Appendix D: Matrices

Vocab/Notation

<u>matrix</u> (plural is matrices): an ordered array of numbers

- characterized by number of rows by number of columns as its <u>size</u>
- typically denoted by capital letters

<u>ex</u> $\begin{array}{l} 2x+3y=11 \\ -x+5y=7 \end{array} \iff \underbrace{\begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 11 \\ 7 \end{bmatrix}}_{B}$

$\iff AX=B$ (matrix eqn)

(A is called the <u>coefficient matrix</u>
X is called the <u>variable matrix</u>
B is called the <u>constant matrix</u>)

- size of A is 2×2, size of B is 2×1

- if # rows = # columns, then the matrix is <u>square</u>

- In above example, the <u>augmented matrix</u> would be $\left[\begin{array}{cc:c} 2 & 3 & 11 \\ -1 & 5 & 7 \end{array}\right]$ (it's the coefficient matrix augmented w/ the constant matrix)

- $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$ (notice subscripts that describe location)

App D (cont)

Ex1 (a) Write the augmented matrix for

$$\begin{aligned} x+y+z &= 10 \\ 3x+2y+z &= 7 \\ x \quad +5z &= 0 \end{aligned}$$

(b) What size is the matrix from part (a)?

Ex2 Perform elementary row operations to get a 1 in a_{11} place.

$$\left[\begin{array}{ccc|c} 5 & 20 & 15 & 6 \\ 7 & -5 & 3 & 2 \\ 12 & 0 & 1 & 4 \end{array}\right]$$

Elementary Row Operations

① Multiply any row by a nonzero constant.

② Exchange order of two rows.

③ Temporarily multiply one row by a nonzero constant and add to another row, replacing one of these rows by the sum.

☆ Using these is called Gauss-Jordan Elimination.

App D (cont)

Ex 3 Solve using Gauss-Jordan elimination.

(a)
$$
\begin{aligned}
5x + z &= 1 \\
x - 5z &= 21 \\
x + y - z &= 0
\end{aligned}
$$

(b)
$$
\begin{aligned}
x + 2y - z &= 3 \\
z - 3x - 6y &= -9 \\
4y + 2x - z &= 6
\end{aligned}
$$

App D (cont)

Ex 3 (c) $x+2y=9$

$x+y=5$

$3x-y=26$

(d) $2x+3y-z=5$

$x+5y-4y=2$

App D (cont)

Arithmetic w/ Matrices

① $A = B$ iff every entry in A = every corresponding entry in B.

② We can add/subtract matrices only if they are the same size. (add/subtract corresponding entries)

③ Multiplication: two types

(a) scalar multiplication: cA, means multiply each entry of A by c $(c \in \mathbb{R})$.

(b) matrix multiplication: AB, can only do this if # cols in A = # rows in B; if A is size $m \times n$ and B is size $n \times p$, then AB is size $m \times p$ (but BA is not doable)

* Note: There is no matrix division.

④ O is the zero matrix (filled w/ zeros) can be any size

⑤ I is the identity matrix; a square $(n \times n)$ matrix w/ zeros on the off-diagonal entries and ones on the diagonal entries.

ex $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

⑥ inverse matrix A^{-1} (read "A inverse") is the square matrix such that $A^{-1}A = AA^{-1} = I$ (where A is also square)

NOTE: $A^{-1} \neq \frac{1}{A}$

App D (cont)

Ex 4 For $A = \begin{bmatrix} 1 & 2 \\ 4 & 0 \\ -1 & 3 \\ 7 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 1 & 3 & 2 \\ 1 & 0 & -2 & 9 \end{bmatrix}$.

$C = \begin{bmatrix} 2 & 1 & 4 & 7 \\ 1 & -3 & 0 & 9 \end{bmatrix}$

(a) Find $-3A$.

(b) Find $2B + C$.

(c) Find AB (if possible).

(d) Find BA (if possible)

App D (cont)

Ex 5 Show that A and B are inverses.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & 1 \\ 3 & 6 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} -16 & -2 & 7 \\ 7 & 1 & -3 \\ -3 & 0 & 1 \end{bmatrix}$$

Ex 6 Find inverse of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (assuming it exists)

App D (cont)

Ex 7 Using formula from last example find A^{-1} for $A = \begin{bmatrix} 1 & 3 \\ -4 & 5 \end{bmatrix}$

Ex 8 Find A^{-1} for $\begin{bmatrix} 6 & 1 & 20 \\ 1 & -1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

App D (cont)

Ex 9 (a) Set up $\begin{aligned} 4x - 7y &= -65 \\ -x + 2y &= 18 \end{aligned}$ as $AX = B$.

(b) Find A^{-1}.

(c) Solve for $X = \begin{bmatrix} x \\ y \end{bmatrix}$

App D (cont)

Ex 10 Show $A = \begin{bmatrix} 6 & 9 \\ 2 & 3 \end{bmatrix}$ is not invertible.

Ex 11 Use the inverse from Ex 8 to solve

$$\begin{aligned} 6x + y + 20z &= 5 \\ x - y &= 2 \\ y + 3z &= -1 \end{aligned}$$