Math5700 Notes
Chapter 4

Starters:

1. Which two equations are equivalent? Explain your reasoning.
   (a) |x| = -5 (b) x = 5 (c) |x| = 5 (d) x = |5|

2. Which two equations are equivalent? Explain your reasoning.
   (a) 3A = 5B (b) 3B = 5A (c) $\frac{A}{B}=\frac{3}{5}$ (d) 5A = 3B

3. Which equation is not equivalent to the others? Why?
   (a) $x^2=9$ (b) $x=\sqrt{9}$ (c) $|x|=3$ (d) $x=\pm 3$

4. Consider this "proof."

Claim: 1 = 2
Proof:

(1) Let x = 1 and y = 1.
(2) They $xy=y^2$ by substitution.
(3) Then, $-xy=-y^2$
(4) and $x^2-xy=x^2-y^2$
(5) and $x(x-y)=(x+y)(x-y)$
(6) So, $x=x+y$. Then from (1), we get
(7) 1 = 1 + 1, which means
(8) 1 = 2.

Where is the mistake?

5. Which pairs of these statements are equivalent for a and b both real? Why?
   (a) $a = 0$ and $b = 0$ (b) $a^2+b^2=0$

   (c) $a = 0$ or $b = 0$ (d) $ab = 0$

6. Show that $3x^2+5xy-2y^2-9x+10y-12=0$ is an equation for a pair of lines.

7. Under what conditions is $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ an equation of a pair of lines?

8. Give a single equation whose graph consists of the union of the x-axis, y-axis, and two lines that bisect the right angles formed by the axes.

Let's go back to the dirt biker problem.

Dirt Biker Problem

A dirt biker must circle a 5-mile track twice. His average speed must be 60 mph. On his first lap, he averaged 30 mph. How fast must he travel on his second lap in order to qualify?

Remember that the answer is that he used up all his time in the first lap and can't ride the second lap to finish in time for the average he wants. Let's continue from here.

Generalize your result by finding a formula for the average rate in the two laps.

Now consider two related problems.

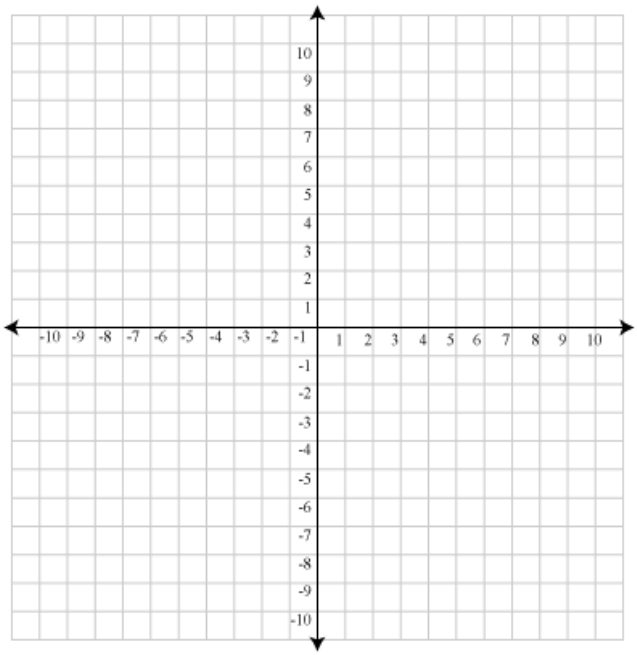
1. Suppose that on a round trip you average 30 mph on the way out and 60 mph on the way back. What is your average speed for the entire trip?	2. On a trip, suppose you average 30 mph for a certain time, and then your friend takes over and averages 60 mph for the same amount of time. What is the average speed for the total time?

These two problems highlight two different types of means, out of three types of means we'll consider.

Means-- assume we are finding the desired mean for two inputs, x and y. Then, in each category, we can generalize further for n inputs.

Arithmetic	Harmonic	Geometric
$a_2(x,y)=\frac{x+y}{2}$	$h_2(x,y)=\frac{1}{\frac{1}{2}\left(\frac{1}{x}+\frac{1}{y}\right)}=\frac{2xy}{x+y}$	$g_2(x,y)=\sqrt{xy}$
$a_n(x_1,x_{2,}\ldots,x_n)=\frac{1}{n}\sum_{i=1}^{n}x_i$	$h_n(x_1,x_{2,}\ldots,x_n)=\frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}}$	$g_n(x_1,x_{2,}\ldots,x_n)=\sqrt[n]{\prod_{i=1}^{n}x_i}$

Graph $y=a(x)=a_2(x,1)$, $y=g(x)=g_2(x,1)$ and $y=h(x)=h_2(x,1)$ on the same axes.



When do all three means produce the same output?

Does h(x) have a limit as x goes toward infinity?