CONIC SECTIONS (Chapter 8)

Conic Sections include parabolas, hyperbolas and ellipses (& circles)

All conic sections have the form

$$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$$

where A, B, C, D, E, F are constants and

$$A^2+B^2+C^2 \neq 0.$$

Parabolas

(1) $Ax^2+Dx+Ey+F=0$

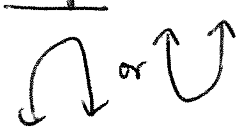
(B and $C=0$)

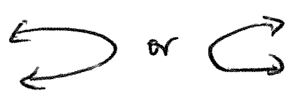
or

(2) $Cy^2+Dx+Ey+F=0$

(A and $B=0$)

shapes:

(1) 

(2) 

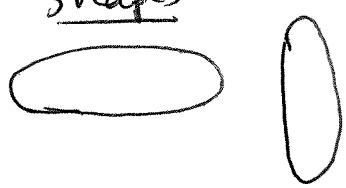
Ellipses

$Ax^2+Cy^2+Dx+Ey+F=0$

$(B=0)$

(A and C have same sign)

shapes



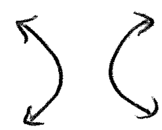
Hyperbolas

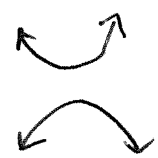
$Ax^2+Cy^2+Dx+Ey+F=0$

$(B=0)$

(A and C have different signs)

shapes

$A<0$ 

$A>0$ 

★ All of the above conics have horizontal and/or vertical symmetry. If $B \neq 0$, this rotates the conic.
$(B=0)$

ex 

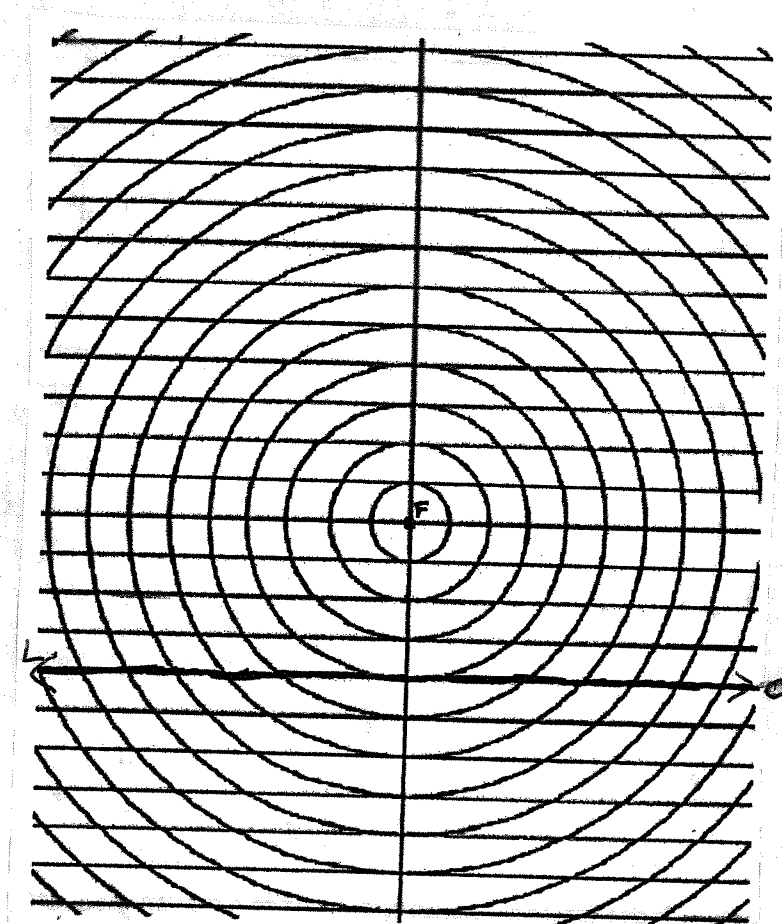
We will not cover this.

8.1 Parabolas

Ingredients: one point (called the focus) and a line (called the directrix)

Defn (Parabola): the set of all points from the given line (directrix) given pt (focus) and the given line (directrix)
F L

Ex 1 Given the focus, F, and directrix, L, that is 4 units from F, draw the parabola that they determine.



More Vocab

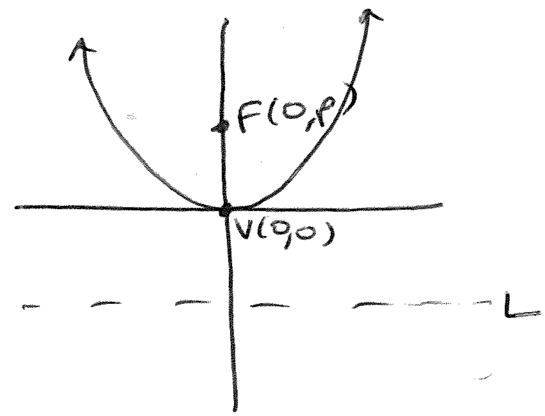
- Vertex is halfway between F and L
- We'll let p = distance from the vertex to the focus

$\Leftrightarrow$ $2p$ is the distance from focus to directrix

8.1 (cont)

Let (x,y) be any pt on the parabola.
Let F be at $(0,p)$, directrix at $y=-p$, $p\in\mathbb{R}^+$.
Then the parabola has vertex $(0,0)$ and opens upward.

Distance from (x,y) to F = distance from (x,y) to directrix



$$\Rightarrow \sqrt{(x-0)^2+(y-p)^2} = |y+p|$$

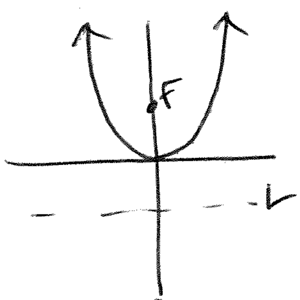
$$\left(\sqrt{x^2+(y-p)^2}\right)^2 = (|y+p|)^2$$

$$x^2+(y-p)^2 = (y+p)^2$$

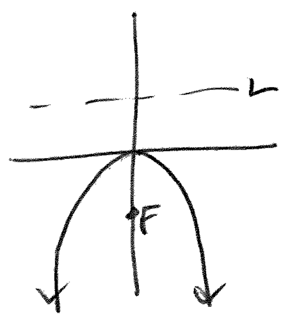
$$x^2+\cancel{y^2}-2py+\cancel{p^2} = \cancel{y^2}+2py+\cancel{p^2}$$

$$x^2 = 4py$$

Standard Form of Parabolas w/ vertex at (0,0)



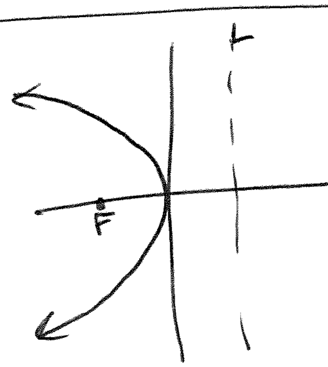
$F(0,p)$ focus $\boxed{p>0}$



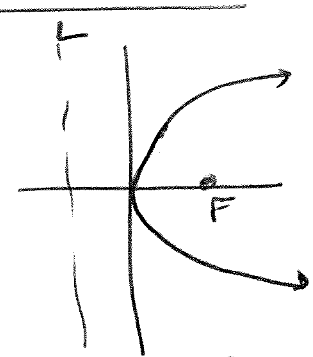
$F(0,p)$ focus $\boxed{p<0}$

eqn: $x^2=4py$

directrix: $y=-p$



$F(p,0)$ focus $\boxed{p<0}$

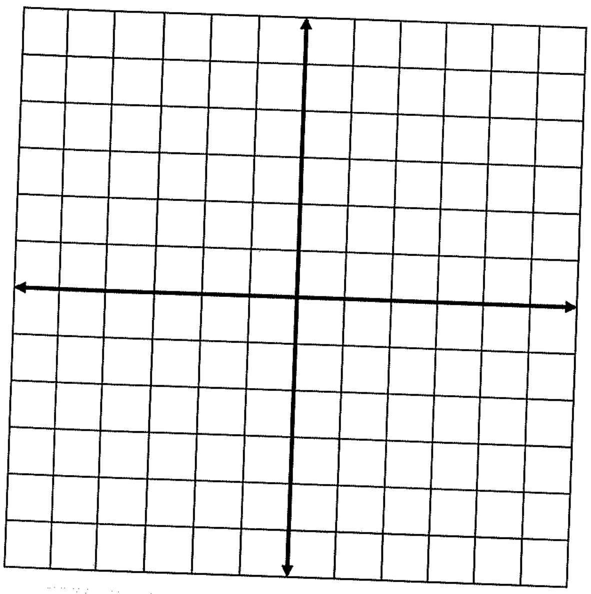


$F(p,0)$ focus $\boxed{p>0}$

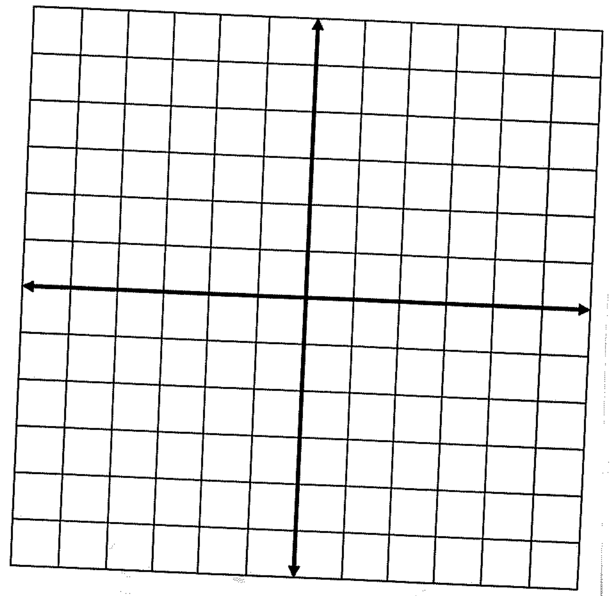
eqn $y^2=4px$

directrix: $x=-p$

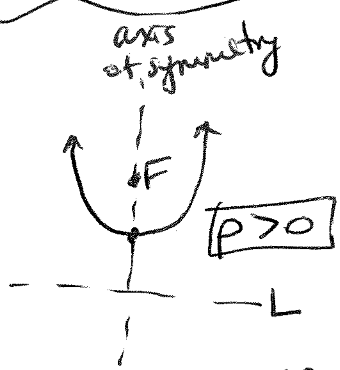
Ex 2 (a) Graph $2x^2 = -4y$

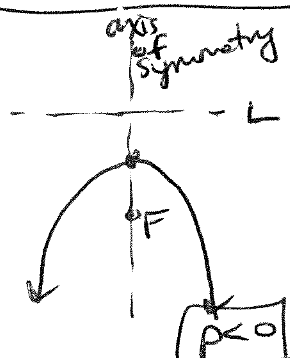


(b) Graph $3y^2 - 12x = 0$



We can transform these graphs like all the others. shifting left/right h and up/down k would move vertex from (0,0) to (h,k).





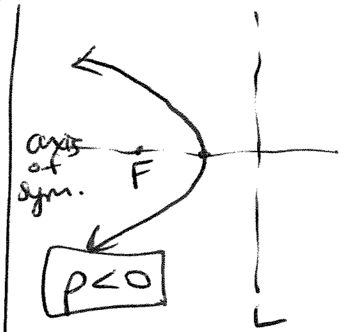
eqn: $(x-h)^2 = 4p(y-k)$

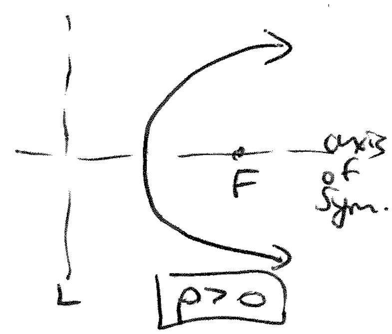
$F(h, k+p)$ focus

directrix: $y = k-p$

axis of symmetry: $x = h$

vertex (h,k)





eqn $(y-k)^2 = 4p(x-h)$

$F(h+c, k)$ focus

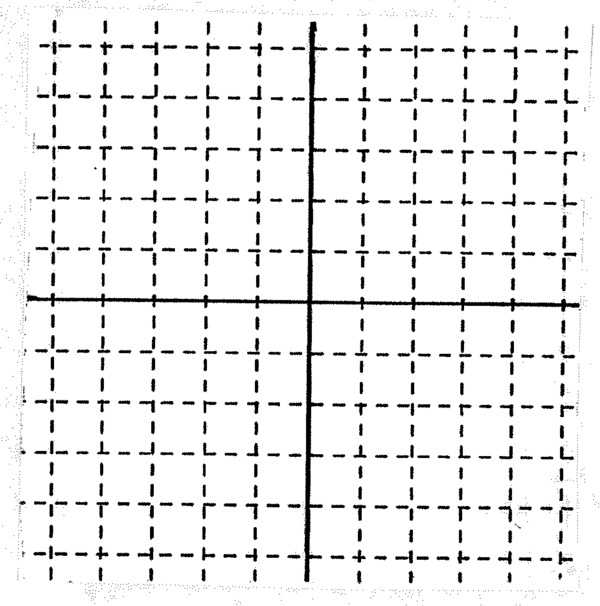
directrix: $x = h-p$

axis of symmetry: $y = k$

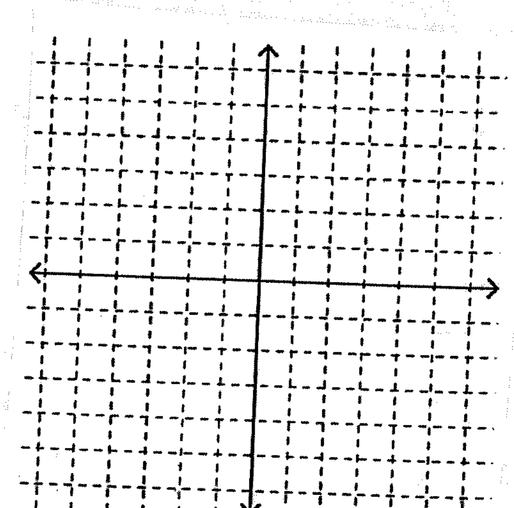
vertex (h,k)

8.1 (cont)

Ex 3 Graph $(x+2)^2 = 2(y-1)$



Ex4 Graph $y^2 - 6y = -4x-11$ (hint: complete the square)



8.1 (cont)

Ex5 Find the eqn of each parabola.

(a) w/ directrix at $y=-4$ and vertex at $(4,-1)$

(b) w/ vertex at $(4,2)$ passing through $(-3,-4)$; axis of symmetry parallel to x-axis