5.6 Vectors

Vector: has both ① magnitude (length) and ② direction; denoted as $\vec{v}$

scalar: a $\mathbb{R}$ constant

zero vector: $\vec{0}$; has zero length and no direction (additive identity for vector addition)

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Vector Arithmetic

(i) addition/subtraction:

$\vec{v} = \langle a, b \rangle \quad \vec{w} = \langle c, d \rangle$

$\Rightarrow \vec{v} \pm \vec{w} = \langle a \pm c, b \pm d \rangle$

(ii) scalar multiplication:

$c \in \mathbb{R}, \quad \vec{v} = \langle a, b \rangle$

$\Rightarrow c\vec{v} = \langle ca, cb \rangle$

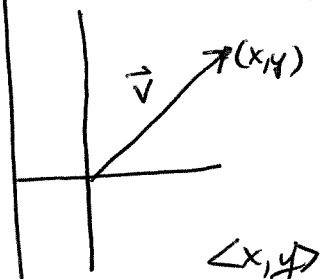
(iii) $\vec{w} = \vec{v} \iff a = c$ and $b = d$

(iv) magnitude $\|\vec{v}\| = \sqrt{a^2 + b^2}$

Notation

vector $\vec{v} = x\hat{i} + y\hat{j}$

$= \langle x, y \rangle$



standard position

$\langle x, y \rangle$ starts at origin and ends at pt (x, y)

unit vector: a vector of length 1; denoted as $\hat{u}$

Ex 1 Compute vector (in standard position) that starts at $(2, 5)$ and ends at $(6, -13)$. Sketch the vector.

5.6 (cont)

Ex2 Compute and sketch, given $\vec{w} = \langle 1, 3 \rangle$, $\vec{u} = \langle 5, 6 \rangle$, $\vec{v} = \langle -2, 7 \rangle$, $\vec{z} = \langle 3, -8 \rangle$.

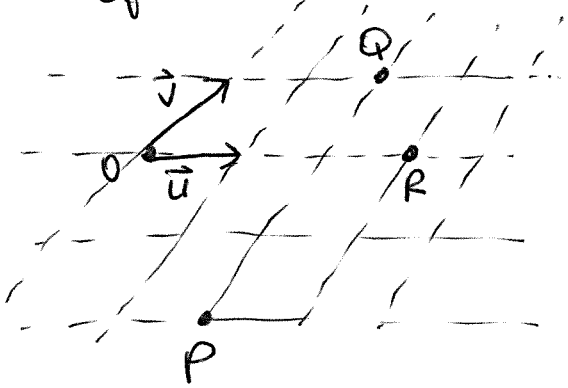
(a) $\vec{w} + 2\vec{u}$

(b) $3\vec{v} - \vec{z}$

(c) $\|\vec{w}\|$

(d) $\hat{\vec{z}}$

Ex3 Write each vector as sum of scalar multiples of $\vec{u}$ and $\vec{v}$.



(a) $\overrightarrow{PQ}$

(b) $\overrightarrow{PR}$

(c) $\overrightarrow{RQ}$

Ex4 Find component form for vector of length 20, pointing 45° below x-axis, in standard position.

Ex5 Find $\vec{u} \cdot \vec{v}$ for $\vec{u} = \langle -3, -5 \rangle$ and $\vec{v} = \langle 1, 4 \rangle$.

Notice: $\|\vec{u}\|^2 = \vec{u} \cdot \vec{u}$

or $\|\vec{u}\| = \sqrt{\vec{u} \cdot \vec{u}}$

Dot Product

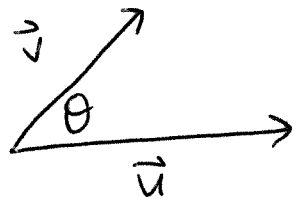
① If $\vec{u} = \langle u_1, u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$, then

$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$

(notice the dot product of 2 vectors produces a scalar)

② $\vec{u} \cdot \vec{v} = \|\vec{u}\| \, \|\vec{v}\| \cos\theta$

where θ is angle between $\vec{u}$ and $\vec{v}$



Ex 6 Find angle between

$\vec{u} = \langle -3, -2 \rangle$ and $\vec{v} = \langle 0, 4 \rangle$.