1.1 Linear Eqns in One Variable

Ex1 Solve

$$2(x-3) = 5(x+1) - 4$$

Ex2 Solve

$$\frac{x-7}{5} + 1 = \frac{x-4}{3} - 1$$

Strategy to Solve Linear Eqns

1. simplify each side of eqn
2. add/subtract from both sides of eqn to get all variables on one side and a number on the other side of eqn
3. finish solving by dividing both sides of eqn by coefficient of the variable.

1.1 (cont)

Be sure to check your answers!

Ex 3 Solve these rational eqns.

(a) $\frac{1}{2x} + \frac{1}{x} = \frac{1}{2}$

(b) $\frac{6}{x-6} + 2 = \frac{x}{x-6}$

Ex 4 The perimeter of a rectangle is 700 ft and the length is four times as long as the width of the rectangle. Find the dimensions of the rectangle.

1.2 Linear Inequalities in One Variable

Ex 1 Solve these inequalities and graph the solutions on a real number line.

Note: Remember to switch the sign if you multiply both sides of the inequality by a negative number!

(a) $\frac{7}{4}x - 2 \geq 5$

(b) $\frac{5x+3}{8} - 1 \geq \frac{x+4}{6} + 1$

1.2 (cont)

Ex 2 Translate to an inequality:

Two times a number is less than the sum of 15 and the number.

Ex 3 A line of t-shirts sells for \$3.50 each. The fixed costs to manufacture them are \$6500 and it costs \$1.30 to make each t-shirt. How many t-shirts must be sold to make a profit?

1.3 Equations of lines

Ex 1 Find the slope of line through $(-2, 1)$ and $(3, -5)$

Ex 2 Find slope and y-intercept of line $3x+4y=8$

Ex 3 If a line has slope $\frac{-3}{2}$, what is slope of every line perpendicular to that line?

Slope: of line between 2 points (x_1, y_1) and (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

parallel lines have same slope

perpendicular lines have negative reciprocal slopes

Slope-intercept form of a line:

$$y = mx + b$$

where $(0, b)$ is y-intercept

point-slope form of a line:

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is pt on line

1.3 (cont)

Ex 4 Find the equation of the line (in slope-intercept form).

(a) $m = 3$, through pt $(9, 1)$

(b) through pts $(1, 5)$ and $(-1, 4)$

(c) a vertical line through pt $(-3, 7)$

(d) a line through $(0, 7)$ and perpendicular to $2x + 3y = 12$

1.4 Systems of Linear Eqns

Ex1 Solve the systems of eqns

(a) $x - 4y = 8$
$4x + y = -2$

(b) $5x + 3y = 1$
$-10x - 6y = -2$

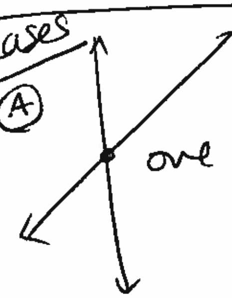
We can use either

① Substitution method

OR

② Elimination method

cases

Ⓐ

one solution

Ⓑ

No solution

Ⓒ

two lines are the same line
$\Rightarrow$ all pts are solutions

1.4 (cont)

Ex 1 (cont)

(c) $2x - y = 4$

$2y = 4x - 3$

Ex 2 Solve $6x - 8y = -3$

$8x + 4y = 7$

1.4 (cont)

EX3 Solve the system of three linear eqns.

$$2x - y = 4$$
$$-2x + y + z = 0$$
$$-3z = 21$$

Ex4 A collection of dimes and quarters contains 100 coins worth \$14.50. How many dimes and quarters are in the collection?

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1.5 Functions

Ex 1 Is this set of ordered pairs a relation or function? State the domain and range.

(a) $\{(3,6), (10,1), (8,4), (6,2), (5,10)\}$

(b) $\{(3,6), (10,1), (8,4), (6,2), (6,1), (3,1)\}$

Ex 2 Is this a function or relation? State the domain and range.

(a) $y = \frac{1}{x+1}$

(b) $y^2 = x^2$

Vocab

Domain - set of allowable inputs

Range - Set of outputs

Relation

a set of ordered pairs that maps inputs to outputs
(x,y) x = input, y = output

Function

a relation that has only one output for every input

Vertical line test - used to determine if a graph is a function or not

1.5 (cont)

Ex3 Evaluate each function at the given input.

(a) $f(x) = \sqrt{x^2+1}$ at $x=0$ and at $x=w$

(b) $f(x) = \dfrac{2x}{x+5}$ at $x=1$ and at $x=b+2$

Ex4 Find the domain for these functions.

(a) $f(x) = \sqrt{5-x}$

(b) $f(x) = \dfrac{2}{x-3}$

1.6 Linear Business Applications

Ex 1 A craftsman makes and sells leather belts for \$15 each. If his tools and machines require \$300 in one-time setup costs and his variable costs are \$9 per belt:

(a) what is the cost function?

(b) what is the revenue function?

(c) what is the profit function?

(d) how many belts must he sell to break even?

Profit = Revenue − Cost

i.e.

$P = R - C$

break-even pt: point where profit is zero, i.e. where $R = C$.

equilibrium pt: pt where supply = demand

1.6 (cont)

Ex 2 Assume supply for a toy is given by $35p - 20q = 350$ and demand is $p + 2q = 100$.

(a) compare the quantity demanded and supplied when price is \$10.

(b) what is the equilibrium pt?

Ex 3 A product has fixed costs of \$1800 and variable costs of \$35 per unit. If the product is sold for \$60 per unit, (a) how many units need to be sold in order to break even? (b) what is marginal cost? (c) what is marginal revenue?

1.7 Linear Inequalities in Two Variables

Ex 1 Graph solution for this inequality (in 2-d).

$$2x - \frac{3}{5}y \geq \frac{-2}{5}$$

Ex 2 Graph the solution set for this system of inequalities

1. $x + 2y < 4$
2. $2x + y < 6$
3. $x > -1$
4. $y > -1$

1.7 (cont)

Ex 3 Graph solution set for these inequalities.

① $2y \leq 3x + 6$

② $7y \geq 2x + 21$

③ $y \leq -\frac{1}{5}x + \frac{32}{5}$

Ex 4 A factory can manufacture 800 vehicles, either cars or trucks, per month. The factory must make at least 300 more cars than trucks, Find the inequalities which represent this situation.

1.8 Graphical Linear Programming

Ex 1 Find both min and max of objective function
$P = 2x - 5y$ given these

constraints:

$3x - 2y \geq -12$
$x + y \leq 9$
$y \geq 0$
$x \geq 0$

1. Graph all constraint lines.
2. Shade in feasible region.
3. Find and label all vertices of that shaded region.
4. Plug in vertices (corner pts) to objective function to find min/max values.

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1.8 (cont)

Ex 2 Find both max & min values of objective function $P = 4y - 2x$ given:

① $y \geq 1$

② $x \geq 0$

③ $y \leq \frac{1}{5}x + 5$

④ $5x + 3y \leq 43$

1.8 (cont)

Ex 3 A farmer can plant 100 acres of either beans or corn. The farmer must plant at least 10 acres of corn. If the farmer can make a profit of \$2,500 per acre of beans and \$2,000 per acre of corn, how much of each crop should they plant to make the most profit?

2.1 Basic Operations with Matrices

Ex 1 For each matrix, tell the size, and find the transpose.

(a) $A = \begin{bmatrix} 1 & 4 & 9 & 5 \\ -2 & 7 & 0 & 1 \end{bmatrix}$

(b) $B = \begin{bmatrix} 3 & 2 & -1 \end{bmatrix}$

(c) $C = \begin{bmatrix} 9 & 8 \\ 7 & -5 \end{bmatrix}$

matrix: an array of numbers

$A = [a_{ij}]$

$= \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$

size is $m \times n$

m = # of rows

n = # of cols

scalar: a constant multiplier

square matrix: when $n = m$

transpose of a matrix: switch cols w/ rows

i.e. if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

2.1 (cont)

Ex2 Use $A = \begin{bmatrix} 1 & 5 \\ 3 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -2 \\ 4 & 0 \\ 1 & 9 \end{bmatrix}$ and $C = \begin{bmatrix} -5 & -2 & 3 \\ 0 & 2 & 7 \end{bmatrix}$

$D = \begin{bmatrix} 11 & 8 \\ 0 & -3 \end{bmatrix}$

to do these computations

(a) $3A + D$

(b) $A^T - 2D^T$

(c) $(A+D)^T$

(d) $2B - C^T$

(e) $C + B^T$

2.2 Matrix Multiplication

Ex 1 For $A = \begin{bmatrix} 2 & -4 \\ 3 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

$C = \begin{bmatrix} 1 & -9 \end{bmatrix}$

do these computations, or state it's not possible.

(a) AB

(b) BA

(c) CB

Properties

① $(AB)C = A(BC)$

② $A(B+C) = AB + AC$

③ $(B+C)A = BA + CA$

④ $(AB)^T = B^T A^T$

$\boxed{\text{WARNING}}$

$AB \neq BA$

Identity Matrix

is always a square matrix

ex $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2.2 (cont)

EX2 For $Q = \begin{bmatrix} -1 & 4 \\ 1 & 2 \\ 0 & 8 \end{bmatrix}$ $P = \begin{bmatrix} 5 & 0 & -2 \\ 10 & -1 & 6 \end{bmatrix}$ and $R = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$

do these computations or state they're not possible.

(a) QP

(b) PR

(c) $P^T R$

(d) R^2

2.3 Gauss-Jordan Elemination

Ex 1 (a) Write the augmented matrix for the system of linear eqns.

$$x + 5y = 48$$
$$3x - 4y = -46$$

(b) Use Gauss-Jordan Elemination to solve the system of eqns in (a).

Elementary Row Ops

① switch two rows

② multiply one row by nonzero constant (permanently)

③ elemination step: multiply one row (temporarily) by a nonzero constant and add to another row and replace one of these 2 rows with the result.

2.3 (cont)

Ex 2 Use Gauss-Jordan Elemenation to solve.

(a) $13x - 1 = 9y$
$6x = 8y + 12$

(b) $2x + 6y = -3$
$x + 3y = 2$

(c) $3x - 2y = 12$
$-3x + 2y = -12$

(d) $2x + 5y - z = 2$
$x + 3y - 5z = -13$
$y - 8z = -25$

2.3 (cont)

Ex 2 (cont)

(e) $x - y - z = 1$

$-x + 2y - 3z = -4$

$3x - 2y - 7z = 0$

2.4 Inverse Matrices

Ex 1 Find A^{-1} or state it does not exist (DNE).

(a) $A = \begin{bmatrix} 3 & \frac{2}{3} \\ \frac{3}{4} & \frac{-1}{4} \end{bmatrix}$

(b) $A = \begin{bmatrix} 6 & 9 \\ -4 & -6 \end{bmatrix}$

- can only find A^{-1} if A is square
- not all square matrices have an inverse

To find A^{-1}:

1. augment A with Identity matrix
2. Do elementary rows ops until left side looks like I
3. right side is now A^{-1}

For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

IF A^{-1} exists

2.4 (cont)

EX2 Find A^{-1} or state it DNE.

(a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & -3 & -4 \\ 25 & 0 & 7 \\ 7 & 2 & 5 \end{bmatrix}$

2.4 (cont)

Ex 3 Solve this system of linear eqns using an inverse matrix

$$2x+y=14$$
$$5x-y=0$$

2.5 Applications with Matrices

Ex1 Using A=1, B=2, C=3, ..., Z=26, encrypt the message "LOVE" using encryption matrix

$$A = \begin{bmatrix} 2 & -3 \\ -2 & 4 \end{bmatrix}$$

Ex2 Decrypt this message "17 -12 -7 10 10 -10" knowing that it was sent w/ encryption matrix from example 1 (above).

2.5 (cont)

EX 3 A movie theater is showing two different movies. Tickets for the first movie cost \$4.50/person while the tickets for the second movie cost \$7.50/person. The total ticket sales for the day was \$10,002. The second movie is newer than the first movie, so there were 100 more tickets sold for the first movie, compared to the second movie. How many tickets of each type were sold?

2.5 (cont)

EX 4 Toshi met their friends at the coffee house. They bought a total of 12 items. The number of drinks purchased was 2 more than the number of danishes. Each mocha latte cost \$3.50, each cup of regular coffee cost \$2, and the danishes were \$1.50 each. They spent a total of \$27.50 for the lattes, coffee, and danishes. How many of each item was purchased?