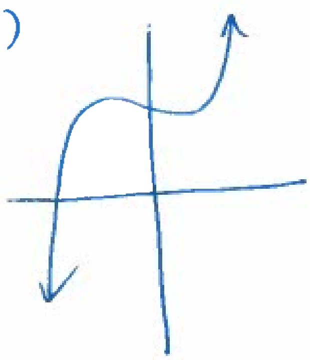
4.1 Inverse Functions

Ex 1 Is $f(x) = \sqrt[3]{x} + 2$ a function? Is it invertible? If so, find the inverse, $f^{-1}(x)$.

Ex 2 Do these fns have an inverse? Why or why not?

(a)



(b)

If $f(g(x)) = g(f(x)) = x$, then $g(x)$ is inverse fn of $f(x)$. It's written as $f^{-1}(x)$ and read "f inverse of x".

Vertical line test:
used to decide if a graph represents a functn.

Horizontal line Test:
used to decide if a graph represents a function that's invertible

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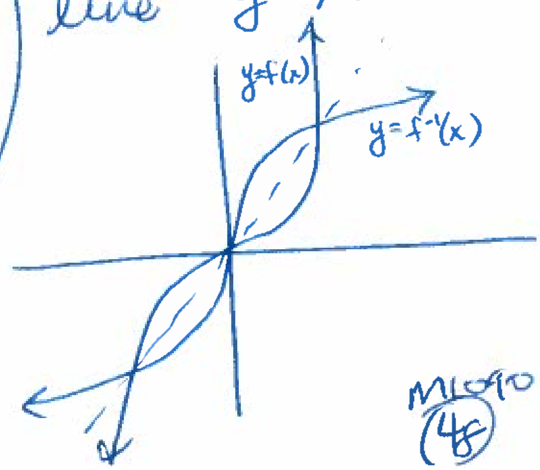
inverse fn graphs are reflected across line $y = x$.



4.1 (cont)

Ex 3 Find $f^{-1}(x)$ for

(a) $f(x) = \dfrac{2x+1}{2x+5}$

(b) $f(x) = 4(x-1)^3 + 7$

Given $f(x)$,
To find the inverse function $f^{-1}(x)$:

① If x is only written down once in fn defn, you can undo the layers of what happened to x in the reverse order to create $f^{-1}(x)$.

OR

② (This technique works all the time.)
(a) swap y and x.
(b) Resolve for y by itself. And what you have is $f^{-1}(x)$.

Ex 4 For $f(x) = (x-1)^2 + 3$, does $f^{-1}(x)$ exist, given the domain of $f(x)$ is $\mathbb{R}$? If not, then how can you restrict the domain of $f(x)$ so $f^{-1}(x)$ exists? Find $f^{-1}(x)$.

Fill in table

	Domain	Range
$f(x)$		
$f^{-1}(x)$		

4.2 Exponential Functions

Ex 1 Simplify each expression.

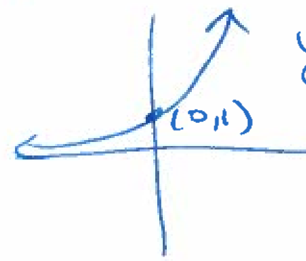
(a) $(3^{2x})^x$

(b) $\frac{4^{x+1}}{4^{2-x}}$

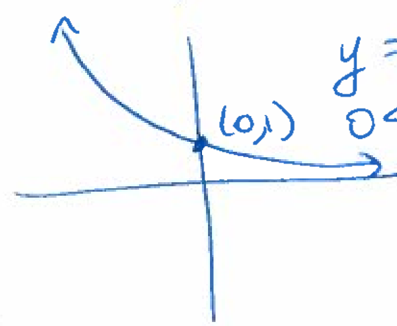
(c) $5^x 5^2 25^4$

$f(x) = a^x$ is an exponential fn; a is fixed, $a \neq 1$, $a > 0$; notice that the variable, x, is in the exponent

General graphs of exponential fns:



$y = a^x$
$a > 1$



$y = a^x$
$0 < a < 1$

(Review) Rules of Exponents:

(1) $a^m a^n = a^{m+n}$

(2) $\frac{a^m}{a^n} = a^{m-n}$

(3) $(a^m)^n = a^{mn}$

(4) $a^0 = 1$, if $a \neq 0$

(5) $a^{-n} = \frac{1}{a^n}$

(6) $(ab)^m = a^m b^m$

4.2 (Cont)

Ex 2 Sketch the graph of these fns.

(a) $f(x) = 2^x$

(b) $f(x) = 2^{x+3} - 1$

(c) $f(x) = \left(\frac{1}{3}\right)^x$

(d) $f(x) = -\left(\frac{1}{3}\right)^x$

4.2 (cont)

Ex 3 The demand fn is given by $p = 400 - 0.3e^{0.005x}$.
Find the price, p, when demand is 60.

Ex 4 A car costs \$25,000 new. Its depreciating value is given by $d(t) = 25{,}000(0.7)^t$ where t is time (in years) after the purchase date.
What is the car worth in 6 years?

4.3 Logarithmic Functions

EX1 Rewrite these statements as their equivalent statement, using defn of log.

(a) $\log_3 81 = 4$

(b) $\log_5 \frac{1}{125} = -3$

(c) $4^3 = 64$

(d) $10^6 = 1000000$

Defn $a > 0,\ a \neq 1$

$$\log_a x = y \iff a^y = x$$

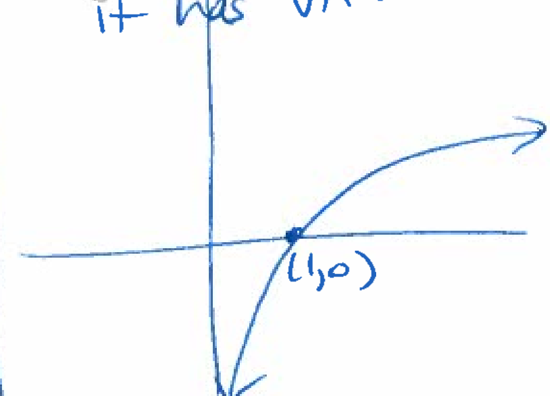
read "log base a of x equals y" is the same information as "a to the y equals x".

$\log x = \log_{10} x$

$\ln x = \log_e x$

General shape of $y = \log_a x$

it has VA at $x = 0$



4.3 (cont)

Ex2 Simplify these expressions, without a calculator.

(a) $\log_8\left(\frac{1}{64}\right)$

(b) $\ln(e^5)$

(c) $\log_3 1$

(d) $\log 1000$

Ex3 For each function (i) find domain, (ii) find x-intercept, (iii) find VA, (iv) sketch graph.

(a) $f(x) = \ln(-x) + 5$

(b) $f(x) = -\log_2(x-1)$

4.4 Properties of Logarithms

Ex1 Use properties of logs to expand these expressions completely.

(a) $\log_5\left(\frac{x^2}{x-1}\right)$

(b) $\ln\left(3x(x+5)\right)^3$

(c) $\log\left(\frac{(x+2)}{10}\right)$

$a>0, a\neq 1, m>0, n>0$

① $\log_a 1 = 0$

② $\log_a a = 1$

③ $\log_a a^x = x$

④ $a^{\log_a x} = x$ (for all $x>0$)

⑤ $\log_a(mn) = \log_a m + \log_a n$

⑥ $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

⑦ $\log_a(m^n) = n\log_a m$

WARNINGS!

(1) $\log_a(m+n) \neq \log_a m + \log_a n$

(logarithms don't distribute)

(2) $(\log_a m)^n \neq \log_a m^n$

(different order of operations)

4.4 (cont)

Ex 2 Use log properties to condense these expressions fully.

(a) $\ln 6 - \ln 3 + \ln(5x)$

(b) $4\log_2 x + 5\log_2(x+2)$

Ex3 Evaluate these expressions without a calculator.

(a) $\log_2 16 - \log 100$

(b) $\log_5\left(\frac{100(15)}{12(125)}\right)$

Ex4 Given $\log_b x = 1.6$, $\log_b y = -2.1$ and $\log_b z = 4.2$,

find $\log_b\left(\frac{xy}{z}\right)$

4.5 Logarithmic and Exponential Equations

Ex 1 Solve these eqns

(a) $\log_2(x^2-9) - \log_2(x+3) = 3$

(b) $\log(x^2) + 1 = \log(3x+1)$

Solve logarithmic Eqn

① Get all log terms on one side

② Use log properties to condense into one log expression

③ Either
(a) use defn of log to rewrite as equivalent exponential statement

OR (b) exponentiate both sides w/ same base as log

④ Continue Solving

⑤ check your answers against domain.

4.5 (cont)

Ex 2 Solve these eqns.

(a) $12e^{2x-1}+4=20$

(b)

Solve Exponential Eqn

① Isolate exponential on one side

② Either
(a) use defn of log to rewrite into equivalent logarithmic statement
OR
(b) take log of both sides

③ Continue solving eqn.

Change of base formula

$$\log_b x = \frac{\log_a x}{\log_a b} = \frac{\ln x}{\ln b} = \frac{\log x}{\log b}$$

4.5 (cont)

Ex 3 Solve these (harder) eqns. (Hint: These may not use strategies for typical log/exponential eqns.)

(a) $\log_2(\log_x 16) = 2$

(b) $\ln e^x + e^{\ln x} + \ln e = 4$

(c) $16^{x+1} = 4^{4x}$

(d) $5^{2x} - 2(5^x) - 3 = 0$

Ex 4 Use change of base formula to evaluate each logarithmic expression.

(a) $\log_3(12)$

(b) $\log_5\left(\frac{1}{20}\right)$

4.6 Logarithmic and Exponential Business Applications

Ex 1 If the population of SLC in 2009 is 300,000 and it was 250,000 in 1995, what is growth rate for that time period?

Ex 2 A population of mosquitoes grows exponentially at a rate of 59% per year. How long does it take for the mosquito population to double? (Hint: If the population originally is 100, then after year 1, it's 159.)

Exponential Growth/Decay can be modeled using

$$\boxed{y = y_0 e^{kt}}$$

y = something that's growing/decaying over time

t = time

y_0 = initial amt of stuff (i.e. y at time 0)

k = growth/decay rate

$k > 0$ growth

$k < 0$ decay

4.6 (cont)

Ex 3 Suppose demand for white-out tape behaves according to the formula $5p\log(q) = 50$, where q = quantity demanded & p = price. What is the price consumers are demanding for a quantity of 100? What is the revenue when 100 units of white-out tape are sold? (Hint: revenue is price times # units sold.)

Ex 4 An investment compounded n times per year with an interest rate of r will double according to the formula

$$2 = \left(1 + \frac{r}{n}\right)^{nt}$$

where t = # yrs it takes to double and n = # compoundings per year.

If the interest rate is 5% and the account is compounded monthly, how long will it take to double?