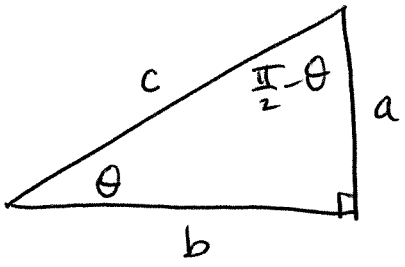
# 6.1 Fundamental Trigonometric Identities

An <u>identity</u> is an equation that's always true, for all allowable variable values.




notice:

$\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$ | $\csc\theta = \sec\left(\frac{\pi}{2}-\theta\right)$

$\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$ | $\sec\theta = \csc\left(\frac{\pi}{2}-\theta\right)$

$\tan\theta = \cot\left(\frac{\pi}{2}-\theta\right)$ | $\cot\theta = \tan\left(\frac{\pi}{2}-\theta\right)$

<u>Ex1</u> Verify

$$1+\sin x = \frac{\cos x}{\sec x - \tan x}$$

## How to Verify an Identity

① choose an expression (either the RHS (right-hand side) or LHS); usually I'll pick the more complicated-looking side

② Use algebra/trig. manipulation to change that expression to look like the other side.

- Keep trying!
- Do <u>NOT</u> "do the same thing to both sides" since we haven't proven yet that the identity is true!

6.1 (cont)

Ex 2 Verify

(a) $\csc x \sin^2 x + \frac{1}{\sec x} = \cos x \tan x + \sin\left(\frac{\pi}{2} - x\right)$

(b) $\frac{1-\cos\theta}{1+\cos\theta} = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

Typical things to try:

1. multiply by some form of 1
2. use Pythagorean Identities
3. add fractions by getting a common denominator

Ex 3 Prove $\dfrac{2\tan^2\theta + 2\tan\theta\sec\theta}{\tan\theta + \sec\theta - 1} = \tan\theta + \sec\theta + 1$

Ex 4 Prove or disprove $\cos(\alpha - \beta) = \cos\alpha - \cos\beta$.

Ex 5 Solve this equation

$3\cos^2 x - 2\cos x = 1 + \sin^2 x \qquad x \in [0, 2\pi)$

# 6.2 Sum, Difference and Double-Angle Identities

**Ex 1** Use identities to help you evaluate (w/o calculator).

(a) $\sin\left(\frac{23\pi}{12}\right)$

(b) $\tan\left(\frac{\pi}{12}\right)$

(c) $\cos^2\left(\frac{9\pi}{8}\right)$

(d) $\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$

## Sum & Difference Identities

$$\cos(\alpha \mp \beta) = \cos\alpha\cos\beta \pm \sin\alpha\sin\beta$$

$$\sin(\alpha \mp \beta) = \sin\alpha\cos\beta \mp \cos\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

## Double-Angle Identities

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$
$$= 2\cos^2\theta - 1$$

Ex2 Verify this identities

- $\dfrac{\cos\theta\tan(2\theta)+\sin\theta\tan(2\theta)}{\cos\theta} = \dfrac{2\tan\theta}{1-\tan\theta}$

Ex3 Solve these equations, for $\theta \in [0, 2\pi)$

(a) $\cos(2\theta) = -\sin^2\theta$

(b) $\sin(2\theta) = 2\cos\theta$

# 6.3 Power-Reducing, Half-Angle and Product-Sum Identities

**Ex1** Use Power Reducing Identities to rewrite $\tan^3 x$ until it has only powers of 1.

## Power-Reducing Identities

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

## Half-Angle Identities

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

* choose "plus or minus" (only one) depending on which quadrant $\theta$ is in.

**Ex2** Evaluate

(a) $\cot\left(\frac{5\pi}{12}\right)$

(b) $\sin\left(\frac{-5\pi}{8}\right)$

## Product-to-Sum Identities

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha-\beta)+\sin(\alpha+\beta)]$$

## Sum-to-Product Identities

$$\cos\alpha+\cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha-\cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha+\sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha-\sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

6.3 (cont)

Ex 3 rewrite as sum or difference

$\sin(4x)\cos(3x)$

Ex 4 rewrite as a product. $\cos(9x) - \cos(4x)$

Ex 5 Evaluate $\cos\left(\frac{\pi}{24}\right)$