4.4 Secant, Cosecant, and Cotangent Fns

<u>Defns</u> $\sec\theta = \dfrac{1}{\cos\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$

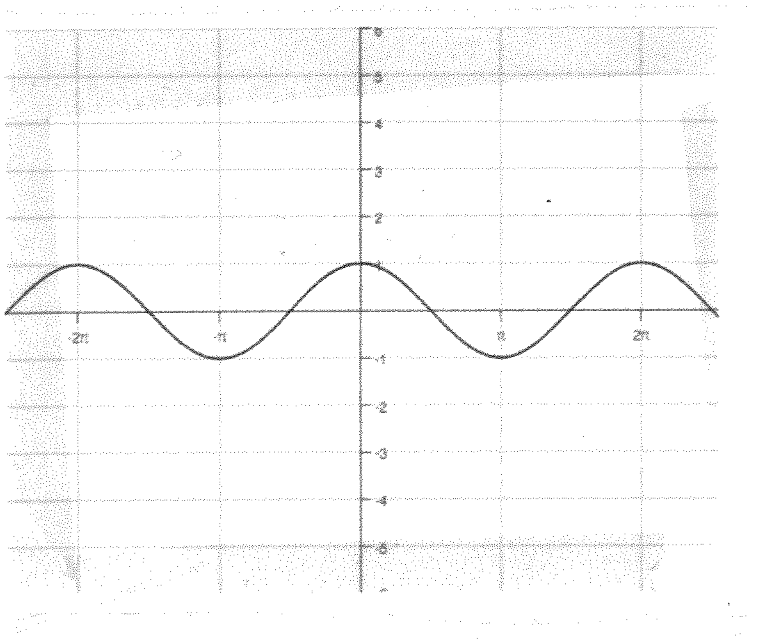
$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

★ these all have restricted domains

★ ★ ★
4.44
4.45
4.46
4.47

<u>Ex1</u> Given the graphs of $y = \cos x$, $y = \sin x$, $y = \tan x$, create the graphs of their reciprocal trig fns.

(a) Graph $y = \sec x$



period =

domain:

range:

We forgot to discuss <u>amplitude</u>: This applies to $y = \sin x$, and $y = \cos x$ curves. Amplitude is affected by vertical stretch.

The typical (untransformed) amplitude of $y = \sin x$ and $y = \cos x$ is 1.

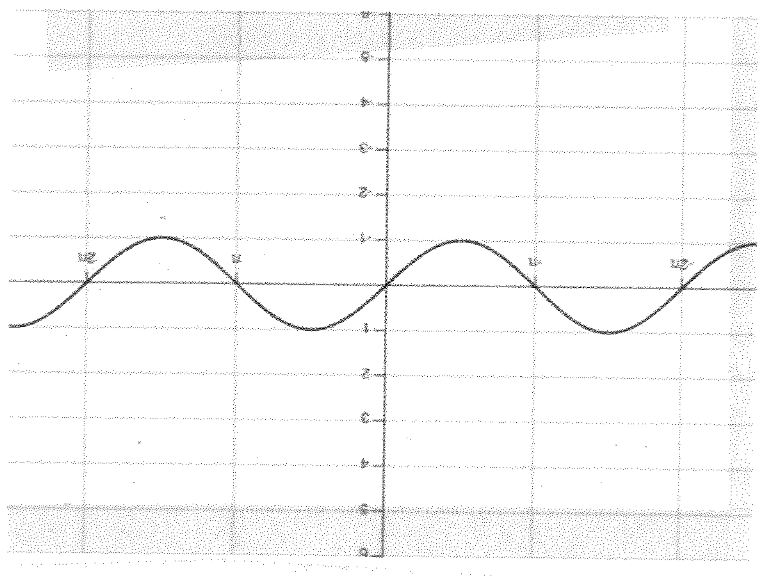
<u>ex</u> What is amplitude of

(a) $y = 3\cos(2x+1)$?

(b) $y = -\frac{1}{4}\sin(5x)$

4.4 (cont)

(b) Graph $y = \csc x$

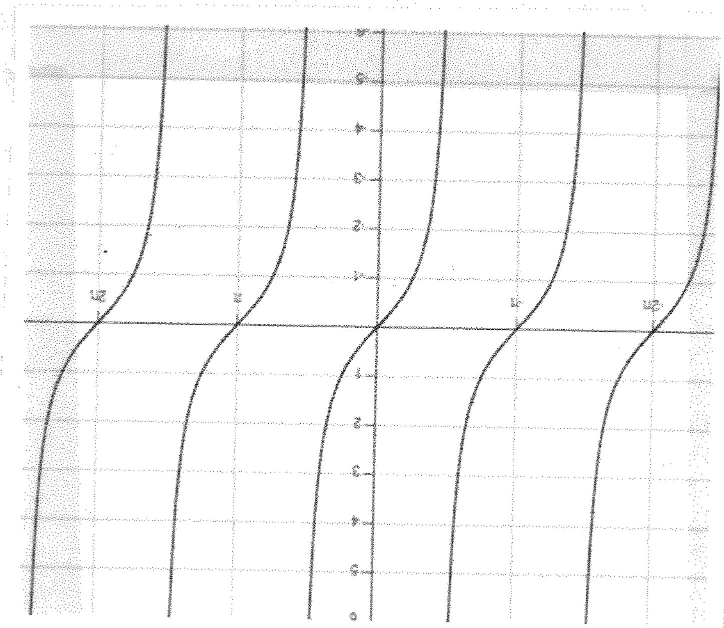


domain:

range:

period =

(c) Graph $y = \cot x$



domain:

range:

period =

4.4 (cont)

Ex2 Evaluate

(a) $\csc\left(\frac{-5\pi}{3}\right)$

(b) $\sec 0$

(c) $\cot\left(-\pi/3\right)$

(d) $\sec\left(\pi/4\right)$

Pythagorean Identities (Reminder)

① $\sin^2\theta + \cos^2\theta = 1$

② $1 + \tan^2\theta = \sec^2\theta$
$(\theta \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z})$

③ $\cot^2\theta + 1 = \csc^2\theta$
$(\theta \neq k\pi,\ k \in \mathbb{Z})$

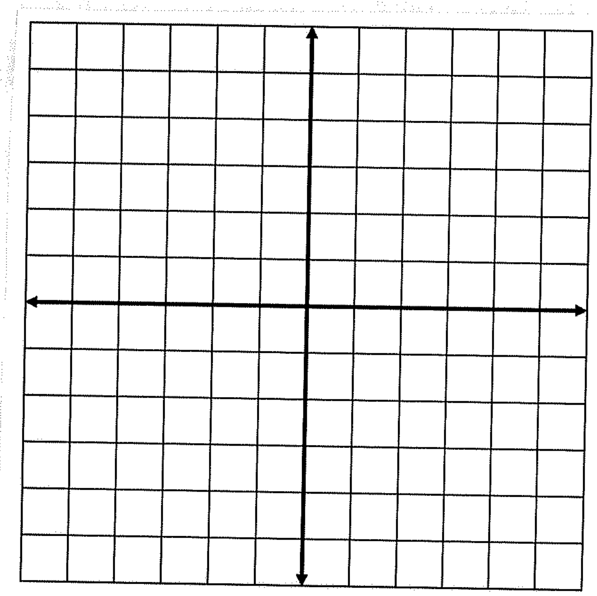
Ex3 Solve, for $\theta \in [0, 2\pi]$

(a) $\sec\theta = \frac{2}{\sqrt{3}}$

(b) $\csc\theta = -\sqrt{2}$

4.4 (cont)

Ex 4 Graph $y = 3\cot(4\theta - 2)$



Ex 5 Given $\csc\theta = \frac{-10}{3}$, θ in Q3, find all other five trig fn values of θ.

4.5 Inverse Trigonometric fns

☆☆☆ 4.55 4.56 4.57

Notation: $\arcsin x = \sin^{-1} x$

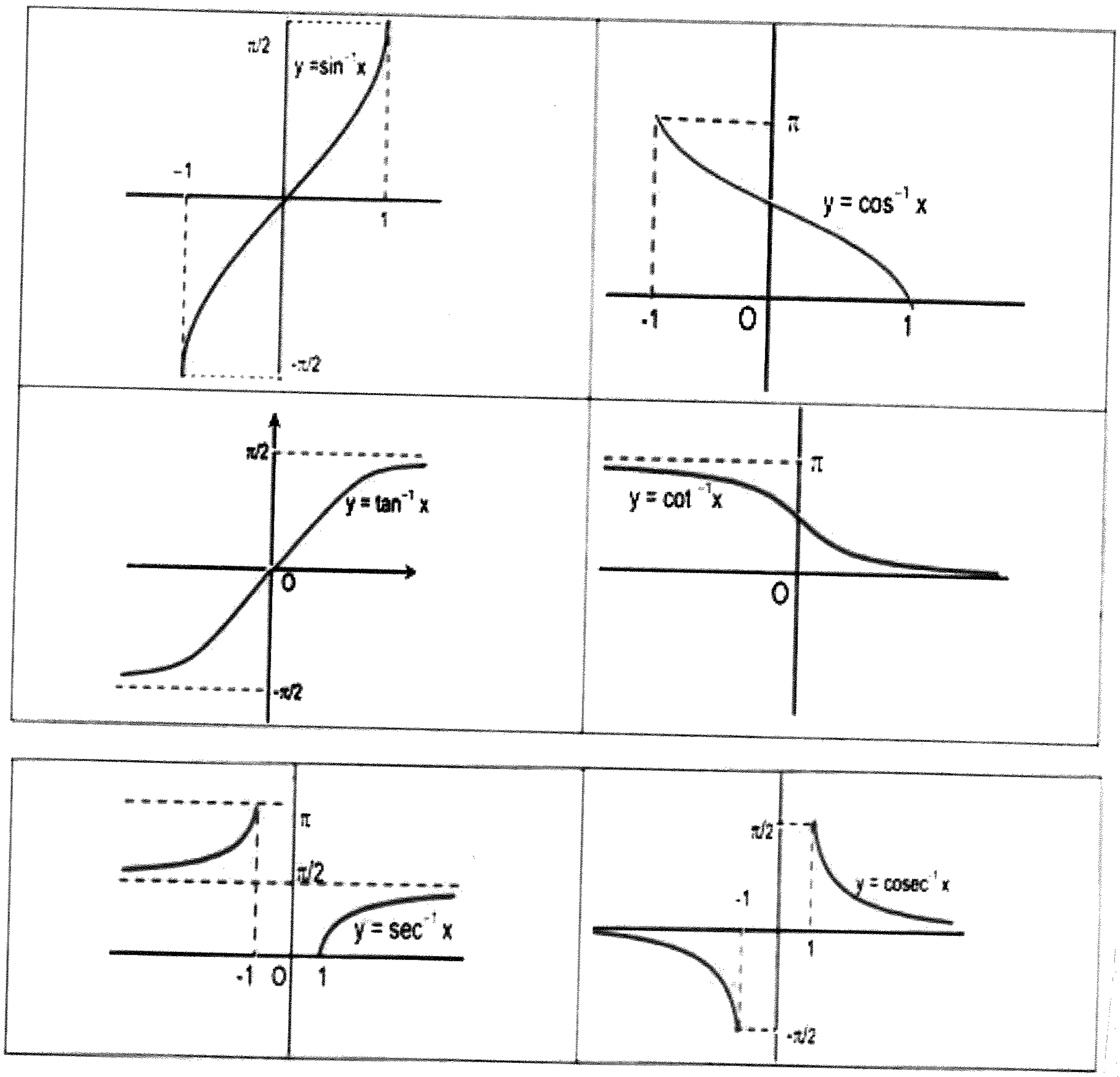
read as "arcsine of x or inverse sine of x or sine inverse of x"

(WARNING: $\sin^{-1} x \neq \frac{1}{\sin x}$)

We must restrict the domain of trig fns to define inverse fns!!!

	domain	range
$y = \sin x$	$[-\pi/2, \pi/2]$	
$y = \sin^{-1} x$		
$y = \cos x$	$[0, \pi]$	
$y = \cos^{-1} x$		
$y = \tan x$	$(-\pi/2, \pi/2)$	
$y = \tan^{-1} x$		
$y = \csc x$	$[-\pi/2, 0) \cup (0, \pi/2]$	
$y = \csc^{-1} x$		
$y = \sec x$	$[0, \pi/2) \cup (\pi/2, \pi]$	
$y = \sec^{-1} x$		
$y = \cot x$	$(0, \pi)$	
$y = \cot^{-1} x$		

4.5 (cont)

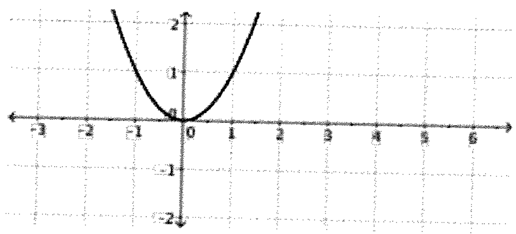

π/2
y = sin⁻¹ x
-1
1
-π/2
π
y = cos⁻¹ x
-1
O
1
π/2
y = tan⁻¹ x
O
-π/2
π
y = cot⁻¹x
O
π
π/2
y = sec⁻¹ x
-1
O
1
π/2
y = cosec⁻¹ x
-1
1
-π/2

Inverse Trig Functions

0. f(x) = ______

   Restrict domain to be 1-1: ____________

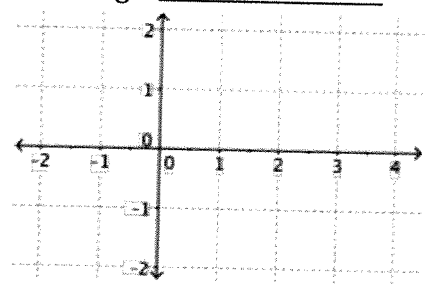
   Range of restricted domain: ____________



$f^{-1}(x)$ = ____________

Domain: ____________

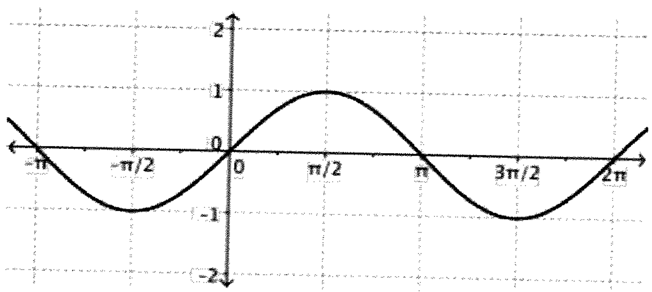
Range: ____________



1. f(x) = ______

   Restrict domain to be 1-1: ____________

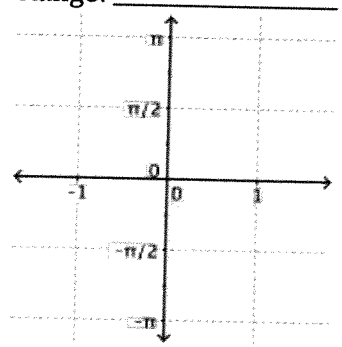
   Range of restricted domain: ____________



$f^{-1}(x)$ = ____________

Domain: ____________

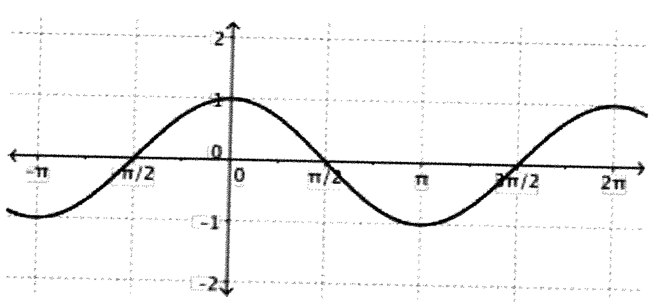
Range: ____________



2. f(x) = ______

   Restrict domain to be 1-1: ____________

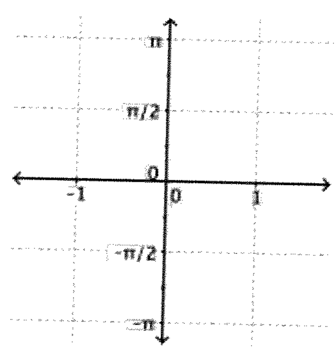
   Range of restricted domain: ____________



$f^{-1}(x)$ = ____________

Domain: ____________

Range: ____________



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3. f(x) = ________

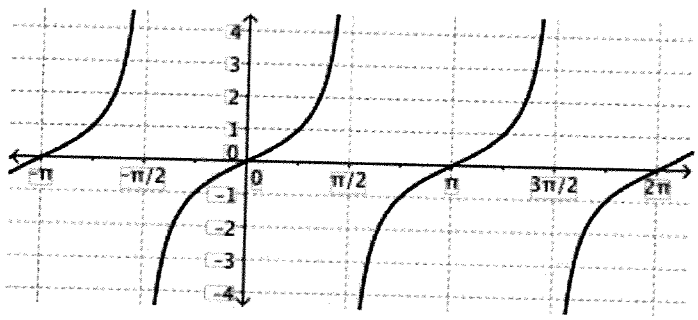
$f^{-1}(x)$ = ______________

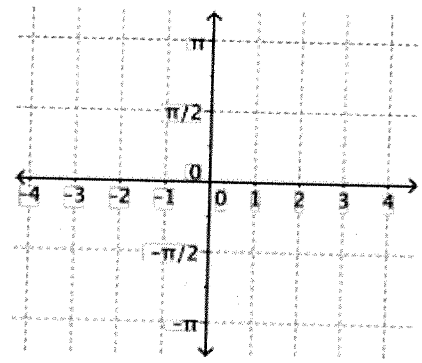
Restrict domain to be 1-1: ______________

Domain: ______________

Range of restricted domain: ____________

Range: ______________





4. f(x) = ________

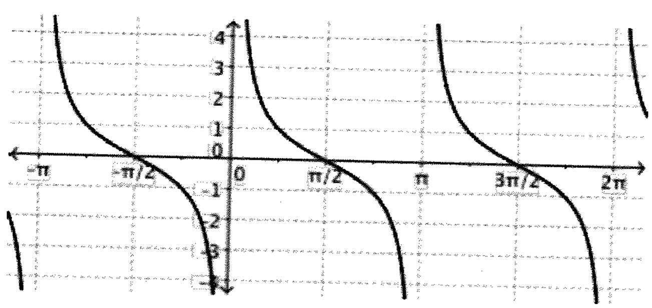
$f^{-1}(x)$ = ______________

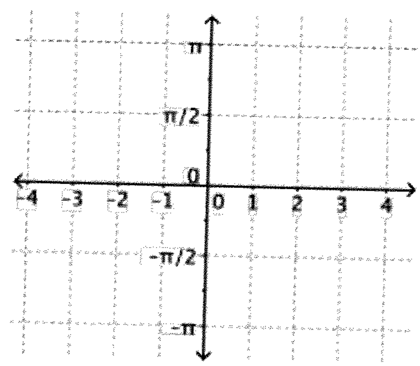
Restrict domain to be 1-1: ______________

Domain: ______________

Range of restricted domain: ____________

Range: ______________





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5. f(x) = ________

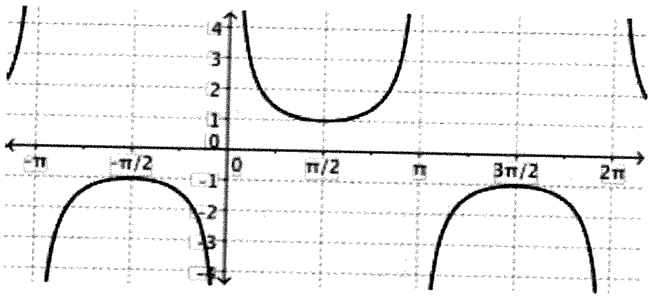
Restrict domain to be 1-1: ______________

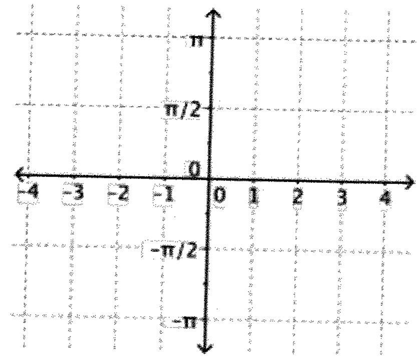
Range of restricted domain: ______________

$f^{-1}(x)$ = ______________

Domain: ______________

Range: ______________





6. f(x) = ________

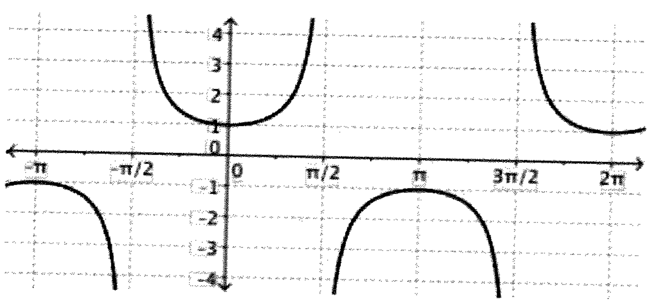
Restrict domain to be 1-1: ______________

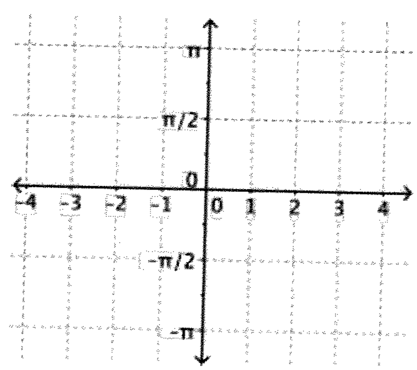
Range of restricted domain: ______________

$f^{-1}(x)$ = ______________

Domain: ______________

Range: ______________





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4.5 (cont)

Ex1 Evaluate

(a) $\arccos\left(\frac{\sqrt{3}}{2}\right)$

(b) $\operatorname{arccot}(1)$

(c) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(d) $\sin^{-1}(1)$

(e) $\cos^{-1}(0)$

(f) $\sin^{-1}(0)$

(g) $\arcsin\left(\sin\left(\frac{3\pi}{4}\right)\right)$

(h) $\arccos\left(\cos\left(\frac{\pi}{2}\right)\right)$

Ex2 Evaluate.

(a) $\sin(\sin^{-1} 0.76)$ (b) $\tan(\tan^{-1} 7)$

Ex3 Which of these (or both) are correct & why?

(a) $\sin(\sin^{-1} x) = x$

(b) $\sin^{-1}(\sin x) = x$

Ex4 Use a right triangle to verify

$$\tan(\arccos x) = \frac{\sqrt{1-x^2}}{x}, \quad 0 < |x| \leq 1$$

Ex5 Solve, for $x \in [0, 2\pi)$

(a) $4 - 2\sin x = 5$

(b) $4 - 2\sin(3x) = 5$

(c) $4 - 2\sin\left(\frac{1}{3}x\right) = 5$

(d) $\tan(2x - 1) = \sqrt{3}$

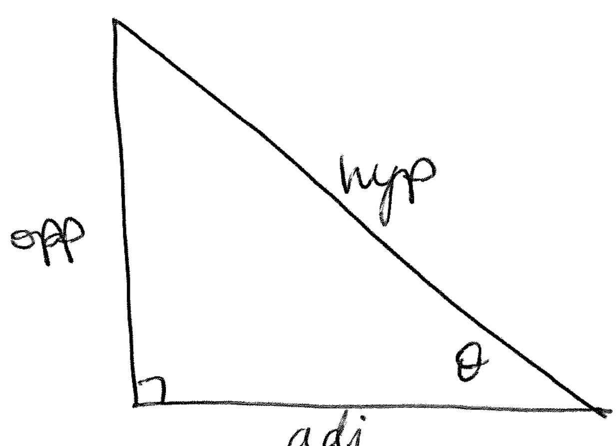
5.1 Right Triangle Trigonometry

$1° = \frac{1}{360}$ of a full circle counterclockwise rotation

1 minute $= 1'$,	conversion: $60' = 1°$
1 second $= 1''$,	conversion: $60'' = 1'$

Ex 1 $\quad 102° \, 24' \, 36''$ = how many degrees?

Remember Right Triangle Defns



$\sin\theta = \frac{opp}{hyp}$ $\qquad$ $\csc\theta = \frac{hyp}{opp}$

$\cos\theta = \frac{adj}{hyp}$ $\qquad$ $\sec\theta = \frac{hyp}{adj}$

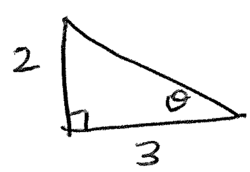
$\tan\theta = \frac{opp}{adj}$ $\qquad$ $\cot\theta = \frac{adj}{opp}$

5.1 (cont)

Ex 1 Given adj $= 4$, hyp $= 5$, find all six trig fns of θ.

Ex 2 Given $\sec\theta = \frac{3}{2}$, use right triangle to find all other five trig fns of θ. Assume $\theta \in$ QI.

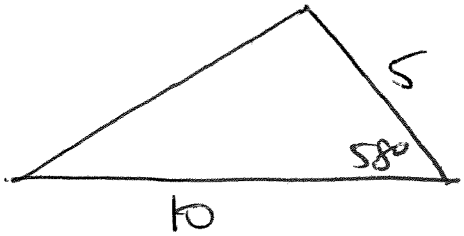
Ex 3 Given



Find expression (simplified) for θ.

5.1 (cont)

Ex 4 Find the area of this triangle.
(Give answer in simplified form, but exact.)



Ex 5 A 6-foot-6-inch-tall person casts a shadow that is 3 ft 9 in. long. Find the angle of elevation from the tip of the shadow to the sun.

5.2 Right Triangle and Unit Circle

<u>Defn</u>: "unit" right triangle means a right triangle w/ hypotenuse length of 1 unit.

Supplementary Angle Identities

$\sin(\pi-\theta)=\sin\theta$, $\cos(\pi-\theta)=-\cos\theta$, $\tan(\pi-\theta)=-\tan\theta$

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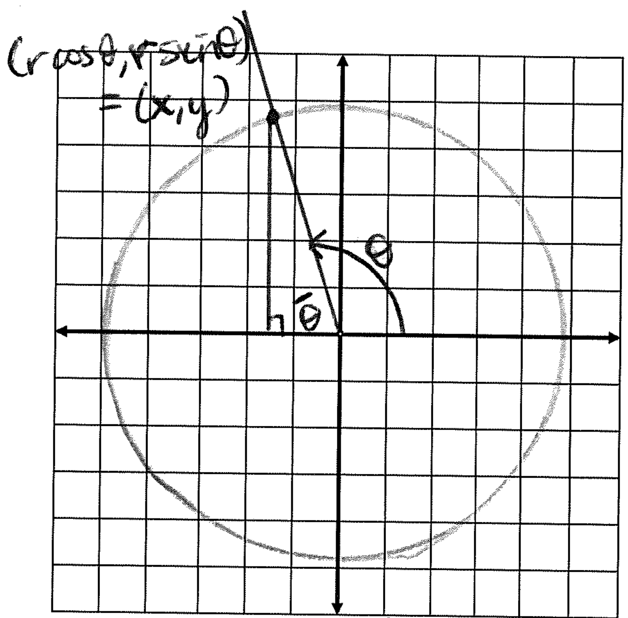
5.16
5.19



This is <u>NOT</u> a unit circle.

$$x^2+y^2=r^2$$

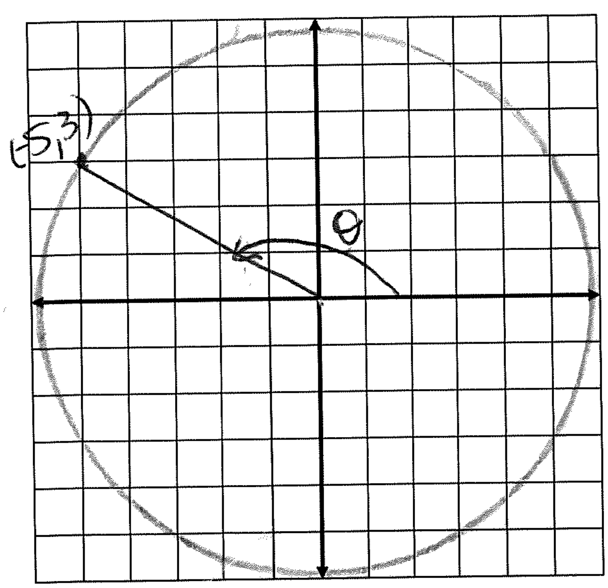
$$\sin\theta=\frac{y}{r}, \quad \cos\theta=\frac{x}{r}$$

<u>Ex1</u> Find $\sin\theta$, $\cos\theta$, $\tan\theta$ for the angle θ in standard position and whose terminal side goes through $(-2,-1)$.

5.2 (cont)

Ex 2 Find the side lengths (using similar triangles) for a unit right triangle similar to a rt. Δ. w/ lengths $2, 4, 2\sqrt{5}$.

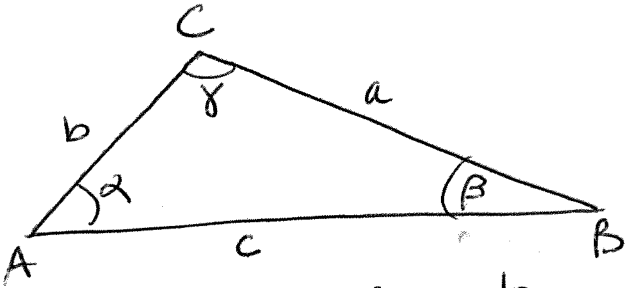
Ex 3 Find all six trig fns of θ as shown.



5.3 Law of Sines

5.23
5.25

Law of Sines

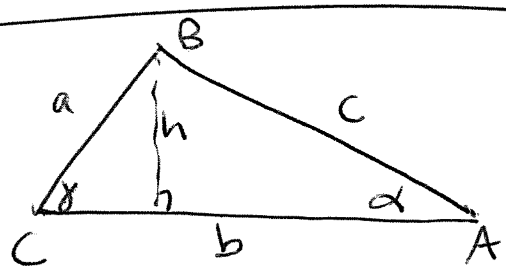


$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \iff \frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

We use this to "solve" triangles (meaning to find all six values $\alpha, \beta, \gamma, a, b, c$ for a triangle) in these cases:

① ASA/SAA:

Proof of Law of Sines:



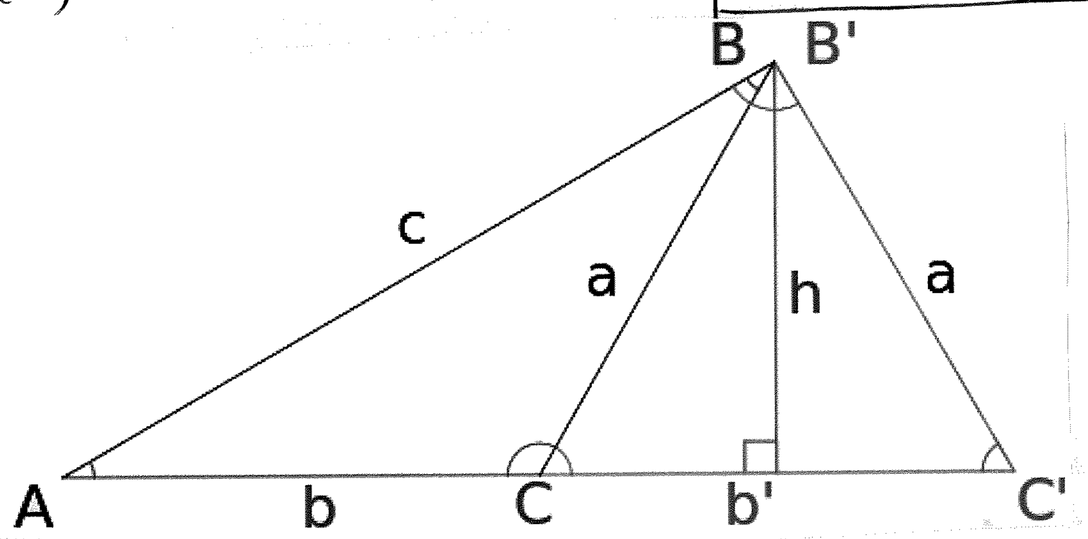
$\sin\gamma = \frac{h}{a}$, $\sin\alpha = \frac{h}{c}$

$\Rightarrow h = a\sin\gamma = c\sin\alpha$

$\Rightarrow \quad a\sin\gamma = c\sin\alpha$

$$\frac{\sin\gamma}{c} = \frac{\sin\alpha}{a} \qquad //$$

② SSA: (this only works sometimes)
(and I think it's easier to use law of cosines on this case)



(SSA ambiguous case)

5.3 (cont)

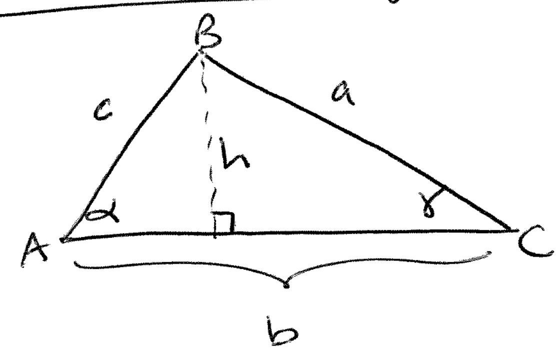
The Law of Sines	
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Use to find ANGLES	Use to find sides

Ex1 Solve these triangles. (if possible)

(a) $a=4$, $\alpha=47^\circ$, $\beta=65^\circ$

(b) $c=6$, $\alpha=35^\circ$, $\gamma=81^\circ$

Area of Oblique Triangle



area $= \frac{1}{2}bh$

but $\frac{h}{c} = \sin\alpha$

$\Rightarrow h = c\sin\alpha$

$\Rightarrow \boxed{\text{area} = \frac{1}{2}b(c\sin\alpha)}$

Similarly (using a different height)

$$\boxed{\begin{array}{l}\text{area} = \frac{1}{2}ac\sin\beta \\ \hline \text{area} = \frac{1}{2}ab\sin\gamma\end{array}}$$

5.3 (cont)

Ex 2. Find the area of the triangle, as given.

(a) $b=5$, $c=8$, $\alpha=46°$

(b) $b=3$, $c=5$, $\beta=36.7°$; $\gamma=95°$

Ex 3 Are these true or false? Why?

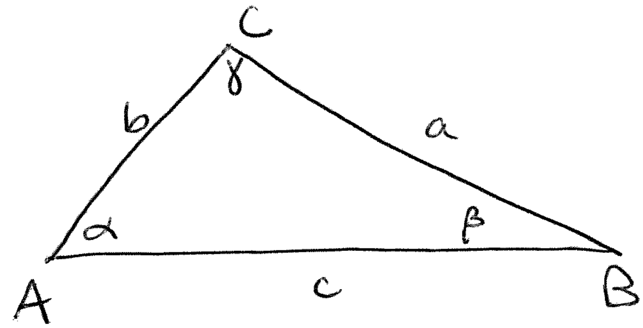
(a) An oblique triangle has 3 acute angles.

(b) An SSA triangle has a unique solution.

(c) An AAA triangle has infinitely many solutions.

5.4 Law of Cosines

Law of Cosines For Δ,



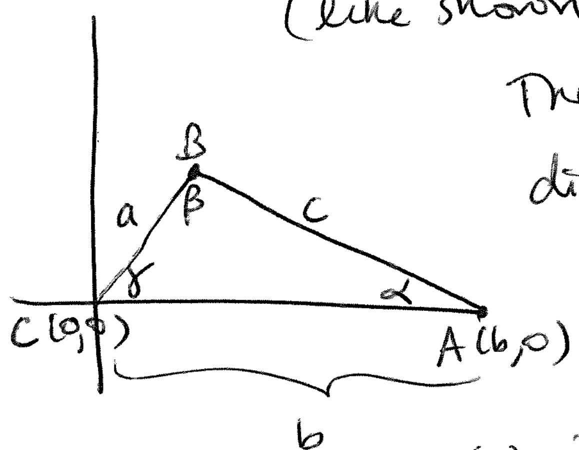
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$b^2 = a^2 + c^2 + 2ac\cos\beta$$

$$a^2 = b^2 + c^2 + 2cb\cos\alpha$$

Note: what happens if $\alpha = 90°$?

Proof Put ΔABC on a Cartesian Coord plane.
(like shown)



Then point $B = (a\cos\gamma, a\sin\gamma)$

distance from A to B:

$$c^2 = (a\cos\gamma - b)^2 + (a\sin\gamma - 0)^2$$

$$c^2 = a^2\cos^2\gamma - 2ab\cos\gamma + b^2 + a^2\sin^2\gamma$$

$$\Leftrightarrow c^2 = a^2(\cos^2\gamma + \sin^2\gamma) - 2ab\cos\gamma + b^2$$

$$\Leftrightarrow$$

//

Use Law of Cosines for these cases:

① SSS:

② SAS:

③ SSA*

* this is case that's not unique

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M1050

5.4 (cont)

Ex 1 Solve these triangles.

(a) $\alpha=12°$, $b=8$, $c=5$

(b) $a=8$, $b=12$, $c=9$

(c) $a=3$, $c=6$, $\alpha=142°$

(d) $a=3$, $c=6$, $\alpha=12°$

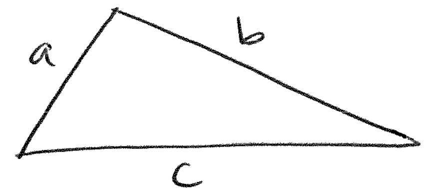
5.4 (cont)

Ex2 Find the area of triangle where $a=11$, $b=2$, $c=10$.

Heron's Area Formula

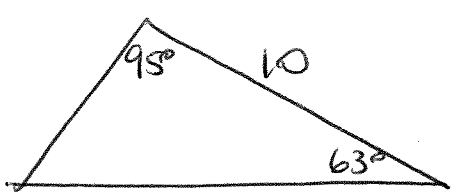
$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where
$$s = \tfrac{1}{2}(a+b+c)$$
(semi-perimeter)

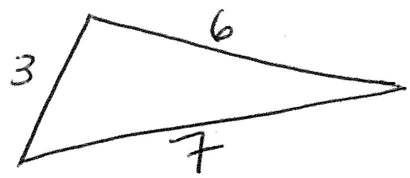


Ex3 Would you start w/ law of Cosines or law of Sines for these Δs?

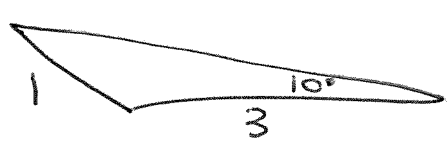
(a)



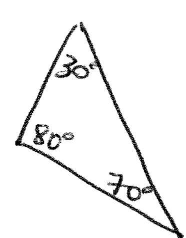
(b)



(c)



(d)



Ex 4 Can we create a Δ w/ sides 1, 3 and 1? Why or why not?

5.5 Applications of Triangles

Ex1 Simplify (i.e. rewrite as an equivalent algebraic expression) $\cos(\arcsin(2x))$.

Ex2 At a distance of 100 ft from the base of a power pole, the angle of elevation to the base is 8°. And the angle of elevation to the top is 55°. Find the height of the power pole.