

2.2 Roots and Factors of Polynomial Functions

Real Roots/zeros of a polynomial

roots of $f(x)$ are the x -values that make $f(x)=0$.



Note: remember difference between:
rational number:

irrational number:

Ex1 for $f(x) = x(x-1)^2(x+3)^3(x+5)$ degree = _____

(a) Name the factors: _____

Name the roots/zeros: _____

total
Number of roots = _____ Number of distinct roots = _____

Multiplicity: $x=a$ is a root/zero of multiplicity k
if, in factored form, the polynomial has $(x-a)^k$ as
a factor.

Note: If k is even, the graph of $y=f(x)$ touches the
 x -axis; If k is odd, the graph of $y=f(x)$ goes through
the x -axis.

Ex1 (b) Multiplicity for each zero/root:

2.2 (cont)

Thm ① Rational Root Theorem

If $p(x)$ is polynomial w/ integer coefficients, then if there are any rational roots/zeros of $p(x)$, they are in the form of factors of constant term over factors of leading coefficients.

Ex 2 List all possible rational roots for
 $p(x) = 3x^5 + 4x^3 - 2x + 10$

Ex 3 (working backwards)

Consider $p(x) = (x-2)(5x-3) = \underline{\hspace{10em}}$

What are roots? and

Ex 4 Show $f(x) = 2x^3 + 5x^2 - 3x + 1$ has no rational roots.

2.2 (cont)

Thm ② Upper/Lower Bound Theorem
 $p(x)$ is polynomial w/ real coefficients and positive leading coefficient.

(a) If $a > 0$ and the last row of synthetic division for $(x-a)$ has all nonnegative numbers, then a is an upper bound for all real roots of $p(x)$.

(b) If $b < 0$ and the last row of synthetic division for $(x-b)$ has numbers that alternate signs, then b is a lower bound for all real roots of $p(x)$.

Ex 5 list all possible rational roots of $p(x) = x^3 - 2x^2 + x + 12$ (from Thm ①). Then use Thm ② to rule out some possibilities. Does this fn have rational roots?

2.2 (cont)

Thm ③: Conjugate Pair Theorem

If $p(x)$ is a polynomial w/ rational coefficients, then when $m+\sqrt{n}$ is a root/zero, so is $m-\sqrt{n}$.

Ex 6 Find a polynomial of degree 3 that has $x=\frac{1}{5}$ and $x=2+\sqrt{3}$ as roots and has rational coefficients. How many such polynomials are there?

Thm ④: Descartes Rule of Signs

$p(x)$ is a polynomial with real coefficients. When written in descending order, count the # of sign changes in $p(x)$ and $p(-x)$.

(a) The # of positive IR zeros/roots of $p(x)$ is the # of sign changes of $p(x)$, or # decreased by an even whole number.

(b) The # of negative IR zeros/roots of $p(x)$ is the # of sign changes in $p(-x)$, or # decreased by an even whole number.

2.2 (cont)

Ex 7 Use Thm ④ to find possible # of positive and negative roots of $p(x) = 2x^4 - 3x^3 + 15x^2 - 30x - 50$.
Then find the roots.

2.3 Complex Numbers

Defn complex number can be written as $a+bi$

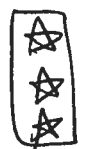
where $a, b \in \mathbb{R}$, $i = \sqrt{-1}$.

- For $z = a+bi$, $a = \operatorname{Re}(z)$ = real part of z
 $b = \operatorname{Im}(z)$ = imaginary part of z

- For $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$, $z_1 = z_2$ iff $a_1 = a_2$ and $b_1 = b_2$.

- \mathbb{C} = set of complex #s; $\mathbb{R} \subset \mathbb{C}$

- modulus of $z = a+bi$ is $|z| = \sqrt{a^2 + b^2}$
(the distance from origin to (a, b) in \mathbb{C} plane)



2.15

Ex1 Find i^2, i^6, i^{34}, i^{83} . Can you find a rule for $i^n, n \in \mathbb{N}$?

Ex2 Find (a) $(5+2i) + (3-4i)$

(b) $(3-7i)(4+2i)$

2.3 (cont)

Ex 3 Simplify, and write in complex form,
 $(\sqrt{15} + \sqrt{-27})(\sqrt{-15} + \sqrt{3})$

Conjugate of $z = a + bi$ is $\bar{z} = a - bi$.

Ex 4 Find conjugate of

(a) $(-3 + 8i)$

(b) 3

(c) $3i$

Ex 5 Let $w = 4 + 5i$ and $z = -2 + 9i$, find

(a) $z - \bar{z}$

(b) $w\bar{w}$

2.3 (cont)

Ex 6 For $z = -2 + 3i$ and $w = 5 + 4i$, find (in complex form) ^{simplified}

(a) $\frac{w}{z+3}$

(b) $\frac{z}{z-i}$

Ex 7 Solve this eqn, allowing \mathbb{C} solutions.
 $25x^2 - 40x + 25 = 0$