SELF TEST *All of the answers for this self test are given in the back of the book.*

1. For each given sequence,
i. Classify as arithmetic, geometric, both, or neither.
ii. If arithmetic, give d; if geometric, give r; if neither, state a pattern using your own words.
iii. Supply the next term.
iv. State the general term (if possible).
a. $6, 11, 16, \ldots$
b. $35, 46, 57, \ldots$
c. $1, \frac{1}{2}, \frac{1}{4}, \ldots$
d. $4, 6, 10, 16, \ldots$
e. $3, -1, \frac{1}{3}, -\frac{1}{9}, \ldots$

2. **i.** State the first four terms of the given sequence.
ii. Either find the limit or show that the sequence diverges.

a. $\left\{\frac{e^n}{n!}\right\}$

b. $\left\{\frac{3n^2-n+1}{(1-2n)n}\right\}$

c. $\left\{\left(1+\frac{1}{n}\right)^n\right\}$

3. **a.** Find the sum of the first ten terms of the sequence
$$a+b,\ a^2+ab,\ a^3+a^2b,\ldots$$
b. Find the sum of the first ten terms of the sequence
$$P,\ P(1+r),\ P(1+r)^2,\ P(1+r)^3,\ldots$$
c. Find the sum of the infinite series 2,000
$$1{,}000+500+250+\cdots$$
d. Find the sum of the infinite series
$$\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\cdots$$

4. Evaluate:

a. $\sum_{k=0}^{5} 3^k$ **b.** $\sum_{k=1}^{8} 3$

c. $\sum_{k=1}^{10} 2(3)^{k-1}$ **d.** $\sum_{k=1}^{100} [5+(k-1)4]$

Solve the systems in Problems 5–9 by ~~the indicated~~ any method.

5. By graphing: $\begin{cases} 2x-y=2 \\ 3x-2y=1 \end{cases}$

6. By addition: $\begin{cases} x+3y=3 \\ 4x-6y=-6 \end{cases}$

7. By substitution: $\begin{cases} x+y=1 \\ x^2+y^2=13 \end{cases}$

8. By Gauss–Jordan: $\begin{cases} 2x+y=1 \\ x+z=1 \\ 2x-y=1 \end{cases}$

9. By using an inverse matrix: $\begin{cases} 2x+7y=13 \\ x+4y=8 \end{cases}$

10. Graph the solution of the system: $\begin{cases} 2x-y+2\le 0 \\ 2x-y+12\ge 0 \\ 3x+2y+10>0 \\ 3x+2y-18<0 \end{cases}$

STOP **STUDY HINTS** *Compare your solutions and answers to the self test.*

Practice for Calculus—*Supplementary Problems: Cumulative Review Chapters 1–8*

***Simplify** the expressions in Problems 1–4.*

1. $\dfrac{(3-x^2)(4)-(4x-7)(-2x)}{(3-x^2)^2}$

2. $\dfrac{(2x^2+x-3)(6x)-(3x^2+5)(4x+1)}{(2x^2+x-3)^2}$

3. $\dfrac{\frac{1}{2}t^{-1/2}\cos t-t^{1/2}(-\sin t)}{\cos^2 t}$

4. $\frac{1}{4}\left(\frac{x}{1-3x}\right)^{-3/4}\left[\frac{(1-3x)(1)-x(-3)}{(1-3x)^2}\right]$

***Factor** the expressions in Problems 5–8.*

5. $-24x^{-4}y^4+32x^{-3}y^3$
6. $2(2x+1)(5x^2+4)+(2x+1)^2(10x)$
7. $16\tan^3 x-2$
8. $4\cos^2 x\sin x+4\cos x\sin x-3\sin x$

***Evaluate** the expressions in Problems 9–12.*

9. $f(x)=\dfrac{x-1}{x+1}$ for $x=0.999$

10. $|x^2-\pi^2|$ for $x=\pi$

11. $\lim\limits_{n\to\infty}\left(\dfrac{2n^2+5n-7}{n^3}\right)$

12. $\dfrac{8}{\sqrt{3}}\left[\dfrac{u(u^2+1)^{3/2}}{4}-\dfrac{u\sqrt{u^2+1}}{8}\right]$ for $u=\sqrt{3x}$

Solve the equations in Problems 13–20.

13. $\dfrac{x(9x^2-2)}{\sqrt{3x^2-1}}=0$

14. $\sqrt{2x-1}+\sqrt{x+4}=6$

15. $2\sin^2 x-2\cos^2 x=1$

16. $I=\frac{E}{R}(1-e^{-Rt/L})$ for L

17. **a.** $\left(\frac{1}{2}\right)^x=\frac{1}{8}$ **b.** $\left(\frac{1}{2}\right)^x=8$

18. **a.** $8^x=300$ **b.** $2^x=1{,}000$

19. $6^{3x+2}=200$

20. $\ln 2+\frac{1}{2}\ln 3+4\ln 5=\ln x$

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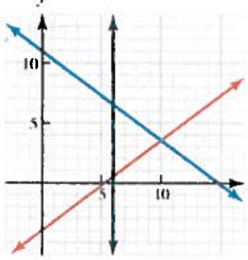

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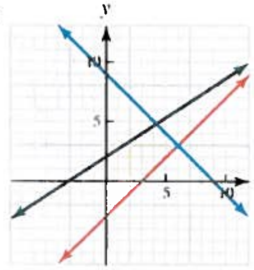

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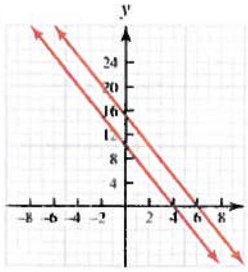

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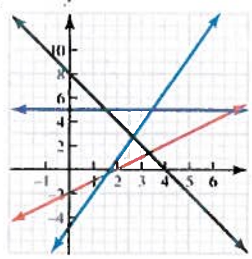

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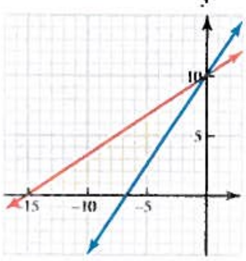

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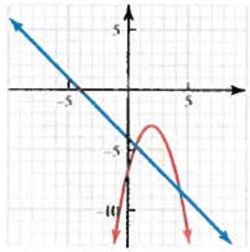

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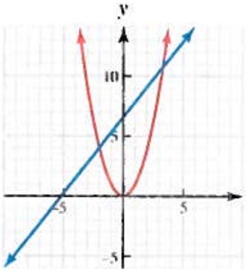

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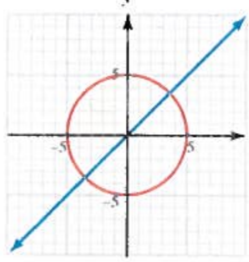

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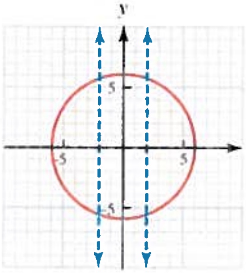

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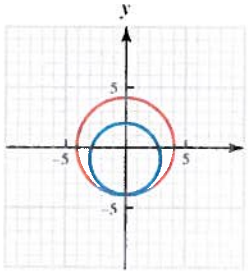

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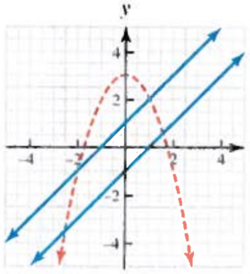

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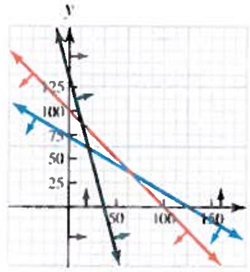

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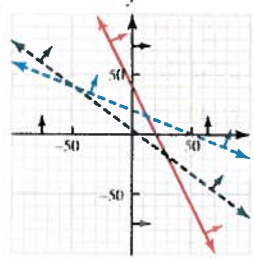

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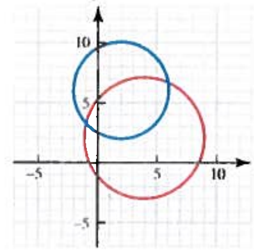

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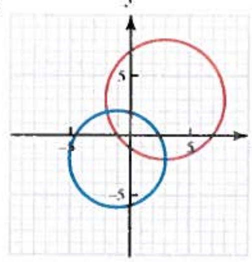

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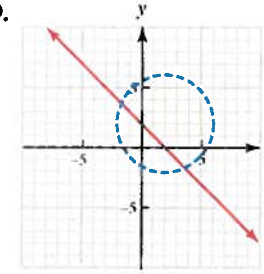
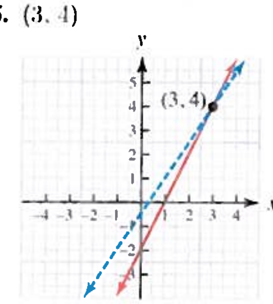
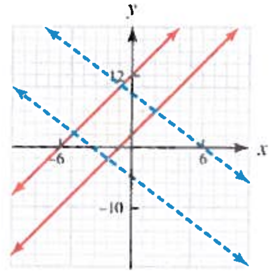


Chapter 8 Self-Test, page 578

1. **a.** arithmetic; $d = 5$; 21; $a_n = 1 + 5n$
   b. arithmetic; $d = 11$; 68; $a_n = 24 + 11n$
   c. geometric; $r = \frac{1}{2}$; $\frac{1}{8}$; $g_n = 2\left(\frac{1}{2}\right)^n$
   d. neither; add 2, then add 4, add 6, add 8, and so on; 24
   e. geometric; $r = -\frac{1}{3}$; $\frac{1}{27}$; $g_n = 3\left(-\frac{1}{3}\right)^{n-1}$
2. **a.** $e, \frac{1}{2}e^2, \frac{1}{6}e^3, \frac{1}{24}e^4$; $\lim_{n\to\infty} \frac{e^n}{n!} = 0$
   b. $-3, -\frac{11}{7}, -\frac{5}{3}, -\frac{15}{7}$; $\lim_{n\to\infty} \frac{3n^2 - n + 1}{(1-2n)n} = -\frac{3}{2}$
   c. $2, \frac{9}{4}, \frac{64}{27}, \frac{625}{256}$; $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$
3. **a.** $G_{10} = \frac{(a+b)(1-a^{10})}{1-a}$ **b.** $G_{10} = \frac{P[1-(1-r)^{10}]}{1-P}$
   c. 2,000 **d.** $\frac{1}{4}$
4. **a.** 364 **b.** 24
   c. 59,048 **d.** 20,300
5. (3, 4) 6. (0, 1)



7. (−2, 3), (3, −2) 8. $\left(\frac{1}{2}, 0, \frac{1}{2}\right)$
9. (−4, 3)
10.



7 Chapter Test

Take this test as you would take a test in class. When you are finished, check your work against the answers given in the back of the book.

In Exercises 1–3, solve the system by the method of substitution.

1. $\begin{cases} x - y = -7 \\ 4x + 5y = 8 \end{cases}$ **2.** $\begin{cases} y = x - 1 \\ y = (x-1)^3 \end{cases}$ **3.** $\begin{cases} 2x - y^2 = 0 \\ x - y = 4 \end{cases}$

In Exercises 4–6, solve the system graphically.

4. $\begin{cases} 2x - 3y = 0 \\ 2x + 3y = 12 \end{cases}$ **5.** $\begin{cases} y = 9 - x^2 \\ y = x + 3 \end{cases}$ **6.** $\begin{cases} y - \ln x = 12 \\ 7x - 2y + 11 = -6 \end{cases}$

In Exercises 7–10, solve the linear system by the method of elimination.

7. $\begin{cases} 2x + 3y = 17 \\ 5x - 4y = -15 \end{cases}$ **8.** $\begin{cases} 2.5x - y = 6 \\ 3x + 4y = 2 \end{cases}$

9. $\begin{cases} x - 2y + 3z = 11 \\ 2x - z = 3 \\ 3y + z = -8 \end{cases}$ **10.** $\begin{cases} 3x + 2y + z = 17 \\ -x + y + z = 4 \\ x - y - z = 3 \end{cases}$

In Exercises 11–14, write the partial fraction decomposition of the rational expression.

11. $\dfrac{2x + 5}{x^2 - x - 2}$ **12.** $\dfrac{3x^2 - 2x + 4}{x^2(2 - x)}$ **13.** $\dfrac{x^2 + 5}{x^3 - x}$ **14.** $\dfrac{x^2 - 4}{x^3 + 2x}$

In Exercises 15–17, sketch the graph and label the vertices of the solution of the system of inequalities.

15. $\begin{cases} 2x + y \le 4 \\ 2x - y \ge 0 \\ x \ge 0 \end{cases}$ **16.** $\begin{cases} y < -x^2 + x + 4 \\ y > 4x \end{cases}$ **17.** $\begin{cases} x^2 + y^2 \le 16 \\ x \ge 1 \\ y \ge -3 \end{cases}$

18. Find the maximum and minimum values of the objective function $z = 20x + 12y$ and where they occur, subject to the following constraints.

$$\left.\begin{aligned} x &\ge 0 \\ y &\ge 0 \\ x + 4y &\le 32 \\ 3x + 2y &\le 36 \end{aligned}\right\} \text{ Constraints}$$

19. A total of $50,000 is invested in two funds paying 8% and 8.5% simple interest. The yearly interest is $4150. How much is invested at each rate?

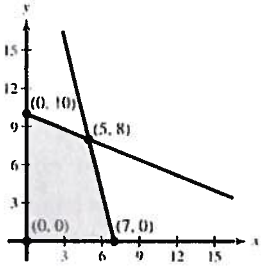
20. Find the equation of the parabola $y = ax^2 + bx + c$ passing through the points $(0, 6)$, $(-2, 2)$, and $\left(3, \frac{9}{2}\right)$.

21. A manufacturer produces two types of television stands. The amounts (in hours) of time for assembling, staining, and packaging the two models are shown in the table at the left. The total amounts of time available for assembling, staining, and packaging are 4000, 8950, and 2650 hours, respectively. The profits per unit are $30 (model I) and $40 (model II). What is the optimal inventory level for each model? What is the optimal profit?

	Model I	Model II
Assembling	0.5	0.75
Staining	2.0	1.5
Packaging	0.5	0.5

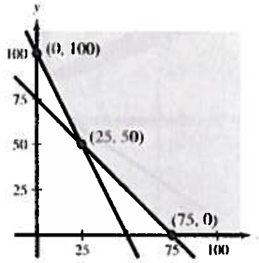
TABLE FOR 21

77.



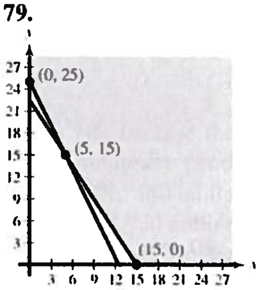
Minimum at (0, 0): 0
Maximum at (5, 8): 47

78.



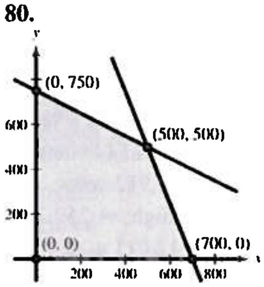
Minimum at (25, 50): 600
No maximum

79.



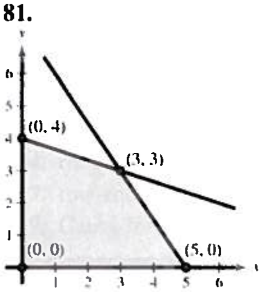
Minimum at (15, 0): 26.25
No maximum

80.



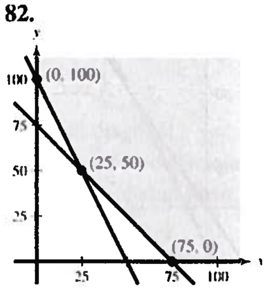
Minimum at (0, 0): 0
Maximum at (500, 500): 60,000

81.



Minimum at (0, 0): 0
Maximum at (3, 3): 48

82.



Minimum at (7, 0): −14
No maximum

83. 72 haircuts, 0 permanents; Optimal revenue: $1800

84. 5 walking shoes
2 running shoes
Optimal profit: $138

85. Three bags of brand X
Two bags of brand Y
Optimal cost: $105

86. $\left(\frac{2}{3}\right)$ regular unleaded gasoline
$\left(\frac{1}{3}\right)$ premium unleaded gasoline
Optimal cost: $1.70

87. False. To represent a region covered by an isosceles trapezoid, the last two inequality signs should be ≤.

88. False. An objective function can have no maximum value, one point where a maximum value occurs, or an infinite number of points where a maximum value occurs.

89. $\begin{cases} x + y = 2 \\ x - y = -14 \end{cases}$ **90.** $\begin{cases} x - y = 9 \\ 3x + y = 11 \end{cases}$

91. $\begin{cases} 3x + y = 7 \\ -6x + 3y = 1 \end{cases}$ **92.** $\begin{cases} -x + 4y = 10 \\ 3x - 8y = -21 \end{cases}$

93. $\begin{cases} x + y + z = 6 \\ x + y - z = 0 \\ x - y - z = 2 \end{cases}$ **94.** $\begin{cases} x - 2y + z = -7 \\ 2x + y - 4z = -25 \\ -x + 3y - z = 12 \end{cases}$

95. $\begin{cases} 2x + 2y - 3z = 7 \\ x - 2y + z = 4 \\ -x + 4y - z = -1 \end{cases}$ **96.** $\begin{cases} 4x + y - z = -7 \\ 8x + 3y + 2z = 16 \\ 4x - 2y + 3z = 31 \end{cases}$

97. An inconsistent system of linear equations has no solution.

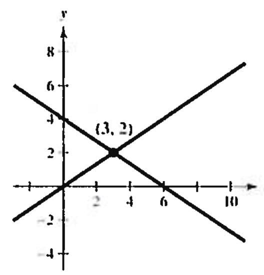
98. The lines are distinct and parallel.
$\begin{cases} x + 2y = 3 \\ 2x + 4y = 9 \end{cases}$

99. Answers will vary.

Chapter Test *(page 567)*

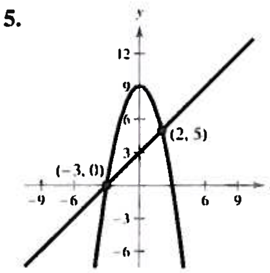
1. (−3, 4) **2.** (0, −1), (1, 0), (2, 1)
3. (8, 4), (2, −2)

4.



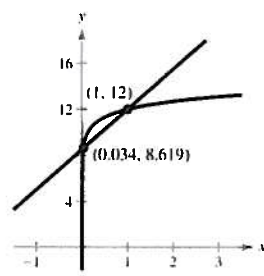
(3, 2)

5.



(−3, 0), (2, 5)

6.



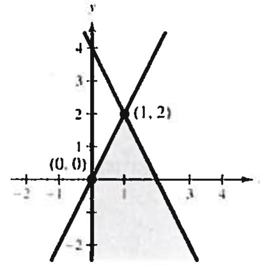
(1, 12), (0.034, 8.619)

7. (1, 5) **8.** (2, −1)
9. (2, −3, 1) **10.** No solution

11. $-\dfrac{1}{x+1} + \dfrac{3}{x-2}$ **12.** $\dfrac{2}{x^2} + \dfrac{3}{2-x}$

13. $-\dfrac{5}{x} + \dfrac{3}{x+1} + \dfrac{3}{x-1}$ **14.** $-\dfrac{2}{x} + \dfrac{3x}{x^2+2}$

15.



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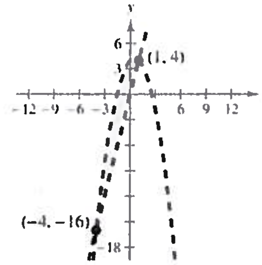


CHAPTER 7

8 Chapter Test

Take this test as you would take a test in class. When you are finished, check your work against the answers given in the back of the book.

In Exercises 1 and 2, write the matrix in reduced row-echelon form.

1. $\begin{bmatrix} 1 & -1 & 5 \\ 6 & 2 & 3 \\ 5 & 3 & -3 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 0 & -1 & 2 \\ -1 & 1 & 1 & -3 \\ 1 & 1 & -1 & 1 \\ 3 & 2 & -3 & 4 \end{bmatrix}$

3. Write the augmented matrix corresponding to the system of equations and solve the system.

$$\begin{cases} 4x + 3y - 2z = 14 \\ -x - y + 2z = -5 \\ 3x + y - 4z = 8 \end{cases}$$

4. Find (a) $A - B$, (b) $3A$, (c) $3A - 2B$, and (d) AB (if possible).

$$A = \begin{bmatrix} 5 & 4 \\ -4 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -1 \\ -4 & 0 \end{bmatrix}$$

In Exercises 5 and 6, find the inverse of the matrix (if it exists).

5. $\begin{bmatrix} -6 & 4 \\ 10 & -5 \end{bmatrix}$

6. $\begin{bmatrix} -2 & 4 & -6 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{bmatrix}$

7. Use the result of Exercise 5 to solve the system.

$$\begin{cases} -6x + 4y = 10 \\ 10x - 5y = 20 \end{cases}$$

In Exercises 8–10, evaluate the determinant of the matrix.

8. $\begin{bmatrix} -9 & 4 \\ 13 & 16 \end{bmatrix}$

9. $\begin{bmatrix} \frac{5}{2} & \frac{13}{4} \\ -8 & \frac{6}{5} \end{bmatrix}$

10. $\begin{bmatrix} 6 & -7 & 2 \\ 3 & -2 & 0 \\ 1 & 5 & 1 \end{bmatrix}$

In Exercises 11 and 12, use Cramer's Rule to solve (if possible) the system of equations.

11. $\begin{cases} 7x + 6y = 9 \\ -2x - 11y = -49 \end{cases}$

12. $\begin{cases} 6x - y + 2z = -4 \\ -2x + 3y - z = 10 \\ 4x - 4y + z = -18 \end{cases}$

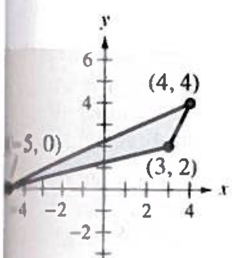


FIGURE FOR 13

13. Use a determinant to find the area of the triangle in the figure.

14. Find the uncoded 1×3 row matrices for the message KNOCK ON WOOD. Then encode the message using the matrix A below.

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$$

15. One hundred liters of a 50% solution is obtained by mixing a 60% solution with a 20% solution. How many liters of each solution must be used to obtain the desired mixture?

83. (36, 11) **84.** (2, −3) **85.** (−6, −1)
86. (−2, 1) **87.** (2, −1, −2) **88.** (−2, 4, 3)
89. (6, 1, −1) **90.** (−3, 5, 0) **91.** (−3, 1)
92. (5, 6) **93.** (1, 1, −2) **94.** (4, −2, 1)
95. −42 **96.** −41 **97.** 550 **98.** 78
99. (a) $M_{11} = 4, M_{12} = 7, M_{21} = -1, M_{22} = 2$
(b) $C_{11} = 4, C_{12} = -7, C_{21} = 1, C_{22} = 2$
100. (a) $M_{11} = -4, M_{12} = 5, M_{21} = 6, M_{22} = 3$
(b) $C_{11} = -4, C_{12} = -5, C_{21} = -6, C_{22} = 3$
101. (a) $M_{11} = 30, M_{12} = -12, M_{13} = -21,$
$M_{21} = 20, M_{22} = 19, M_{23} = 22, M_{31} = 5,$
$M_{32} = -2, M_{33} = 19$
(b) $C_{11} = 30, C_{12} = 12, C_{13} = -21,$
$C_{21} = -20, C_{22} = 19, C_{23} = -22,$
$C_{31} = 5, C_{32} = 2, C_{33} = 19$
102. (a) $M_{11} = 19, M_{12} = -24, M_{13} = 26, M_{21} = 2,$
$M_{22} = 32, M_{23} = 20, M_{31} = -47, M_{32} = -96,$
$M_{33} = 22$
(b) $C_{11} = 19, C_{12} = 24, C_{13} = 26, C_{21} = -2,$
$C_{22} = 32, C_{23} = -20, C_{31} = -47, C_{32} = 96,$
$C_{33} = 22$
103. 130 **104.** −117 **105.** 279 **106.** −255
107. (4, 7) **108.** (3, −2) **109.** (−1, 4, 5)
110. (6, 8, 1) **111.** 16 **112.** 24 **113.** 10
114. $\frac{25}{8}$ **115.** Collinear **116.** Collinear
117. $x - 2y + 4 = 0$ **118.** $3x + 2y - 16 = 0$
119. $2x + 6y - 13 = 0$ **120.** $10x - 5y + 9 = 0$
121. Uncoded: [12 15 15], [11 0 15], [21 20 0], [2 5 12], [15 23 0]
Encoded: −21 6 0 −68 8 45 102 −42 −60 −53 20 21 99 −30 −69
122. Uncoded: [18 5 20], [21 18 14], [0 20 15], [0 2 1], [19 5 0]
Encoded: 66 28 10 −24 −59 −22 −75 −90 −25 −9 −10 −3 8 −11 −10
123. SEE YOU FRIDAY
124. MAY THE FORCE BE WITH YOU
125. False. The matrix must be square.
126. True. This is the correct sum of the two determinants.
127. The matrix must be square and its determinant nonzero.
128. If A is a square matrix, the cofactor C_{ij} of the entry a_{ij} is $(-1)^{i+j}M_{ij}$, where M_{ij} is the determinant obtained by deleting the ith row and jth column of A. The determinant of A is the sum of the entries of any row or column of A multiplied by their respective cofactors.
129. No. The first two matrices describe a system of equations with one solution. The third matrix describes a system with infinitely many solutions.
130. The part of the matrix corresponding to the coefficients of the system reduces to a matrix in which the number of rows with nonzero entries is the same as the number of variables.
131. $\lambda = \pm 2\sqrt{10} - 3$

Chapter Test *(page 637)*

1. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

3. $\left[\begin{array}{ccc|c} 4 & 3 & -2 & 14 \\ -1 & -1 & 2 & -5 \\ 3 & 1 & -4 & 8 \end{array}\right]$, $(1, 3, -\frac{1}{2})$

4. (a) $\begin{bmatrix} 1 & 5 \\ 0 & -4 \end{bmatrix}$
(b) $\begin{bmatrix} 15 & 12 \\ -12 & -12 \end{bmatrix}$
(c) $\begin{bmatrix} 7 & 14 \\ -4 & -12 \end{bmatrix}$
(d) $\begin{bmatrix} 4 & -5 \\ 0 & 4 \end{bmatrix}$

5. $\begin{bmatrix} \frac{1}{2} & \frac{2}{5} \\ 1 & \frac{3}{5} \end{bmatrix}$

6. $\begin{bmatrix} -\frac{5}{2} & 4 & -3 \\ 5 & -7 & 6 \\ 4 & -6 & 5 \end{bmatrix}$

7. (13, 22) **8.** −196 **9.** 29 **10.** 43
11. (−3, 5) **12.** (−2, 4, 6) **13.** 7
14. Uncoded: [11 14 15], [3 11 0], [15 14 0], [23 15 15], [4 0 0]
Encoded: 115 −41 −59 14 −3 −11 29 −15 −14 128 −53 −60 4 −4 0
15. 75 liters of 60% solution
25 liters of 20% solution

Chapter Test

Take this test as you would take a test in class. When you are finished, check your work against the answers given in the back of the book.

1. Write the first five terms of the sequence $a_n = \frac{(-1)^n}{3n + 2}$. (Assume that n begins with 1.)

2. Write an expression for the nth term of the sequence.

$\frac{3}{1!}, \frac{4}{2!}, \frac{5}{3!}, \frac{6}{4!}, \frac{7}{5!}, \ldots$

3. Find the next three terms of the series. Then find the fifth partial sum of the series.

$6 + 17 + 28 + 39 + \cdots$

4. The fifth term of an arithmetic sequence is 5.4, and the 12th term is 11.0. Find the nth term.

5. Write the first five terms of the sequence $a_n = 5(2)^{n-1}$. (Assume that n begins with 1.)

In Exercises 6–8, find the sum.

6. $\sum_{i=1}^{50}(2i^2 + 5)$. 7. $\sum_{n=1}^{7}(8n - 5)$ 8. $\sum_{i=1}^{\infty}4\left(\frac{1}{2}\right)^i$.

9. Use mathematical induction to prove the formula.

$$5 + 10 + 15 + \cdots + 5n = \frac{5n(n+1)}{2}$$

10. Use the Binomial Theorem to expand the expression $(x + 2y)^4$.

11. Find the coefficient of the term a^3b^5 in the expansion of $(2a - 3b)^8$.

In Exercises 12 and 13, evaluate each expression.

12. (a) ${}_9P_2$ (b) ${}_{70}P_3$

13. (a) ${}_{11}C_4$ (b) ${}_{66}C_4$

14. How many distinct license plates can be issued consisting of one letter followed by a three-digit number?

15. Eight people are going for a ride in a boat that seats eight people. The owner of the boat will drive, and only three of the remaining people are willing to ride in the two bow seats. How many seating arrangements are possible?

16. You attend a karaoke night and hope to hear your favorite song. The karaoke song book has 300 different songs (your favorite song is among the 300 songs). Assuming that the singers are equally likely to pick any song and no song is repeated, what is the probability that your favorite song is one of the 20 that you hear that night?

17. You are with seven of your friends at a party. Names of all of the 60 guests are placed in a hat and drawn randomly to award eight door prizes. Each guest is limited to one prize. What is the probability that you and your friends win all eight of the prizes?

18. The weather report calls for a 75% chance of snow. According to this report, what is the probability that it will *not* snow?

9 Cumulative Test for Chapters 7–9

Take this test to review the material from earlier chapters. When you are finished, check your work against the answers given in the back of the book.

In Exercises 1–4, solve the system by the specified method.

1. Substitution

$$\begin{cases} y = 3 - x^2 \\ 2(y - 2) = x - 1 \end{cases}$$

2. Elimination

$$\begin{cases} x + 3y = -1 \\ 2x + 4y = 0 \end{cases}$$

3. Elimination

$$\begin{cases} -2x + 4y - z = 3 \\ x - 2y + 2z = -6 \\ x - 3y - z = 1 \end{cases}$$

4. Gauss-Jordan Elimination

$$\begin{cases} x + 3y - 2z = -7 \\ -2x + y - z = -5 \\ 4x + y + z = 3 \end{cases}$$

In Exercises 5 and 6, sketch the graph of the solution set of the system of inequalities.

5. $\begin{cases} 2x + y \geq -3 \\ x - 3y \leq 2 \end{cases}$

6. $\begin{cases} x - y > 6 \\ 5x + 2y < 10 \end{cases}$

7. Sketch the region determined by the constraints. Then find the minimum and maximum values, and where they occur, of the objective function $z = 3x + 2y$, subject to the indicated constraints.

$$x + 4y \leq 20$$
$$2x + y \leq 12$$
$$x \geq 0$$
$$y \geq 0$$

8. A custom-blend bird seed is to be mixed from seed mixtures costing \$0.75 per pound and \$1.25 per pound. How many pounds of each seed mixture are used to make 200 pounds of custom-blend bird seed costing \$0.95 per pound?

9. Find the equation of the parabola $y = ax^2 + bx + c$ passing through the points $(0, 4)$, $(3, 1)$, and $(6, 4)$.

$$\begin{cases} -x + 2y - z = 9 \\ 2x - y + 2z = -9 \\ 3x + 3y - 4z = 7 \end{cases}$$

SYSTEM FOR 10 AND 11

In Exercises 10 and 11, use the system of equations at the left.

10. Write the augmented matrix corresponding to the system of equations.

11. Solve the system using the matrix found in Exercise 10 and Gauss-Jordan elimination.

In Exercises 12–15, use the following matrices to find each of the following, if possible.

$$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 3 \\ 1 & 0 \end{bmatrix}$$

12. $A + B$

13. $-2B$

14. $A - 2B$

15. AB

$$\begin{bmatrix} 8 & 0 & -5 \\ 1 & 3 & -1 \\ -2 & 6 & 4 \end{bmatrix}$$

MATRIX FOR 16

16. Find the determinant of the matrix at the left.

17. Find the inverse of the matrix (if it exists): $\begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ -5 & -7 & -15 \end{bmatrix}$.

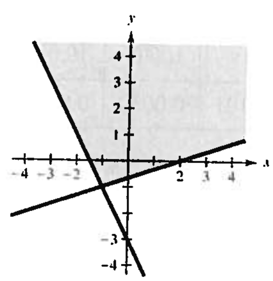
34. 4.2, 4.6, 5.0, 5.4, 5.8 **35.** $a_n = 12n - 5$
36. $a_n = -3n + 28$ **37.** $a_n = 3ny - 2y$
38. $a_n = nx - 3x$ **39.** $a_n = -7n + 107$
40. $a_n = -\frac{1}{3}n + \frac{31}{3}$ **41.** 80 **42.** 52 **43.** 88
44. 250 **45.** 25,250 **46.** 3050
47. (a) \$43,000 (b) \$192,500 **48.** 676 bales
49. Geometric sequence, $r = 2$
50. Geometric sequence, $r = -\frac{1}{3}$
51. Geometric sequence, $r = -2$
52. Not a geometric sequence **53.** $4, -1, \frac{1}{4}, -\frac{1}{16}, \frac{1}{64}$
54. 2, 4, 8, 16, 32 **55.** $9, 6, 4, \frac{8}{3}, \frac{16}{9}$ or $9, -6, 4, -\frac{8}{3}, \frac{16}{9}$
56. $2, 2\sqrt{6}, 12, 12\sqrt{6}, 72$ or $2, -2\sqrt{6}, 12, -12\sqrt{6}, 72$
57. $a_n = 16\left(-\frac{1}{2}\right)^{n-1}$; $\approx 3.052 \times 10^{-5}$
58. $a_n = 1296\left(\frac{1}{6}\right)^n$; 3.545×10^{-13}
59. $a_n = 100(1.05)^{n-1}$; ≈ 252.695
60. $a_n = 5(0.2)^{n-1}$; 2.621×10^{-13} **61.** 127 **62.** 121
63. $\frac{15}{16}$ **64.** $\frac{364}{243}$ **65.** 31 **66.** 720 **67.** 24.85
68. 25 **69.** 5486.45 **70.** 1493.50 **71.** 8
72. $\frac{3}{2}$ **73.** $\frac{10}{9}$ **74.** 2 **75.** 12 **76.** $\frac{13}{9}$
77. (a) $a_t = 120{,}000(0.7)^t$ (b) \$20,168.40
78. \$32,939.75; \$32,967.03 **79–82.** Answers will vary.
83. $S_n = n(2n + 7)$ **84.** $S_n = 4n(18 - n)$
85. $S_n = \frac{5}{2}\left[1 - \left(\frac{3}{5}\right)^n\right]$ **86.** $S_n = \frac{144}{13}\left[1 - \left(-\frac{1}{12}\right)^n\right]$
87. 465 **88.** 385 **89.** 4648 **90.** 12,110
91. 5, 10, 15, 20, 25
First differences: 5, 5, 5, 5
Second differences: 0, 0, 0
Linear
92. −3, −7, −13, −21, −31
First differences: −4, −6, −8, −10
Second differences: −2, −2, −2
Quadratic
93. 16, 15, 14, 13, 12
First differences: −1, −1, −1, −1
Second differences: 0, 0, 0
Linear
94. 0, 1, 1, 2, 2
First differences: 1, 0, 1, 0
Second differences: −1, 1, −1
Neither
95. 15 **96.** 120 **97.** 56 **98.** 220 **99.** 35
100. 126 **101.** 28 **102.** 10
103. $x^4 + 16x^3 + 96x^2 + 256x + 256$
104. $x^6 - 18x^5 + 135x^4 - 540x^3 + 1215x^2 - 1458x + 729$
105. $a^5 - 15a^4b + 90a^3b^2 - 270a^2b^3 + 405ab^4 - 243b^5$
106. $2187x^7 + 5103x^6y^2 + 5103x^5y^4 + 2835x^4y^6 + 945x^3y^8 + 189x^2y^{10} + 21xy^{12} + y^{14}$
107. $41 + 840i$ **108.** $-236 - 115i$ **109.** 11
110. 180 **111.** 10,000 **112.** 72 **113.** 720
114. 225,792,840 **115.** 56 **116.** 327,680 **117.** $\frac{1}{9}$
118. $\frac{1}{120}$ **119.** (a) 43% (b) 82%
120. (a) 41.6% (b) 80% (c) 7.4% **121.** $\frac{1}{216}$
122. $\frac{5}{324}$ **123.** $\frac{3}{4}$ **124.** $\frac{31}{32}$
125. True. $\frac{(n+2)!}{n!} = \frac{(n+2)(n+1)n!}{n!} = (n+2)(n+1)$
126. True by Properties of Sums
127. True by Properties of Sums
128. True. The sums are equal because
$2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 =$
$2^{3-2} + 2^{4-2} + 2^{5-2} + 2^{6-2} + 2^{7-2} + 2^{8-2}$.
129. False. When r equals 0 or 1, then the results are the same.
130. The set of natural numbers
131. In the sequence in part (a), the odd-numbered terms are negative, whereas in the sequence in part (b), the even-numbered terms are negative.
132. (a) Arithmetic. There is a constant difference between consecutive terms.
(b) Geometric. Each term is a constant multiple of the preceding term. In this case, the common ratio is greater than 1.
133. Each term of the sequence is defined in terms of preceding terms.
134. Increasing powers of real numbers between 0 and 1 approach zero.
135. d **136.** a **137.** b **138.** c
139. 240, 440, 810, 1490, 2740
140. $0 \le p \le 1$; Closed interval

Chapter Test *(page 719)*

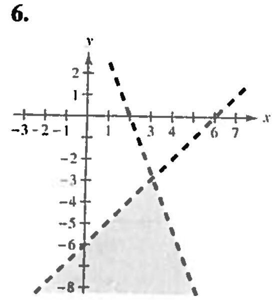
1. $-\frac{1}{5}, \frac{1}{8}, -\frac{1}{11}, \frac{1}{14}, -\frac{1}{17}$ **2.** $a_n = \frac{n+2}{n!}$
3. 50, 61, 72; 140 **4.** $a_n = 0.8n + 1.4$
5. 5, 10, 20, 40, 80 **6.** 86,100 **7.** 189
8. 4 **9.** Answers will vary.
10. $x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$ **11.** −108,864
12. (a) 72 (b) 328,440 **13.** (a) 330 (b) 720,720
14. 26,000 **15.** 720 **16.** $\frac{1}{15}$ **17.** 3.908×10^{-10}
18. 25%

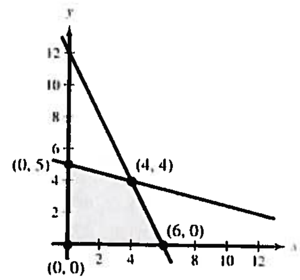
Cumulative Test for Chapters 7–9 *(page 720)*

1. $(1, 2), \left(-\frac{3}{2}, \frac{3}{4}\right)$ **2.** $(2, -1)$
3. $(4, 2, -3)$ **4.** $(1, -2, 1)$
5.



6.



7. 

Maximum at (4, 4): $z = 20$

Minimum at (0, 0): $z = 0$

8. \$0.75 mixture: 120 pounds; \$1.25 mixture: 80 pounds

9. $y = \frac{1}{3}x^2 - 2x + 4$

10. $\begin{bmatrix} -1 & 2 & -1 & \vdots & 9 \\ 2 & -1 & 2 & \vdots & -9 \\ 3 & 3 & -4 & \vdots & 7 \end{bmatrix}$ **11.** $(-2, 3, -1)$

12. $\begin{bmatrix} 3 & 3 \\ 0 & 2 \end{bmatrix}$ **13.** $\begin{bmatrix} 2 & -6 \\ -2 & 0 \end{bmatrix}$ **14.** $\begin{bmatrix} 6 & -6 \\ -3 & 2 \end{bmatrix}$

15. $\begin{bmatrix} -4 & 12 \\ 3 & -3 \end{bmatrix}$ **16.** 84 **17.** $\begin{bmatrix} -175 & 37 & -13 \\ 95 & -20 & 7 \\ 14 & -3 & 1 \end{bmatrix}$

18. Gym shoes: \$198.36 million
Jogging shoes: \$358.48 million
Walking shoes: \$167.17 million

19. $(-5, 4)$ **20.** $(-3, 4, 2)$ **21.** 9

22. $\frac{1}{5}, -\frac{1}{7}, \frac{1}{9}, -\frac{1}{11}, \frac{1}{13}$ **23.** $a_n = \frac{(n+1)!}{n+3}$

24. 920 **25.** (a) 65.4 (b) $a_n = 3.2n + 1.4$

26. 3, 6, 12, 24, 48 **27.** $\frac{13}{9}$ **28.** Answers will vary.

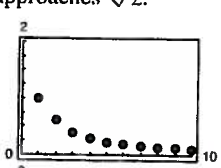
29. $z^4 - 12z^3 + 54z^2 - 108z + 81$ **30.** 210 **31.** 600

32. 70 **33.** 120 **34.** 453,600 **35.** 151,200

36. 720 **37.** $\frac{1}{4}$

Problem Solving *(page 725)*

1. 1, 1.5, $1.41\overline{6}$, 1.414215686, 1.414213562, 1.414213562, . . .
x_n approaches $\sqrt{2}$.

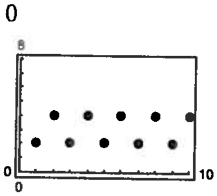
2. (a) 

(b) 0

(c)

n	1	10	100	1000	10,000
a_n	1	0.1089	0.0101	0.0010	0.0001

(d) 0

3. (a) 

(b) If n is odd, $a_n = 2$, and if n is even, $a_n = 4$.

(c)

n	1	10	101	1000	10,001
a_n	2	4	2	4	2

(d) It is not possible to find the value of a_n as n approaches infinity.

4. (a) Arithmetic sequence, difference $= d$
(b) Arithmetic sequence, difference $= dC$
(c) Not an arithmetic sequence

5. (a) 3, 5, 7, 9, 11, 13, 15, 17
$a_n = 2n + 1$
(b) To obtain the arithmetic sequence, find the differences of consecutive terms of the sequence of perfect cubes. Then find the differences of consecutive terms of this sequence.
(c) 12, 18, 24, 30, 36, 42, 48
$a_n = 6n + 6$
(d) To obtain the arithmetic sequence, find the third sequence obtained by taking differences of consecutive terms in consecutive sequences.
(e) 60, 84, 108, 132, 156, 180
$a_n = 24n + 36$

6. $s_d = \frac{a_1}{1-r} = \frac{20}{1-\frac{1}{2}} = 40$
This represents the total distance Achilles ran.
$s_t = \frac{a_1}{1-r} = \frac{1}{1-\frac{1}{2}} = 2$
This represents the total amount of time Achilles ran.

7. $s_n = \left(\frac{1}{2}\right)^{n-1}$
$a_n = \frac{\sqrt{3}}{4}s_n^2$

8. (a) 7, 22, 11, 34, 17, 52, 26, 13, 40, 20
(b) Eventually the terms repeat: 4, 2, 1 if a_1 is a positive integer and $-2, -1$ if a_1 is a negative integer.

9. Answers will vary.

10. (a) P_n is true for integers $n \geq 3$.
(b) P_n is true for integers $1 \leq n \leq 50$.
(c) P_1, P_2, and P_3 are true.
(d) P_{2n} is true for any positive integer n.

11. (a) Answers will vary. (b) 17,710

12. (a) 30 marbles (b) 3 to 7; 7 to 3
(c) $P(E) = \frac{\text{odds in favors of } E}{\text{odds in favor of } E + 1}$
(d) Odds in favor of event $E = \frac{P(E)}{P(E')}$

13. $\frac{1}{3}$ **14.** $\frac{\pi - 1}{\pi}$

15. (a) $-\$0.71$ (b) 2.53, 24 turns