Math5700 Notes
Chapter 9 Notes—Trigonometry

Definitions of Trigonometric Functions

Unit Circle	Right Triangle

Prove the Pythagorean Identities.

Example 1: Given $\sin\theta = \frac{3}{5}$ and $\cos\beta = \frac{-5}{13}$ and both θ and β are in quadrant 2, find $\cos(\theta + \beta)$.

Example 2: Find $\sin(15^{\circ})$.

Example 3: Verify the trigonometric identity.

$$2\sec^2 x - \cos^2 x - 2\sec^2 x \sin^2 x - \sin^2 x = 1$$

Example 4: Solve the equation.

$$2\sin^2(3x)+\cos(3x)=2$$

Example 5: Evaluate.

(a) $\cos(\tan^{-1}\sqrt{3})$

(b) $\sin(\cos^{-1}\frac{2}{3})$

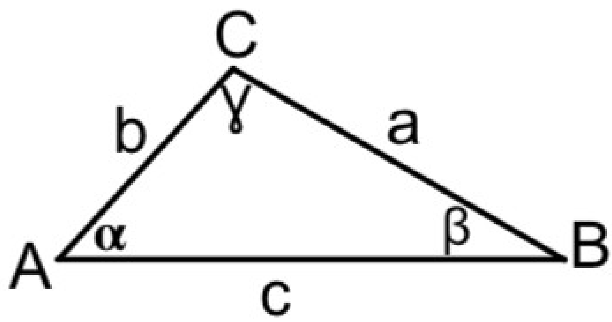
(c) $\tan(sec^{-1}x)$

(d) $\sin(\cos^{-1}\left(\frac{1}{3}\right)+\sin^{-1}\left(\frac{1}{2}\right))$

Example 6: Is $\sec^{-1}x = \frac{1}{\cos^{-1}x}$? Why or why not and if not, how can you fix it?

Example 7: Graph y = csc(x).

Example 8: Find $\sin(2\theta)$, $\cos(2\theta)$, $\sin(\frac{\theta}{2})$, $\cos(\frac{\theta}{2})$ if $\tan(\theta) = \frac{-3}{4}$ and θ in quadrant 2.



Law of Cosines	Law of Sines
$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$ Proof: *(Hint: Put triangle on coordinate axes with vertex C at the origin.)*	$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ Proof:

For what triangle cases are these useful (to finish solving the triangle)?
And which case is ambiguous? Why?

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Example 1: The Pythagorean Theorem is a special case of the Law of Cosines.

Proof:

Example 2: What conditions are necessary for $a^2+b^2<c^2$ (acute triangle) and $a^2+b^2>c^2$ (obtuse triangle)?

Example 3: Given a , b , and α in this triangle, do the following problems.
(a) Prove that if $a \geq b$ the there exists exactly one triangle.
Proof:

(b) For what values of $a < b$ is there exactly one triangle?

(c) When is there no triangle with the given information?

9.1.3 Indirect Measurement Problems

Example 1: We want to measure the height of a building.
(a) We know the angle of elevation to the top of the building and the distance on the ground to the base of the building.

(b) Suppose the building is across the freeway and we can't measure the distance along the ground to it. Instead, we know the angle of elevation from point A, and the angle of elevation from point B, and the distance from A to B is 100 feet.

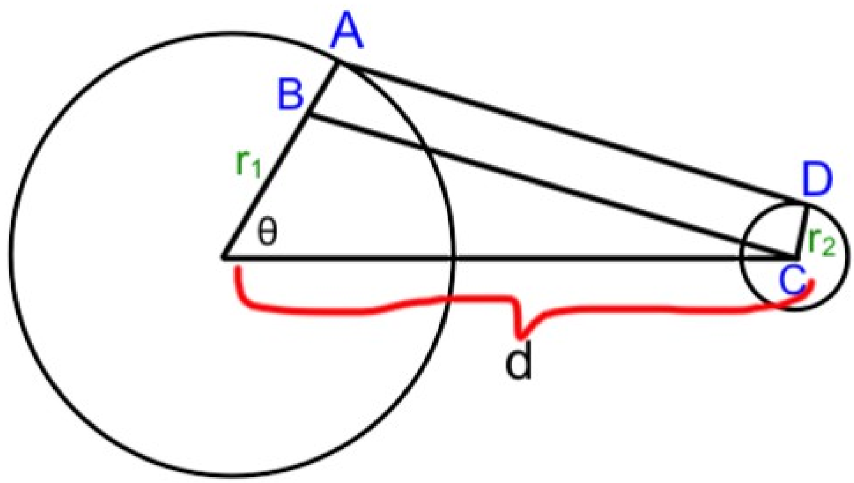
(c) Suppose the building is still across the freeway. You can measure the angle of elevation to the top of the building from point A. Then, you walk along a line perpendicular to line AP to reach another point B, then measure the angle (angle from the perpendicular line to the base of the building).

(d) Suppose there exists line AB of known length b, such that P, Q, A and B are not in the same plane.

Example 2: Belt problem. (9.1.3 #7) Suppose that a belt is stretched tightly over two pulleys of radii as indicated and whose centers are d units apart. Find a formula for the total length L of the belt.

(a) $d=6$, $r_1=r_2=3$

(the other parts of this problem are assigned as homework)



A Brief look at Complex Numbers

$z=a+bi \Leftrightarrow z=r(\cos\theta+i\sin\theta)$

such that $r^2=a^2+b^2$ and $\tan\theta=\frac{b}{a}$

1. Prove that if $z_1=r_1(\cos\theta_1+i\sin\theta_1)$ and $z_2=r_2(\cos\theta_2+i\sin\theta_2)$, then
   (a) $z_1 z_2=r_1 r_2(\cos(\theta_1+\theta_2)+i\sin(\theta_1+\theta_2))$
   (b) $\frac{z_1}{z_2}=\frac{r_1}{r_2}(\cos(\theta_1-\theta_2)+i\sin(\theta_1-\theta_2))$

2. Find the direct formula for z^n (for integer n).

3. Use the formula $\sqrt[n]{z}=\sqrt[n]{r}(\cos\left(\frac{\theta+2\pi k}{n}\right)+i\sin\left(\frac{\theta+2\pi k}{n}\right))$ for all $k=0,1,2,\ldots,n-1$ to find all the

(a) square roots of $z=\frac{25}{2}-\frac{25\sqrt{3}}{2}i$

(b) cube roots of -1.