7.4 DeMoivre's Theorem

Polar form of complex #: $z = a+bi = r(\cos\theta + i\sin\theta)$

☆☆☆ 7.33 where $r = |z| = \sqrt{a^2+b^2}$ (called the modulus of z)

and $\tan\theta = \frac{b}{a}$ (θ is called the argument of z)

Ex 1 Find polar form of $3+3i$.

Products & Quotients in Polar Form

If $w = r(\cos\alpha + i\sin\alpha)$, $z = s(\cos\beta + i\sin\beta)$, then

$$wz = rs(\cos(\alpha+\beta) + i\sin(\alpha+\beta))$$

and $$\frac{w}{z} = \frac{r}{s}(\cos(\alpha-\beta) + i\sin(\alpha-\beta))$$

$(z \neq 0)$

7.4 (cont)

Ex2 Given $w=-4i$ and $z=8\left(\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right)$

find (a) wz and (b) $\frac{w}{z}$

De Moivre's Thm For $n\in\mathbb{Z}$, $z=r(\cos\theta+i\sin\theta)$,

$\star\star\star$ we have $z^n=r^n(\cos(n\theta)+i\sin(n\theta))$

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Why? Think about $z^2=z\cdot z=r^2(\cos(\theta+\theta)+i\sin(\theta+\theta))$

$=r^2(\cos(2\theta)+i\sin(2\theta))$

$\Rightarrow z^3=z^2\cdot z=(r^2[\cos(2\theta)+i\sin(2\theta)])(r(\cos(\theta)+i\sin\theta))$

$=$

7.4 (cont)

Ex 3 If $z = 4\left(\cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)\right)$, find z^{10}.

Roots of Complex #s

If $z = r(\cos\theta + i\sin\theta)$, for $-\pi < \theta \leq \pi$, n^{th} roots of z are

$$w_k = \sqrt[n]{r}\left(\cos(\beta_k) + i\sin(\beta_k)\right) \quad \text{where } \beta_k = \frac{\theta}{n} + \frac{2\pi k}{n}$$

$k = 0, 1, \ldots, n-1$.

($k=0$ gives principal n^{th} root of z).

Ex 4 Find all the 4^{th} roots of 81.