

1.1 What Is A Function?

Defn A function (f) is a relation between two sets assigning to each element in the first set (domain) exactly one element in the second set. (range)

independent variable ^(input) comes from domain (usually x)

dependent variable ^(output) " " range (usually y)

All f 's are relations

Notation: $f(x)$ read "f of x"
is a function f , where x is the name of the input variable.

Ex 1: Is this relation a f ? Give domain and range.

(a)

input	-4	-3	-2	-1	0	1	2	3
output	4	0	4	0	2	3	2	1

(b)

input	1	1	2	2	3	4	5	6
output	0	2	0	2	1	1	1	1

1.1 (cont)

Ex 2: let $f(x) = 2 - 4x$ and $g(x) = \sqrt{4 - x}$

Find/evaluate:

(a) $f(0)$

(e) $g(x+h)$

(b) $g(0)$

(f) $g(x) + h$

(c) $f(y+5)$

(d) $g(1/x)$

(g) $f(x+h)$

Ex 3: Find the difference quotient for $f(x) = x^2 + 1$

Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

1.1 (cont)

Ex 4 Find the natural domain for each function.

natural domain:

(a) $n(y) = \frac{1}{\sqrt{4-3y}}$

(b) $w(x) = 3x^2 + \frac{1}{x}$

(c) $y(p) = 4p^2 + 9p^4 - 1$

(d) $t(a) = 1 + \sqrt{a+9}$

(e) $f(b) = \frac{\sqrt[3]{5-b}}{4}$

1.2 Graphs

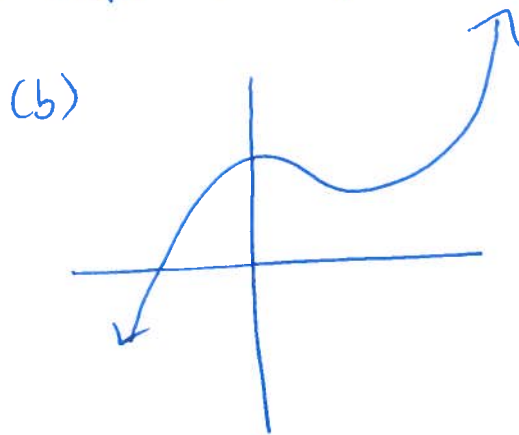
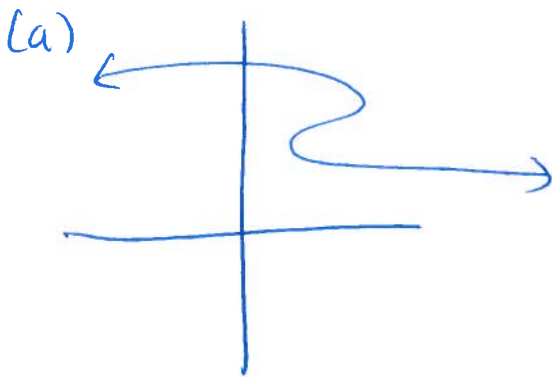
Vertical line Test: A graph represents a fn iff it passes Vertical line Test, meaning that every vertical line intersects the graph at most once.

Ex1 Graph this fn by plotting points.

$$f(x) = x^2 + 3$$

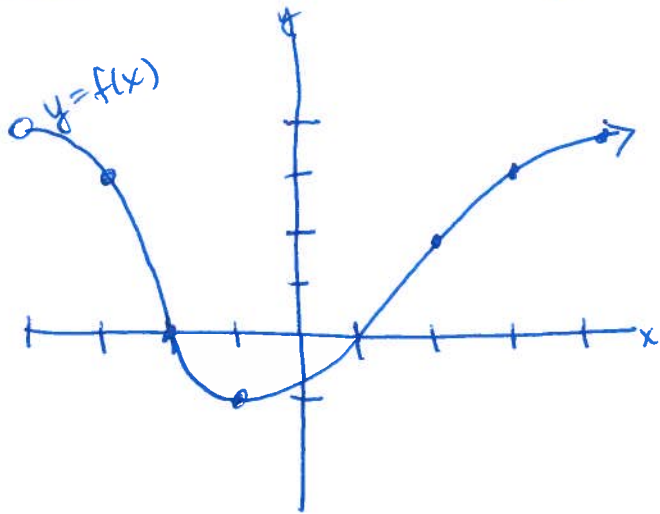
Default way to graph a function/relation:
PLOT pts and connect the dots

Ex2 Do these graphs represent fns?



1.2 (cont)

Ex3 Answer the questions about the given graph.



(a) $f(1)$

(b) x such that $f(x)=3$

(c) $2f(-1)$

(d) domain of f

(e) range of f

Symmetry

wrt = with respect to

Symmetry wrt

if (x,y) is on graph, then so is

quick graph example

even

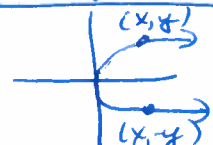
y-axis

$(-x, y)$



x-axis

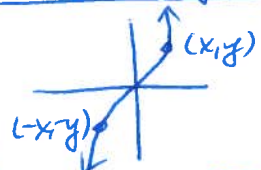
$(x, -y)$



odd

origin

$(-x, -y)$



1.2 (cont)

\forall means "for all"

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★
★
1.10
1.11

Algebraically, a fn is even if $f(-x) = f(x) \quad \forall x$ in domain
a fn is odd if $f(-x) = -f(x) \quad \forall x$ in domain.

Ex 4 List symmetry and whether fn is odd, even, or neither.

(a) $p(x) = \frac{5x^2}{x^3+x}$

(c) $m(x) = 3x^2 - x + 4$

(b) $f(x) = |x| + 5$

1.2 (cont)

Ex 5 Is each relation a fn?

(a) $5x^2 + y = 3x + 2$

(b) $y^2 - x^2 = 0$

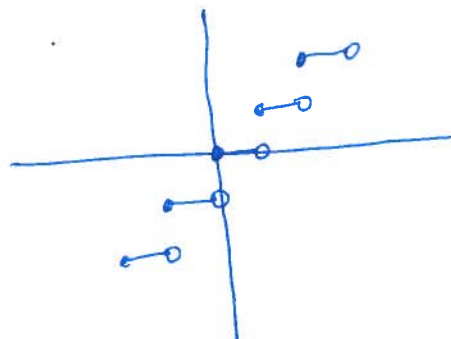
(c) $y^3 + x^3 = 0$

(d) $f(x) = \lfloor x \rfloor$

★★★
1.16

$\lfloor x \rfloor$ is the greatest integer of x , i.e. the largest integer that is $\leq x$

$y = \lfloor x \rfloor$



1.3 Linear Fns

Defn • Any fn that can be written as $f(x) = mx + b$,

where $m \neq 0$, is a linear fn.

• $f(x) = b$ (where b is a constant) is a constant fn and graphs into a horizontal line

• The slope of a line is $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$ for a line that goes through (x_1, y_1) and (x_2, y_2)

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1.22

Ex 1 Find the slope of the line representing the linear fn whose graph goes through $(1, 5)$ and $(-2, 7)$.

m, b, y_1, x_1, h, k constants

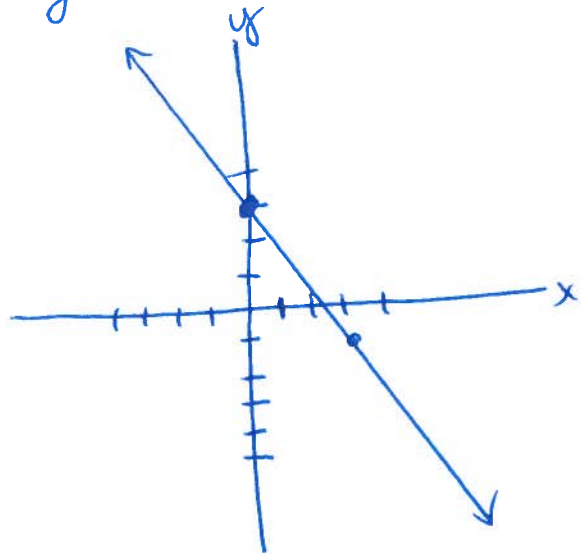
Eqn of lines	Linear Fn
slope-intercept form $y = mx + b$	$f(x) = mx + b$ slope-intercept form
point-slope form $y - y_1 = m(x - x_1)$	$f(x) = m(x - h) + k$ transformation form

1.3 (cont)

Ex 2 Find transformation form of linear f , whose graph passes through $(4, -3)$ and $(-5, 2)$.

Ex 3 Find slope-intercept form of line in Ex 2.

Ex 4 Find the f represented by the following line.



1.3 (cont)

Two lines are parallel (\parallel) if their slopes are the same. Two lines are perpendicular (\perp) when they intersect at right angles to one another.

(Note: f_{\perp} is read "f perp" for the f that is \perp to f)

f is \perp to f_{\perp} if the product of their slopes is -1 .

Ex 5 Find a linear f_{\perp} , that is \perp to $f(x) = 2 - 4x$ passing through the pt $(0, -3)$.

More about the difference quotient:

$$\frac{f(x+h) - f(x)}{h} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{\Delta y}{\Delta x} = \text{slope of line that passes}$$

through pts $(x, f(x))$ and $(x+h, f(x+h))$. Cool!

this is the average rate of change (a.k.a. slope of secant line through graph)

1.3 (cont)

Ex 6 Find the average rate of change for
 $f(x) = \sqrt{x+1} - 3$ on the interval $[3, 8]$.