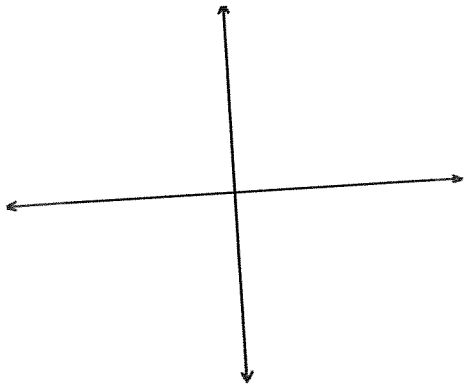
1.7 Families of Functions

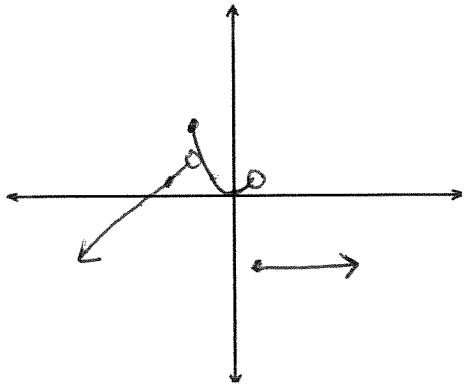
Piecewise Fns: fns defined in pieces; the domain is split into pieces.

Ex1 Graph the fn

$$g(x) = \begin{cases} \frac{1}{2}x + 4 & \text{if } x \le -2 \\ 2 - x & \text{if } x > -2 \end{cases}$$



Ex2 Find $f(x)$ given the graph.



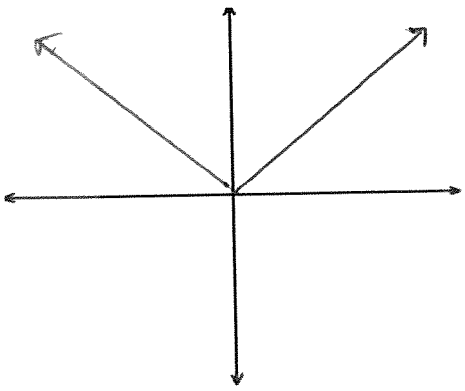
Absolute Value fn

$$f(x) = |x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$$

domain:

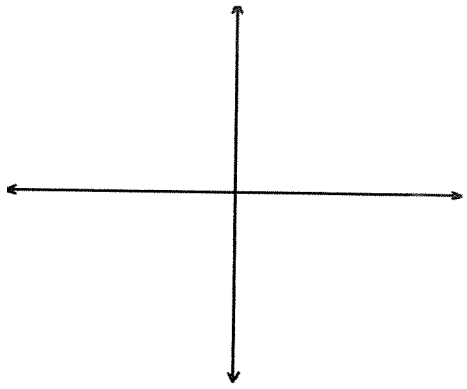
range:

graph:



1.7 (cont)

Ex3 Graph the fn. $f(x) = |(x+4)^2 - 3|$



Power Fns

$f(x) = x^n \quad n \in \mathbb{Z}^+$

domain: $x \in \mathbb{R}$

range: $y \in \mathbb{R}$, if n odd
$y \in [0, \infty)$ if n even

1.83

Root Fns

$f(x) = x^{1/n}$, $n \in \{2, 3, 4, \ldots\}$

domain: $x \in \mathbb{R}$, if n odd
$x \in [0, \infty)$, if n even

range: $y \in \mathbb{R}$, if n odd
$y \in [0, \infty)$ if n even

1.84

Recall Rules of exponents:

1) $a^b a^c = a^{b+c}$
2) $(a^b)^c = a^{bc}$
3) $\frac{a^b}{a^c} = a^{b-c}$
4) $a^0 = 1, \; a \neq 0$
5) $a^{1/n} = \sqrt[n]{a}$
6) $a^{-n} = \frac{1}{a^n}$

$$(x^n)^{\frac{1}{n}} = \begin{cases} x & \text{if } n \text{ odd} \\ |x| & \text{if } n \text{ even} \end{cases}$$

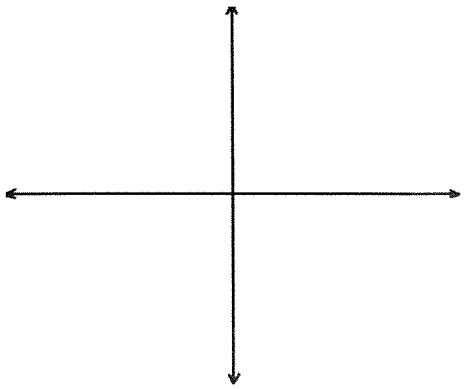
$$(x^{1/n})^n = \begin{cases} x & \text{if (a) } n \text{ odd} \\ & \text{or (b) } n \text{ even} \\ & \text{and} \\ & x \geq 0 \\ \text{undefined, if } n \text{ even} & \\ \text{and } x < 0 & \end{cases}$$

Ex 4 Find (a) $(f \circ g)(x)$ and (b) $(g \circ f)(x)$ + state domain + range.

(a) $f(x) = x^6$, $g(x) = \sqrt[6]{x}$

(b) $f(x) = \sqrt[3]{3x+4}$, $g(x) = x^3$

Ex 5 Graph $y = \left\lceil \frac{x}{2} \right\rceil + 3$



$y = \lceil x \rceil$ is the ceiling fn;
it returns the smallest integer that's greater than or equal to the input

1.87

2.1 Polynomial Functions

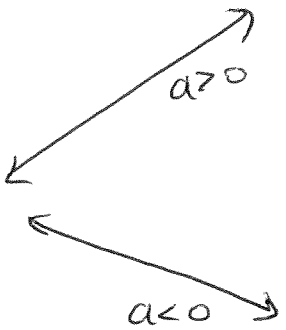
in standard form $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

coefficients $a_0, a_1, \ldots, a_n$ degree $= n$

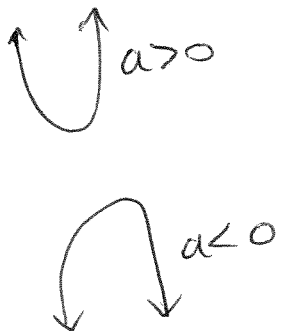
leading coefficient a_n

Generic Graphs of Polynomial fns

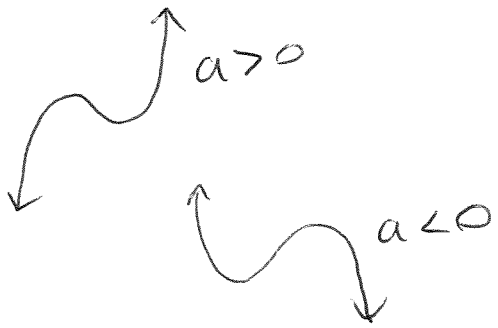
$y = ax + b$



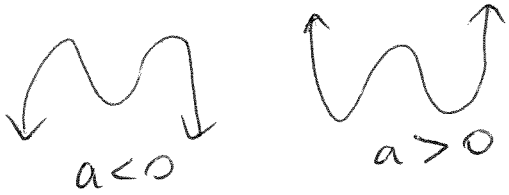
$y = ax^2 + bx^2 + c$



$y = ax^3 + bx^2 + cx + d$

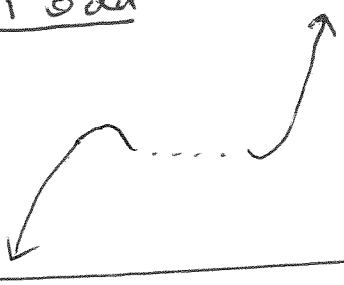
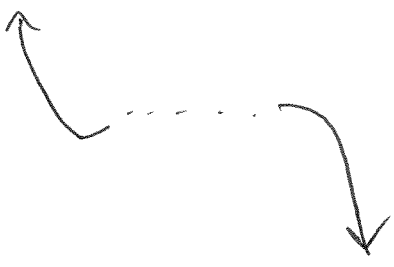


$y = ax^4 + bx^3 + cx^2 + dx + e$



2.1

what do we mean by "end behavior?"

	n odd	n even
$a_n > 0$		
$a_n < 0$		

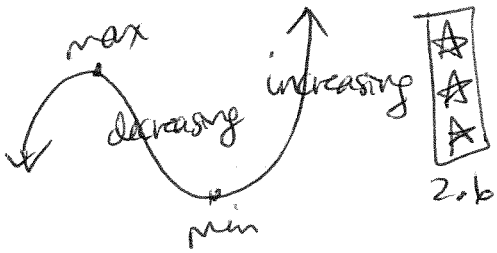
2.1 (cont)

Ex 1 Find the end behavior for

(a) $f(x) = (5-2x)^2$

(b) $f(x) = -x^3+4x^2+9x-7$

Min/Max points: occur where slope changes from positive to negative or negative to positive



2.6

A fn is increasing on an interval if $x_1 < x_2$ (on the interval) $\Rightarrow f(x_1) < f(x_2)$.

A fn is decreasing on an interval if $x_1 < x_2$ (on the interval) $\Rightarrow f(x_1) > f(x_2)$.

Arithmetic w/ polynomials

Ex 2 Multiply $f(x) = x^5+x^2-x$ by $g(x) = x^4+x^2+x+1$.

Ex 3 Divide $f(x)$ by $g(x)$.

$f(x) = 4x^5 + x^3 + 13x^2 + 4x - 2$ $\quad$ $g(x) = 2x^2 + x - 1$.