2.2 Roots and Factors of Polynomial Functions

Real Roots/zeros of a polynomial

roots of fn $f(x)$ are the x-values that make $f(x)=0$.

☆☆☆ Note: remember difference between:

2.8

rational number:

irrational number:

Ex1 for fn $f(x) = x(x-1)^2(x+3)^3(x+5)$ degree = ____

(a) Name the factors: ________________

Name the roots/zeros: ________________

total Number of roots = ________ Number of distinct roots = ____

Multiplicity: $x=a$ is a root/zero of multiplicity k if, in factored form, the polynomial has $(x-a)^k$ as a factor.

Note: If k is even, the graph of $y=f(x)$ touches the x-axis; If k is odd, the graph of $y=f(x)$ goes through the x-axis.

Ex1 (b) Multiplicity for each zero/root:

____

2.2 (cont)

Thm ① Rational Root Theorem

If $p(x)$ is polynomial w/ integer coefficients, then if there are any rational roots/zeros of $p(x)$, they are in the form of factors of constant term over factors of leading coefficients.

Ex2 List all possible rational roots for

$p(x) = 3x^5 + 4x^3 - 2x + 10$

Ex3 (working backwards)

Consider $p(x) = (x-2)(5x-3) =$ ____________

What are roots? ________ and ________

Ex4 Show $f(x) = 2x^3 + 5x^2 - 3x + 1$ has no rational roots.

2.2 (cont)

Thm ② Upper/lower Bound Theorem
p(x) is polynomial w/ real coefficients and positive leading coefficient.

(a) If $a>0$ and the last row of synthetic division for $(x-a)$ has all nonnegative numbers, then a is an upper bound for all real roots of $p(x)$.

(b) If $b<0$ and the last row of synthetic division for $(x-b)$ has numbers that alternate signs, then b is a lower bound for all real roots of $p(x)$.

Ex 5 list all possible rational roots of $p(x) = x^3-2x^2+x+12$ (from Thm ①). Then use Thm ② to rule out some possibilities. Does this fn have rational roots?

M1080

2.2 (cont)

Thm ③: Conjugate Pair Theorem

If $p(x)$ is a polynomial w/ rational coefficients, then when $m+\sqrt{n}$ is a root/zero, so is $m-\sqrt{n}$.

Ex6 Find a polynomial of degree 3 that has $x=\frac{1}{5}$ and $x=2+\sqrt{3}$ as roots and has rational coefficients. How many such polynomials are there?

Thm ④: Descartes Rule of Signs

$p(x)$ is a polynomial with real coefficients. When written in descending order, count the # of sign changes in $p(x)$ and $p(-x)$.

(a) The # of positive $\mathbb{R}$ zeros/roots of $p(x)$ is the # of sign changes of $p(x)$, or # decreased by an even whole number.

(b) The # of negative $\mathbb{R}$ zeros/roots of $p(x)$ is the # of sign changes in $p(-x)$, or # decreased by an even whole number.

2.2 (cont)

Ex 7 Use Thm ④ to find possible # of positive and negative roots of $p(x) = 2x^4 - 3x^3 + 15x^2 - 30x - 50$.

Then find the roots.

2.3 Complex Numbers

Defn complex number can be written as $a+bi$ where $a,b \in \mathbb{R}$, $i=\sqrt{-1}$.

- For $z=a+bi$, $a=\mathrm{Re}(z)$ = real part of z
  $b=\mathrm{Im}(z)$ = imaginary part of z
- For $z_1=a_1+b_1 i$ and $z_2=a_2+b_2 i$, $z_1=z_2$ iff $a_1=a_2$ and $b_1=b_2$.
- $\mathbb{C}$ = set of complex #s ; $\mathbb{R} \subset \mathbb{C}$
- modulus of $z=a+bi$ is $|z|=\sqrt{a^2+b^2}$ (the distance from origin to (a,b) in $\mathbb{C}$ plane)

☆☆☆

2.15

Ex1 Find i^{24}, i^{16}, i^{34}, i^{83}. Can you find a rule for i^n, $n \in \mathbb{N}$?

Ex2 Find (a) $(5+2i)+(3-4i)$ | (b) $(3-7i)(4+2i)$

2.3 (cont)

Ex3 Simplify, and write in complex form,

$(\sqrt{15}+\sqrt{-27})(\sqrt{-15}+\sqrt{3})$

Conjugate of $z=a+bi$ is $\bar{z}=a-bi$.

Ex4 Find conjugate of

(a) $(-3+8i)$

(b) 3

(c) $3i$

Ex5 let $w=4+5i$ and $z=-2+9i$, find

(a) $z-\bar{z}$

(b) $w\bar{w}$

2.3 (cont)

Ex 6 For $z = -2+3i$ and $w = 5+4i$, find (in simplified complex form)

(a) $\dfrac{w}{z+3}$

(b) $\dfrac{z}{z-i}$

Ex 7 Solve this eqn, allowing $\mathbb{C}$ solutions.

$25x^2 - 40x + 25 = 0$