4.2 Unit Circle Defns of Trigonometric Fns

$\boxed{(x, y)}$

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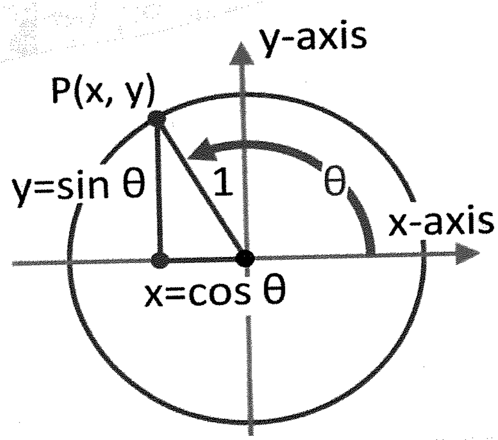


On the unit circle:

$y = \sin\theta$

$x = \cos\theta$

$\tan\theta = \frac{y}{x}$

Also: $\sec\theta = \frac{1}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$\mathbb{Z}$ = set of integers

	domain	range
$\sin x$	$x \in (-\infty, \infty)$	$y \in [-1, 1]$
$\cos x$	$x \in (-\infty, \infty)$	$y \in [-1, 1]$
$\tan x$	$x \in (-\infty, \infty)$, $x \neq \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$	$y \in (-\infty, \infty)$

Even-Odd

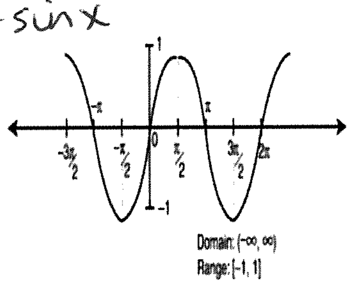
$\sin(-x) = -\sin x$ (odd)

$\cos(-x) = \cos x$ (even)

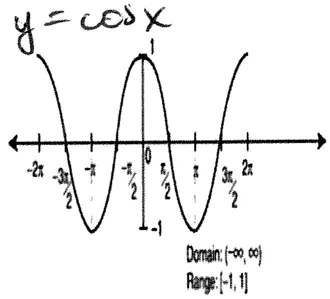
$\tan(-x) = -\tan x$ (odd)

Function Graphs

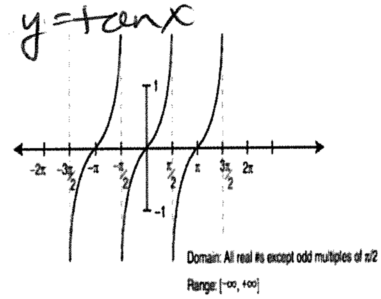
$y = \sin x$



$y = \cos x$



$y = \tan x$



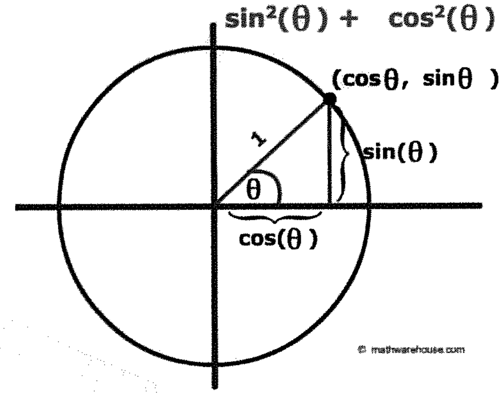
4.2 (cont)

Ex1 Specify which quadrant (or quadrants) θ can be in, given the specified information.

(a) $\sin\theta = -0.74$, $\cos\theta > 0$

(b) $\cos\theta < 0$ and $\tan\theta > 0$

Pythagorean Identities

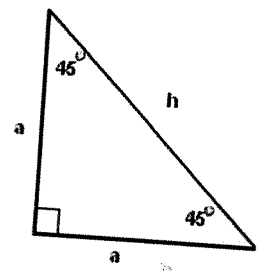


Pythagorean Identities	
$\sin^2\theta + \cos^2\theta = 1$	
$1 + \tan^2\theta = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$

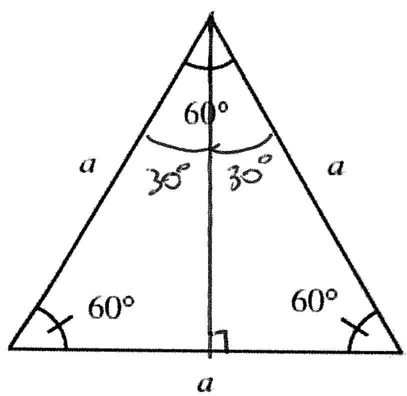
Ex2 Given $\cos\theta = \frac{-5}{13}$ and $\tan\theta < 0$, find $\sin\theta$ and $\tan\theta$.

4.2 (cont) Common Angles

① (isosceles right Δ)
(assume hypotenuse has length 1, like in unit circle)
Find $\cos 45°$ and $\sin 45°$.



② Find $\cos 60°$, $\cos 30°$, $\sin 60°$, $\sin 30°$, $\tan 60°$, $\tan 60°$.



4.2 (cont)

Ex 3 ① Sketch each angle, ② give its reference angle, ③ find $\sin\theta$ and $\cos\theta$.

(a) $\theta = -225°$

(b) $\frac{5\pi}{3} = \theta$

(c) $\theta = \frac{7\pi}{6}$

(Not in book - but important)

Reference Angles

$\bar{\theta}$ is the reference angle for θ;

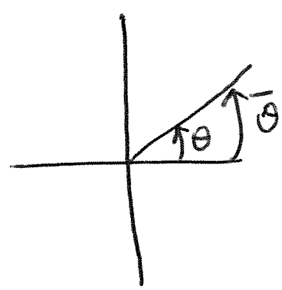
$0 < \bar{\theta} < \pi/2$ (always)

(☆ we can put $\bar{\theta}$ in a right triangle)

$\bar{\theta}$ has the same terminal side as θ & the other side of the angle is the closest piece of the x-axis

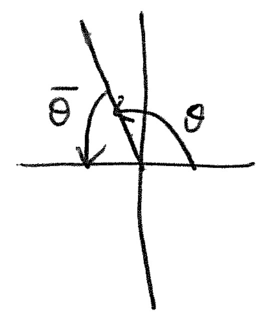
Ex if $0 < \theta < 2\pi$

θ in Q1



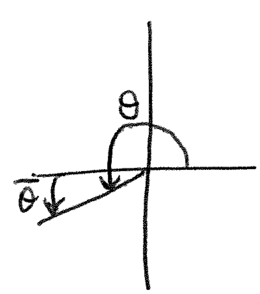
$\bar{\theta} = \theta$

θ in Q2



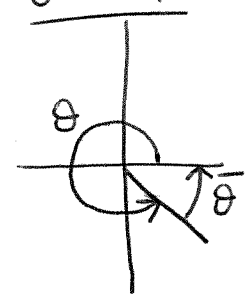
$\bar{\theta} = \pi - \theta$

θ in Q3



$\bar{\theta} = \theta - \pi$

θ in Q4



$\bar{\theta} = 2\pi - \theta$

4.3 Sine, Cosine and Tangent Fns

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Periodic Fns

("Fns that repeat themselves")

f is periodic if $f(x+P) = f(x)$ $\forall x$ in the domain and for some $P \in \mathbb{R}$, $P \neq 0$.

The period of such a fn is P.

What is the period of $y = \sin x$?

" " $y = \cos x$?

" " $y = \tan x$?

Let's bring together information we know about transformations of graphs to see how period gets affected.

Ex) Find all θ values such that

(a) $\sin\theta = \frac{-\sqrt{3}}{2}$

(b) $\cos\theta = \frac{\sqrt{2}}{2}$

4.3 (cont)

Ex2 Find the period.

(a) $y = \cos(-5\theta)$

(b) $y = \tan(\frac{1}{2}\theta)$

Ex 3 Find A such that the fn has the given period P.

(a) $y = \tan(A\theta)$ $\quad P = 3\pi$

(b) $y = \sin(A\theta)$ $\quad P = 3$

Ex4 List all transformations of $y = \frac{-1}{3}\sin(4x - \frac{\pi}{2})$ compared to base graph of $y = \sin x$

EX5 Use transformation information to give period and "phase shift" (horizontal shift) & then sketch the graph.

(a) $y = \frac{1}{2}\cos(2x+\pi) - 1$

(b) $y = -\tan\left(\frac{1}{2}x + \frac{\pi}{4}\right)$