9.5 Permutations and Combinations

Permutation-->An ordered arrangement of objects.

Combination-->A collection of objects, in no particular order.

Factorial--> $n!=n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot3\cdot2\cdot1$ By definition, $1!=1$ and $0!=1$.

The number of permutations of n distinct objects, taken all together, is n!, i.e. "n factorial."

Example 1: Simplify the following expressions.

(a) $\frac{10!}{3!}$

(b) $\frac{3!5!}{2!}$

Let's make a table to record these next several results and try to generalize them to a formula.

Permutation or Combination	***# things to choose from***	***# things we chose***	***# permutations or combinations***	***Formula guess***

Example 2: How many 4-letter "words" can we form (where we count any distinct grouping of letters as a word) with no repeating letters.

Permutation or Combination? _____

Example 3: I want to arrange three marbles in a row. In how many ways can this be done?

Permutation or Combination? _____

Example 4: If 6 horses are entered in a race and there can be no ties, how many different orders of finish are there?

Permutation or Combination? _____

Example 5: In your sorority, you need to choose a president, treasurer and secretary out of a group of 15 women. How many ways can this occur?

Permutation or Combination? _____

Example 6: My friend made 5 cakes and she has offered to let me take three of them home. How many different groups of cakes can I choose?

Permutation or Combination? _____

Example 7: I have 10 marbles and I want to choose 4 to give to my best friend. How many different groupings of marbles can I give her?

Permutation or Combination? _____

Example 8: How many 5-card poker hands are there?

Permutation or Combination? _____

Number of Permutations:

The number of permutations of r objects chosen from n objects, where $0 \le r \le n$, is

$$_nP_r = \frac{n!}{(n-r)!}$$ read "n permute r."

Number of Combinations:

The number of combinations of r objects chosen from n objects, where $0 \le r \le n$, is

$$_nC_r = \frac{n!}{(n-r)!\,r!} = \binom{n}{r}$$ read "n choose r."

$$= \frac{number\ of\ permutations}{number\ of\ permutations\ per\ combination}$$

$$= \frac{number\ of\ permutations}{(number\ chosen)!}$$

Example 9: You have a class of 20 students. You need to select two students to be hall monitors. How many groups of two students can you choose from your class?

Permutation or Combination? _____

Example 10: I go to an ice cream store that has 30 flavors of ice cream.

(a) I want to get a bowl with three scoops to eat. How many different groupings of three flavors can I get?

Permutation or Combination? _____

(b) I want to get my three scoops on a cone instead. How many different ways can I arrange three flavors on my cone?

Permutation or Combination? _____

Example 11: I want to choose 3 candies out of 5 different candies. How many choices do I have?

Permutation or Combination? _____

Example 12: I have 4 different cookies. How many ways can I put them in a line?

Permutation or Combination? _____

Example 13: How many 12-person juries can be chosen from 30 candidates?

Permutation or Combination? _____

Example 14: On a 10-question True/False test, how many ways can 8 or more answers be correct?

Permutation or Combination? _____

Example 15: At a pizza restaurant, you have 10 choices for toppings, 3 sauce choices, and 4 types of crust. They are running a special today for $6 you can get a 1-topping pizza with your choice of topping, sauce and crust. How many different pizzas can be ordered?

Permutation or Combination? _____

Example 16: For a family of 9 children, with 4 girls, how many boy-girl arrangements can there be?

Permutation or Combination? _____

Example 17: You toss a coin 6 times and record the result—either H=heads or T=tails.

Permutation or Combination? _____

(a) How many ways can you get 2 heads?

(b) How many ways can you get 5 tails?

Example 18: You have 3 red (triangular) flags, 2 green flags and 4 blue flags. How many different ways can you order the flags on your flagpole?

Permutation or Combination? _____