus, if we know the probability of E (or the probability of its complement), then we can get to the odds in favor or against E. This is useful, especially for unequally likely outcomes.

Example 3: Find the odds in favor of an event E if

(a) $P(E)=\frac{1}{2}$

(b) $P(E)=\frac{2}{5}$

(c) $P(E)=\frac{3}{7}$

(a) odds in favor 1:1

(b) " " 2:3

(c) " " 3:4

Example 4: If $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$, and $P(C)=\frac{1}{6}$ and events A, B and C are mutually exclusive, compute the odds in favor of A or C.

odds in favor of A 1 : 1
" " C 1 : 5

$P(A \cup C) = P(A) + P(C) = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

Odds in favor of A or C 2 : 1

1
1 1
1 2 1
1 3 3 1
0H 1H 2H 3H

Example 5: Three coins are tossed.
(a) What are the odds in favor of getting two heads and one tail?
(b) What are the odds against getting three heads?

(a) odds in favor of 2H 1T 3 : 5
(b) odds against 3H 7 : 1

Example 6: A jar contains 5 yellow, 4 blue, and 8 green marbles. What are the odds in favor of selecting a yellow marble if a single marble is drawn from the jar?

$5:12$

Example 7: The odds against the Heatwaves winning the Crabgrass Derby are 7 to 1. Express the probability of winning.

odds in favor of winning 1:7

$P(\text{win}) = \frac{1}{8}$

<u>Conditional Probability</u>-->Suppose A and B are events in sample space S such that $P(B) \neq 0$. The conditional probability that event A occurs, given that B occurs, is

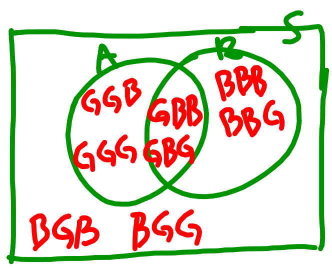
$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

"prob of A given B"

Example 8: A family has three children. Let A be the event that they have a girl first and B be the event that they have a boy second.

(a) Draw a Venn Diagram to represent A and B.
(b) Use the Venn Diagram to calculate the following:
  (1) $P(A|B)$
  (2) $P(B|A)$

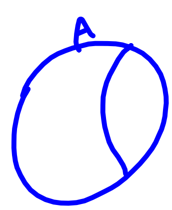
(a)



S = { BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG }

(b) (1) $P(A|B) = \frac{2}{4} = \frac{1}{2}$

(2) $P(B|A) = \frac{2}{4} = \frac{1}{2}$



Example 9: All 24 students in Ms. Henry's preschool class are either three or four years old, as shown in the following table. A student is selected at random.

	Age Three	***Age Four***	
Boys	8	3	11
Girls	6	7	13
	14	10	24

$P(\text{boy} \mid \text{4-yr-old}) = \frac{3}{10}$

(a) What is the probability that the student is three years old?
(b) What is the probability that the student is three years old, given that a boy was selected?
(c) What is the probability that the student is three years old, given that a girl was selected?

(a) $P(\text{3-yr-old}) = \frac{14}{24} = \frac{7}{12}$

(b) $\frac{8}{11} = P(\text{3-yr-old} \mid B)$

(c) $P(\text{3-yr-old} \mid G) = \frac{6}{13}$

Example 10: Five black balls numbered 1, 2, 3, 4, and 5 and seven white balls numbered 1 through 7 are placed in a bag. One is chosen at random. $n = 12$

(a) What is the probability it is numbered 1 or 2?
(b) What is the probability it is numbered 5 or is white?
(c) What is the probability it is numbered 5, given that it is white?
(d) What is the probability it is numbered 3, given that it is black?
(e) What is the probability it is black, given that it's numbered 2?
(f) What are the odds in favor of getting a black ball?

(a) $P(1 \text{ or } 2) = \frac{4}{12} = \frac{1}{3}$

(b) $P(5 \text{ or } W) = \frac{8}{12} = \frac{2}{3}$

(c) $P(5 | W) = \frac{1}{7}$

(d) $P(3 | B) = \frac{1}{5}$

(e) $P(B | 2) = \frac{1}{2}$

(f) $5 : 7$

Example 11: Mrs. Ricco has seven brown-eyed and two blue-eyed brunettes in her fifth grade class. She also has eight brown-eyed and three blue-eyed children with black hair. A child is selected at random.

(a) What is the probability that the child is a brown-eyed brunette?

(b) What is the probability that the child has brown eyes or is brunette?

(c) What is the probability that the child has brown eyes, given that he or she is a brunette?

(d) What is the probability that the child has black hair, given that the child has blue eyes?

(e) What are the odds in favor of the child being a brunette?

(f) What are the odds against the child having brown eyes?

	blue eyes	brown eyes	
brunette	2	7	9
black hair	3	8	11
	5	15	20

(a) P(brown eyes & brunette) $= 7/20$

(b) P(brn eyes or brunette $= 1 - P($blue eyes & black hair$)$ $= 1 - \frac{3}{20} = \frac{17}{20}$

(c) P(brn eyes | brunette) $= \frac{7}{9}$

(d) P(blk hair | blue eyes) $= \frac{3}{5}$

(e) 9:11

(f) 5 : 15 = 1:3

<u>Expected Value</u>: If, in an experiment, the possible outcomes are $a_1, a_2, a_3, \ldots, a_n$ along with corresponding probabilities of $p_1, p_2, p_3, \ldots, p_n$, then the expected value is given by

$$\mu = E = a_1 \cdot p_1 + a_2 \cdot p_2 + \ldots + a_n \cdot p_n .$$

Example 12: A dog with three puppies can have 0, 1, 2, or 3 females. What is the expected value for the number of females in this group assuming the probability of a female puppy is 50%?

$$E = \bar{x} = 0\left(\frac{1}{8}\right) + 1\left(\frac{3}{8}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{1}{8}\right)$$

$$= \frac{3}{8} + \frac{6}{8} + \frac{3}{8} = \frac{12}{8} = 1.5$$

1

1 1

1 2 1

1 3 3 1

0F 1F 2F 3F

$P(0F) = \frac{1}{8}$

9.4 A #16) $E = ?$ \$8 to roll pair of dice

get paid sum of dice

sums $=\{2,3,\dots,12\}$

Expected value (payoff) $= 2P(2) + 3P(3) + 4P(4) + 5P(5) + \dots + 12P(12)$

$$= 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + 4\left(\frac{3}{36}\right) + 5\left(\frac{4}{36}\right) + 6\left(\frac{5}{36}\right) + 7\left(\frac{6}{36}\right) + 8\left(\frac{5}{36}\right) + 9\left(\frac{4}{36}\right) + 10\left(\frac{3}{36}\right) + 11\left(\frac{2}{36}\right) + 12\left(\frac{1}{36}\right)$$

$$= \frac{1}{36}(2+6+12+20+30+42+40+36+30+22+12)$$

$$= \frac{252}{36} = \frac{63}{9} = 7$$

$\Rightarrow$ not a fair game

9.4 A #15) odds favor 5:2 $P(\text{win}) = \frac{5}{7}$

prize \$14000

$$E = \underset{\text{win}}{14000(P(\text{win}))} + \underset{\text{lose}}{0\,P(\text{lose})}$$

$$E = 14000\left(\frac{5}{7}\right) = \$10000$$

9.4 B #15) \$1 for every dot on die

$$E = 1P(1) + 2P(2) + 3P(3) + 4P(4) + 5P(5) + 6P(6)$$

$$= \frac{1}{6}(1+2+3+4+5+6)$$

$$= \frac{21}{6} = \frac{7}{2} = \$3.50$$

$\Rightarrow$ charge \$3.50 for a fair game

9.4A #14)

5 Q
5 D
5 N
10 P
25 coins

$$E = 0.25\,P(Q) + 0.10\,P(D) + 0.05\,P(N) + 0.01\,P(P)$$
$$= 0.25\left(\tfrac{1}{5}\right) + 0.1\left(\tfrac{1}{5}\right) + 0.05\left(\tfrac{1}{5}\right) + 0.01\left(\tfrac{2}{5}\right)$$
$$= 0.01\left(\frac{25+10+5+2}{5}\right) = 0.01\left(\frac{42}{5}\right) = 0.01(8.4) = 0.084$$

9.4B #1) (a) odds in favor of black card
1:1

(b) this means for every 1 black there is 1 red card also in deck

B #14) E = ? win \$10 for sum of 7, otherwise lose \$2

$$E = \overset{\text{win}}{10(P(7))} + \overset{\text{lose}}{-2(P(\text{not }7))}$$
$$= 10\left(\frac{6}{36}\right) + -2\left(\frac{30}{36}\right) = 10\left(\frac{1}{6}\right) - 2\left(\frac{5}{6}\right) = \frac{0}{6} = 0$$

$\Rightarrow$ we expect to come out even

B#4) odds against winning 55,000,000 : 1

$$P(win) = \frac{1}{55,000,001}$$