**Math5700 Notes**
**Section 2.1**
The Real Numbers

Number Systems:

**N** =

**W** =

**Z** =

**Q** =

**R** =

Rational Numbers: Must a rational number be a fraction?

Theorem: 1. Q is closed under addition, subtraction and multiplication.
2. $Q - \{0\}$ is closed under division.

Proof of 2:

Irrational Numbers:

Theorem: Let s be in Q – {0} and v be in R – Q. Then $s+v$ , $s-v$ , $sv$ and $\frac{s}{v}$ are irrational.

Proof:

What about adding, subtracting, multiplying and dividing two irrational numbers?

Prove that $\log_{10} 2$ is irrational.

Theorem: Let $n \in \boldsymbol{Z}^+$ . Then $\sqrt{n}$ is either an integer or is irrational.

Proof:

What is the value of $\sqrt{5+\sqrt{5+\sqrt{5+...}}}$ ?

So, is $\sqrt{n+\sqrt{n+\sqrt{n+...}}}$ always irrational? (What about n = 2?)