2.4 Complex Roots of Polynomial Fns

Fundamental Thm of Algebra

Form 1 Every polynomial w/ real coefficients (and degree $n>0$) has at least one complex root.

Form 2 Every polynomial w/ real coefficients (and degree $n>0$) has exactly n roots, counting multiplicities.

Conjugate Pair Thm ③ (cont)

For polynomial $p(x)$ with real coefficients, if $z=a+bi$ is a root of $p(x)$, then so is $\bar{z}=a-bi$.
(i.e. complex roots come in conjugate pairs)

2.24

Ex 1 Find all the roots of

$$f(x)=x^3-2x^2+9x-18$$

2.4 (cont)

Ex2 Find the standard form of the polynomial fn of smallest degree w/ real coefficients such that $f(0) = 58$ and it has $2+5i$ as one of its roots

Ex3 Find a polynomial of smallest degree that has roots of $5, -1$ and $1+2i$. (leave in factored form.)

2.4 (cont)

Ex 4 Factor completely.

$f(x) = 3x^5 - 12x^4 + 3x^3 - 12x^2$

★ Every polynomial can be factored into linear and/or irreducible quadratic factors.

2.5 Rational Fns

Defn: A fn $f(x) = \frac{p(x)}{q(x)}$ is a rational fn if both $p(x)$ and $q(x)$ are polynomials, $q(x) \neq 0$. The domain of $f(x)$ is all real numbers except where $q(x) = 0$.

2.30 2.37 2.38

We want to graph rational fns in a sophisticated way. Look at

① x-intercepts : points where $f(x) = 0 \Leftrightarrow$ points that make $p(x) = 0$ but NOT $q(x)$.

② VA (vertical asymptotes)

If $q(b) = 0$ and $p(b) \neq 0$, then $x = b$ is VA.

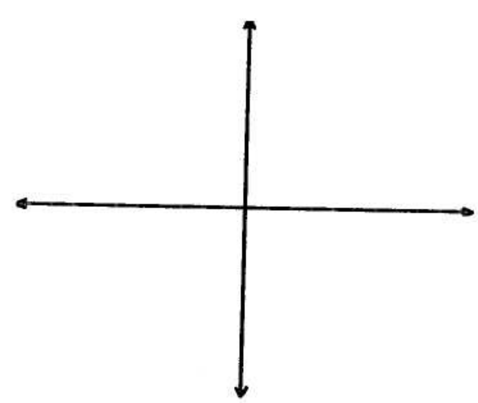
vs

③ Holes

If $q(b) = 0$ and $p(b) = 0$, then there is a hole at the point where $x = b$. (We can find the hole point by simplifying $f(x)$ and then plugging in $x = b$ to get the y-coordinate.)

Ex1 Analyze and graph

$$f(x) = \frac{(4-x)(x+1)}{(x+1)(x-1)}$$



2.5 (cont) End behavior $f(x)=\frac{p(x)}{q(x)}$

④ HA (horizontal asymptote) vs ⑤ OA (oblique asymptotes)

(a) If degree of $p(x)$ < degree of $q(x)$, then $y=0$ is HA.

why?

(b) If degree of $p(x)$ = degree of $q(x)$, then

$$y=\frac{\text{leading coefficient of } p(x)}{\text{leading coefficient of } q(x)}$$

is HA.

(sometimes also called slant asymptotes)

(c) If degree of $p(x)$ > degree of $q(x)$, then

(1) do long division!!!

you'll get

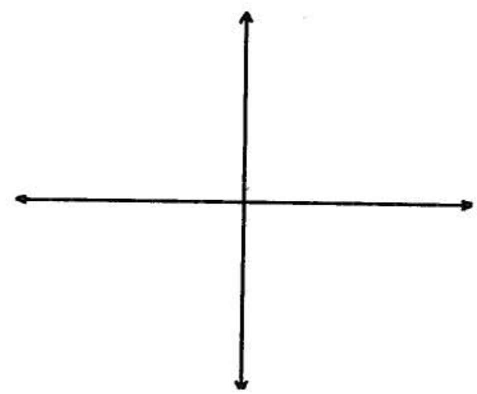
$$f(x)=\text{quotient polynomial} + \frac{\text{remainder}}{q(x)}$$

(2) y = quotient polynomial is the OA.

Ex 2 For $f(x)=\frac{(4-x)(x+1)}{(x+1)(x-1)}$, find the end behavior, i.e. HA or OA.

2.5 (cont)

Ex 3 Analyze and graph $f(x) = \dfrac{4x^2(x-2)(x+1)}{(x+1)^2(2x+3)}$



Ex 4 Analyze and graph $f(x) = \dfrac{(x+3)(x+2)}{x(x-1)(x+5)}$