Equivalent Inequalities
Math5700

Let's start with three numerical inequalities that are all true and apply different mathematical operators to both sides of the inequalities to decide if the new inequalities are still true.

Apply these operators to both sides of the true inequality:
- (a) multiply by -3
- (b) take the absolute value
- (c) exponentiate with a base of 10
- (d) take log (base 10)
- (e) take sine
- (f) square
- (g) take cube root
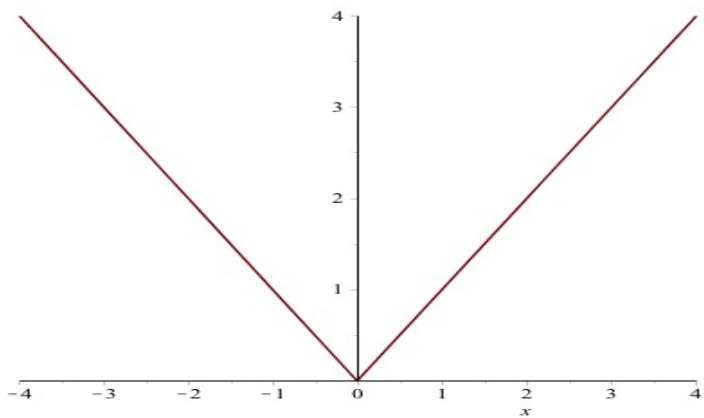- (h) subtract 5
- (i) multiply by 3

Operator	$-1 < 2$	True or False?	$1.5 < 6.2$	True or False?	$-2 < 1$	True or False?
(a)	$-3(-1) < -3(2)$					
(b)						
(c)						
(d)						
(e)						
(f)						
(g)						
(h)						
(i)						

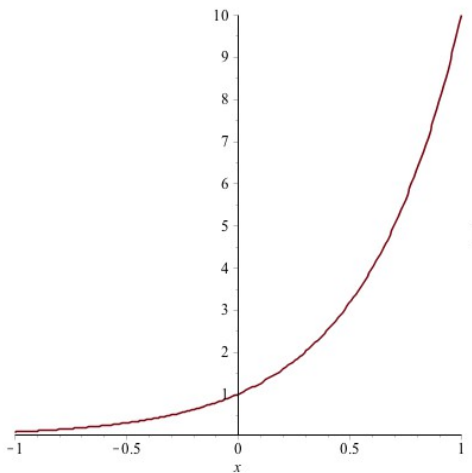Now, let's look at graphs of the functions that correspond to the operators given in (a) – (h).

(a) f(x) = -3x
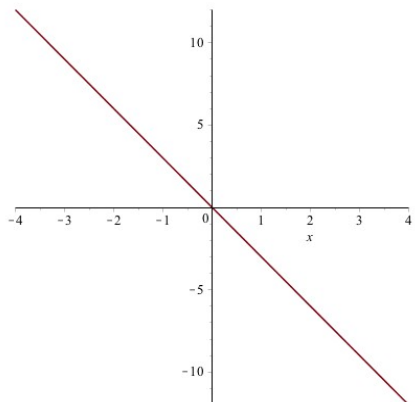
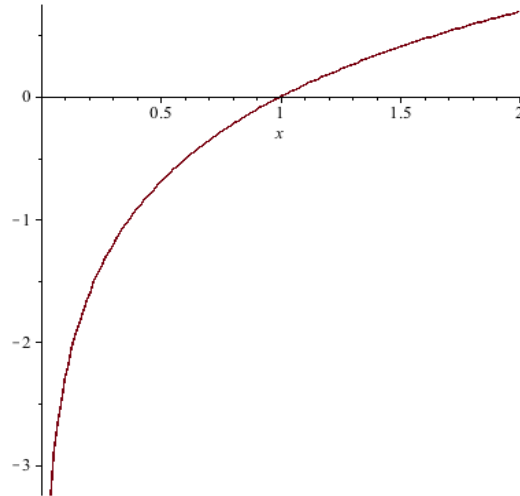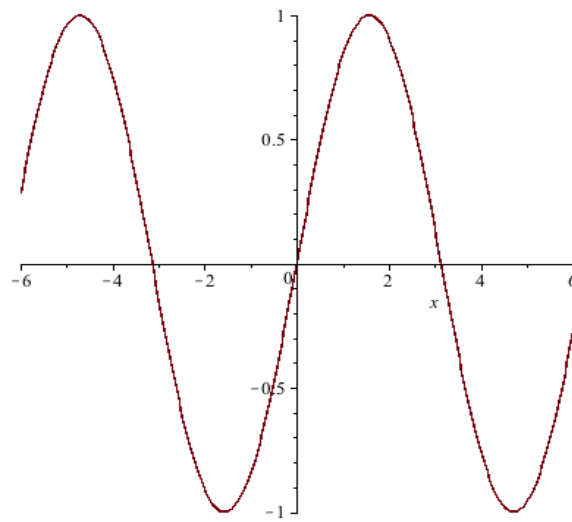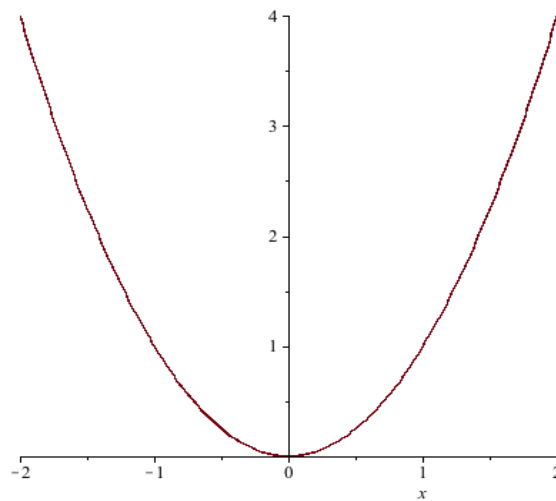(b) $f(x)=|x|$
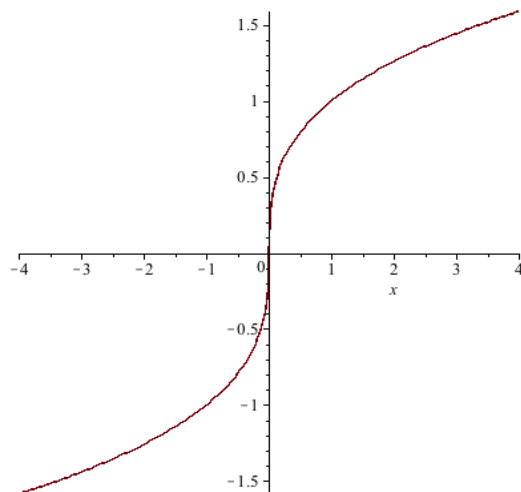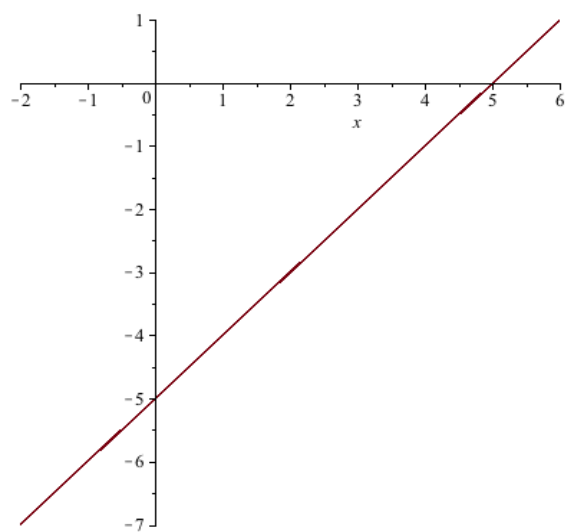
(c) $f(x)=10^x$
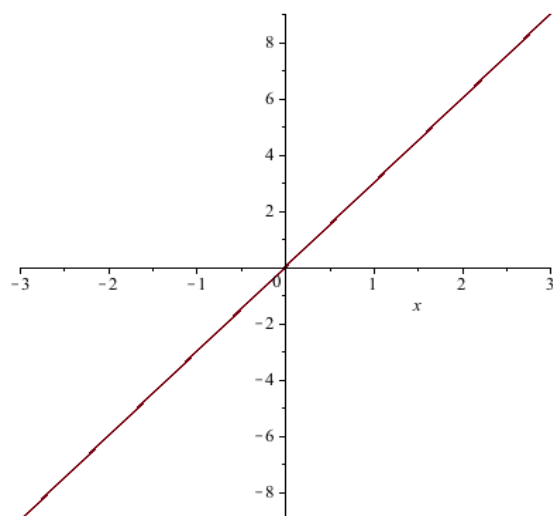
(d) $f(x) = \log x$

(e) $f(x) = \sin x$

(f) $f(x) = x^2$

(g) $f(x)=\sqrt[3]{x}$

(h) $f(x)=x-5$

(i) $f(x)=3x$

What conjecture can we make about how these graphs of functions that correspond to the operators applied to the inequalities give us information about whether or not we have to "switch the sign" in the inequalities?

Remember monotonicity of functions?

(1) A function, f(x), is strictly monotonically increasing if a < b implies f(a) < f(b) also.
(2) A function, f(x), is strictly monotonically decreasing if a < b implies f(a) > f(b).

Conclusion about equivalent inequalities:

(1) If we apply a monotonically increasing operator to both sides of a true inequality, then

(2) If we apply a monotonically decreasing operator to both sides of a true inequality, then

(3) If we apply an operator to both sides of a true inequality that is sometimes decreasing and sometimes increasing, then

<u>Examples to practice this:</u>
(Note: You also may need to pay attention to domain restrictions when solving these.)

1. $2\left(4^{3x^2}\right) \leq 10$

2. $(x-1)^3 - 8 < 0$

3. $\log_5(y-3)^2 \leq -3$

4. $3|4x - 5| > 6$

5. $\log_2(w+1)^4 + 2 \le 4$

6. $\log_2(w+1)^3 + 2 \le 4$

7. $\dfrac{x^2+10x+25}{x+5} \le 4$