

Math5700 Notes
Section 8.1 Notes
Distance

Euclidean Distance in \mathbb{R}^2 .

$$d_E(P, Q) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad \text{for points } P(x_1, y_1) \text{ and } Q(x_2, y_2)$$

Euclidean Distance in \mathbb{R}^3 .

$$d_E(P, Q) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \quad \text{for points } P(x_1, y_1, z_1) \text{ and } Q(x_2, y_2, z_2)$$

Example 1:

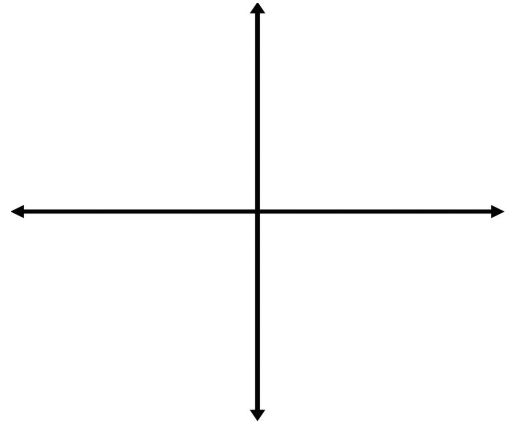
(a) Find the equation for the sphere in \mathbb{R}^3 with center at $Q(1, 3, -4)$ and radius of 8.

(b) Find the equation of intersection of the above sphere with the xy -plane and describe the intersection set.

Example 2: In \mathbb{R}^3 , explain why $P(1, 0, 0)$ and $Q(0, 1, 0)$ are on the unit sphere centered at the origin. Compare $d_E(P, Q)$ and $d_S(P, Q)$ (the spherical distance).

Taxicab Distance

You can only get from any point A to any other point B in a linear piecewise fashion, going strictly horizontal or vertical.



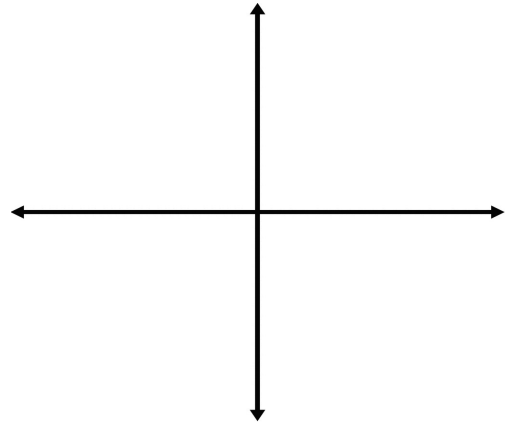
So, $d_T(A, B) = |x_1 - x_2| + |y_1 - y_2|$.

Example 3: Given any two points P and Q in \mathbb{R}^2 , what is the relationship between $d_E(P, Q)$ and $d_T(P, Q)$?

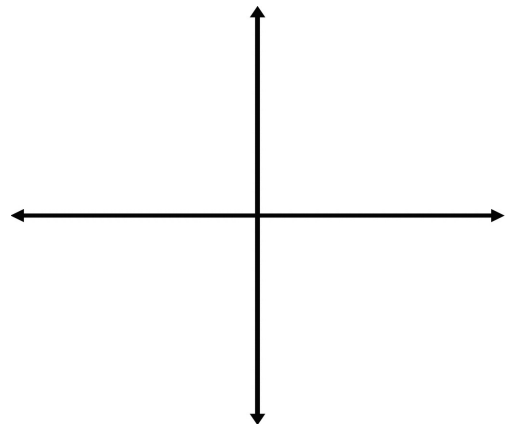
When are the Euclidean distance and the taxicab distance the same?

Circle—the set of points that are at a fixed distance (called the radius) from a fixed point O , called the center.

- (1) Give the equation for the unit circle with Euclidean distance, centered at the origin, and draw that shape.



- (2) Give the equation for the unit circle with the taxicab distance, centered at the origin, and draw that shape.



Both equations of circles have equations of the form:

Definition of Distance:

A distance function or metric on a set S is a function $d: S \times S \rightarrow \mathbb{R}$, such that $\forall P, Q, R \in S$

(d1) $d(P, Q) \geq 0$

(d2) $d(P, Q) = 0$ iff $P = Q$

(d3) $d(P, Q) = d(Q, P)$

(d4) $d(P, Q) + d(Q, R) \geq d(P, R)$

Minkowski Distance:

8.1.2

The distance means the shortest length between two points, in whatever form of distance that we're using.

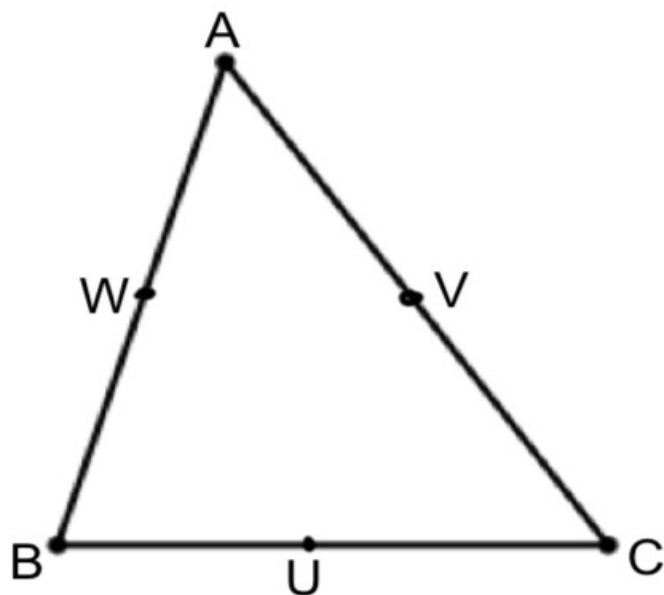
Example 4: Find the Euclidean and taxicab distance from $P(2,2)$ to $Q(x,y)$ such that Q is on $x^2 + y^2 = 1$.

Theorem 8.1:

The Euclidean distance between a point P and a line l , not containing P , is the length of the line segment from P to Q that is perpendicular to line l at Q .

Fagnano's Problem:

In a given acute-angled triangle ABC , inscribe a triangle UVW whose perimeter is as small as possible.



Example 5: Consider a triangle with vertices $A(-2,0)$, $B(2,0)$ and $C(1,3)$.

(a) Determine the vertices U , V and W of smallest perimeter inscribed in triangle ABC .

(b) What is its perimeter?

8.1.3

Problem 1: A dog on a leash is tied to a rectangular-shaped barn (from an aerial view) that is 20 meters long and 10 meters wide. The leash is 8 meters long and fastened 6 meters from the end of the longer side of the barn (at dog's neck level). In what region can the dog roam and what is the area of that region?

Where should the leash be tethered to maximize area that the dog can roam?