Math1090 Final Exam
Spring, 2016

Instructor: ______________________

Name ______________________ Date __________ Student id #: ______________

Instructions:

- Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown.

- All answers should be completely simplified, unless otherwise stated.

- Make sure all cell phones (and all other electronics that can connect to the internet) are put away and out of sight. If you have a cell phone/other electronic device out at any point, for any reason, you will receive a zero on this exam.
- You may use a scientific calculator only. No graphing nor programmable calculators allowed.

- You will be given an opportunity to ask clarifying questions about the instructions within the first fifteen minutes of the time the scheduled final exam is advertised by the University to begin. The questions will be answered for the entire class. After that, no further questions will be allowed, for any reason.

- You must show us your U of U student ID card when finished with the exam.

- You can ask the proctor/instructor for scratch paper. You may use NO other scratch paper. Please transfer all finished work onto the proper page in the test for us to grade there. We will not grade the work on the scratch pages.

- You are allowed to use one 8.5x11 inch sheet of paper (front and back) for your reference during the exam.

STUDENT—PLEASE DO NOT WRITE BELOW THIS LINE. THIS TABLE IS TO BE USED FOR GRADING.

Page	Problems	Score
2	#1	
3	#2	
4	#3a	
5	#3b	
6	#4	
7	#5	
8	#6	
9	#7	
10	#8	
11	#9	
12	#10	
12	#11	
13	#12	

Raw Total (out of 200):	
Total Percentage:	

1. (a) (points) Graph the region of the plane (the feasible region) that satisfies the following inequalities.

$$x \geq 0$$
$$y \geq 0$$
$$x + y \leq 5$$
$$5y - 3x \leq 15$$

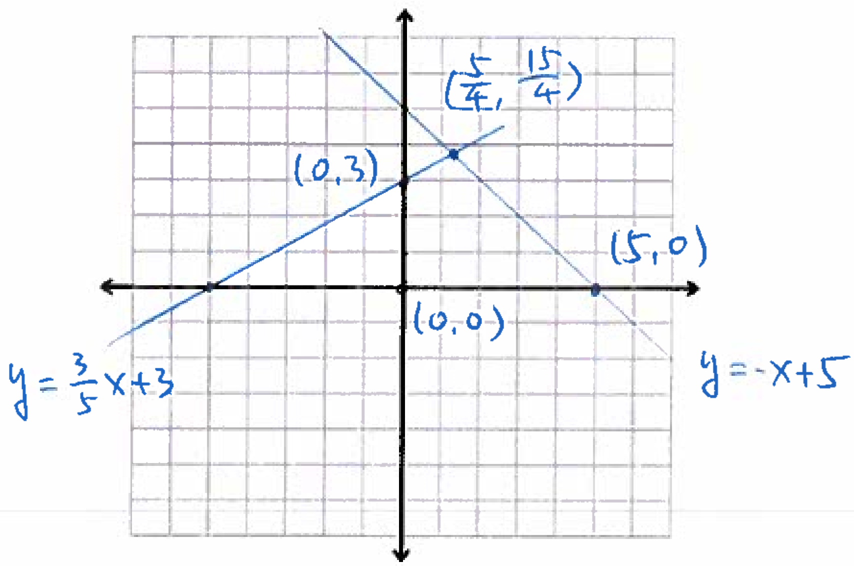
Make sure to label the vertices of the region and shade in the desired region.

$x \geq 0$

$y \geq 0$

$y \leq -x + 5$

$y \leq \frac{3}{5}x + 3$



$$\begin{cases} y = -x + 5 \\ y = \frac{3}{5}x + 3 \end{cases}$$

$$-x + 5 = \frac{3}{5}x + 3$$
$$2 = \frac{8}{5}x$$
$$\Rightarrow x = \frac{10}{8} = \frac{5}{4}$$

$$y = 5 - x = 5 - \frac{5}{4} = \frac{15}{4}$$

(b) (points) Find the maximum and minimum value of the objective function $P = 5 - x + y$ subject to the constraints in part (a) (above).

$(0,0) \rightarrow P(0,0) = 5$

$(0,3) \rightarrow P(0,3) = 8$

$(5,0) \rightarrow P(5,0) = 0$

$(\frac{5}{4}, \frac{15}{4}) \rightarrow P(\frac{5}{4}, \frac{15}{4}) = \frac{15}{2} = 7.5$

Max P-value: 8

Min P-value: 0

2. The supply equation for IKEA tables is $3p - 2q = 200$ and the demand equation is $p = 200 - 2q$.
(Remember that p = price and q = quantity.)
(a) (points)What is the equilibrium quantity and price?

$$\begin{cases} 3p - 2q = 200 \\ p = 200 - 2q \end{cases} \Rightarrow 3 \cdot (200 - 2q) - 2q = 200$$

$$\Rightarrow 600 - 8q = 200$$

$$\Rightarrow 8q = 400$$

$$q = 50$$

$$p = 200 - 2 \cdot 50$$

$$p = 100$$

Equilibrium quantity: ___50___ tables equilibrium price: $___100___

(b) (points) If the market price of IKEA tables is $200 per table, then how many will the company make (i.e. how many tables will IKEA be willing to supply)?

supply ~~function~~ equation: $3p - 2q = 200$

$\& \ p = 200 \Rightarrow 3 \cdot 200 - 2q = 200$

$\Rightarrow q = 200$

Number of tables: ___200___

(c) (points) If the market price of IKEA tables is $50 per table, then how many tables will customers buy (i.e. how many tables will be demanded)?

demand equation: $p = 200 - 2q$

$p = 50 \qquad q = \dfrac{200 - p}{2} = \dfrac{150}{2} = 75$

Number of tables: ___75___

3. Given the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.

(a) (points) Find the inverse matrix A^{-1}. *(Note: To get credit for this problem, you must show all your work here, using Gauss-Jordan elimination or elementary row operations, not just use a formula to give your final answer.)*

Augmented matrix: $\left(\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right) \longrightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array}\right)$ (-1)

$$\xrightarrow{(-1)} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & -1 & 1 \end{array}\right) \xrightarrow{(-1)} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right) \xrightarrow{(-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} -1 & 1 & 0 \\ 2 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

Note: This is #3 continued.

(b) (points) Solve this system of equations. *(Notice that you can choose to use the inverse matrix from part a for this problem, or you can solve this using Gauss-Jordan elimination, substitution, or some other method, but not by guessing.)*

$$y+z=0$$
$$x+y+z=1$$
$$x+2y+3z=2$$

method 1:

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}^{-1}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}=\begin{pmatrix} -1 & 1 & 0 \\ 2 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$=\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

method 2: $\begin{cases} y+z=0 & (1) \\ x+y+z=1 & (2) \end{cases}$ $\quad (2)-(1) \Rightarrow x=1$

$\Rightarrow 2y+3z=1$ $\qquad \Rightarrow z=1$

& $2y+2z=0$ (from (1)) & $y+z=0 \Rightarrow y=-1$

method 3: G-J elimination (no space) Similar to the procedu
in 3.(a)

Solution: $x=1 \quad y=-1 \quad z=1$

4. Use these matrices to compute the following or state why that the computation is not possible.

$$A=\begin{bmatrix}5 & 2\\ -2 & 3\end{bmatrix},\ B=\begin{bmatrix}7 & 4\\ 0 & 0\\ 1 & -9\end{bmatrix},\ C=\begin{bmatrix}3 & 8 & -1\\ 0 & 3 & 5\end{bmatrix},\ D=\begin{bmatrix}-2 & 7 & 0\\ 4 & 1 & -1\end{bmatrix}$$

(a) (points) $D-B^{T}$

$$\begin{pmatrix}-2 & 7 & 0\\ 4 & 1 & -1\end{pmatrix}-\begin{pmatrix}7 & 0 & 1\\ 4 & 0 & -9\end{pmatrix}=\begin{pmatrix}-9 & 7 & -1\\ 0 & 1 & 8\end{pmatrix}$$

Answer: $\begin{pmatrix}-9 & 7 & -1\\ 0 & 1 & 8\end{pmatrix}$

(b) (points) AD

$$\begin{pmatrix}5 & 2\\ -2 & 3\end{pmatrix}\begin{pmatrix}-2 & 7 & 0\\ 4 & 1 & -1\end{pmatrix}=\begin{pmatrix}-2 & 37 & -2\\ 0 & -11 & -3\end{pmatrix}$$

Answer: $\begin{pmatrix}-2 & 37 & -2\\ 0 & -11 & -3\end{pmatrix}$

(c) (points) Using the matrices listed above, answer the following true/false questions.

We can compute the product of CA. True or False (circle one)

We can compute the product of AC. True or False (circle one)

5. For this quadratic function $y=-x^2+2x+3$, answer the following questions.

(a) (points) Find the vertex and axis of symmetry.

(Be sure to report your vertex as a point with two coordinates, and list the axis of symmetry as a line equation.)

(Hint: To answer this question most efficiently, you'll need to complete the square.)

$$y = -x^2+2x+3$$
$$= -x^2+2x-1+4$$
$$= -(x-1)^2+4$$

Axis of symmetry: $x=1$

Vertex: $(1, 4)$

(b) (points) Find the x- and y-intercepts. If there are none, then write NONE as your answer.

(Be sure to list them as points, with two coordinates.)

x-intercept: $y=0$. $-(x-1)^2+4=0 \Rightarrow (x-1)^2=4 \Rightarrow x-1=\pm 2$

$\Rightarrow x=3$ or -1

y-intercept: $x=0$. $y=3$

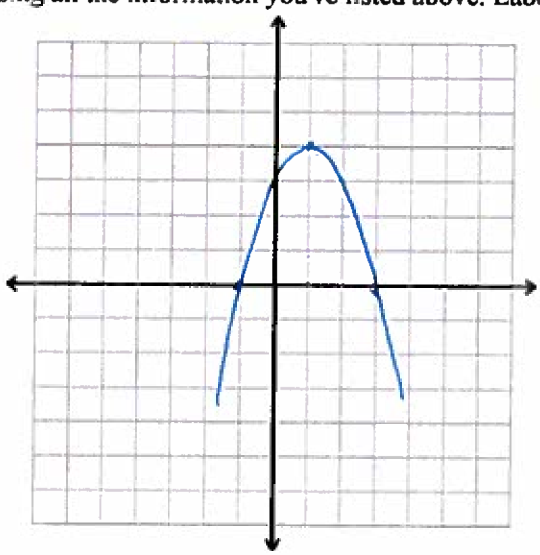
x-intercept(s): $(3,0)$ $(-1,0)$

y-intercept: $(0,3)$

(c) (points) Is the parabola concave up or down?

concave up   concave down   *(circle one)*

(d) (points) Graph the parabola, using all the information you've listed above. Label at least three points on the graph.



6. A tailor manufactures customized jumpers. Suppose the tailor sells each jumper for \$100. Call x the number of jumpers sold by the tailor. The business has fixed costs of \$1,500 and each jumper manufactured costs (x+20) dollars.

(a) (points) Find the revenue function.

Revenue $R(x) =$ $100x$

(b) (points) Find the cost function.

Cost $C(x) =$ $(x+20)x + 1500$

(c) (points) Find the profit function.

$$P(x) = R(x) - C(x)$$
$$= 100x - x^2 - 20x - 1500$$

Profit $P(x) =$ $-x^2 + 80x - 1500$

(d) (points) How many jumpers need to be sold to maximize the profit?

$$P(x) = -x^2 + 80x - 1500$$
$$= -x^2 + 80x - 1600 + 100$$
$$= -(x-40)^2 + 100.$$

Number of jumpers to maximize profit: 40.

(e) (points) What is the maximum possible profit?

Max profit: 100.

7. Solve these equations. If there is no solution, write NS or NONE. *(Note: Be sure to check your answers and state only the valid solutions.)*

(a) (points) $\log_2 x - \log_2(x+3) = 2$

$$\log_2 \frac{x}{(x+3)} = 2.$$

domain:

$$\left.\begin{array}{l} x > 0 \\ x > -3 \end{array}\right\} \Rightarrow x > 0.$$

$$\Rightarrow 2^2 = \frac{x}{x+3}$$

This means $x = -4$ is not an allowable solution.

$$\Rightarrow 4(x+3) = x$$

$$\Rightarrow 4x + 12 = x$$

$$\Rightarrow 3x = -12$$

$$\Rightarrow x = -4$$

Solution(s): NS

(b) (points) $2e^{3x+1} - 5 = 0$ *(Give EXACT answer—no calculator approximations.)*

$$e^{3x+1} = \frac{5}{2}$$

$$3x + 1 = \ln\frac{5}{2}$$

$$x = \frac{1}{3}\left(\ln\frac{5}{2} - 1\right)$$

Solution(s): $\frac{1}{3}\left(\ln\frac{5}{2} - 1\right)$

8. If I invest $5,000 in an account earning 6% interest, how long will it take to double my investment, with each of these situations? *(Use your calculator for this question.)*

(a) (points) The account earns interest compounded quarterly.

$$2 \cdot 5000 = 5000 \cdot \left(1 + \frac{6\%}{4}\right)^{4 \cdot t}.$$

$$\Rightarrow \quad 2 = 1.015^{4t}.$$

take log: $\Rightarrow \quad 4t = \frac{\log 2}{\log 1.015} \approx 46.56$

$t \approx 11.64 \approx 11$ yrs and 3 quarters

Time to double my investment: (11.64 years) or (11 years 3 quarters

(b) (points) The account earns interest compounded continuously.

$$2 \cdot 5000 = 5000 \cdot e^{6\% t}.$$

$$6\% t = \ln 2$$

$$t = \frac{\ln 2}{0.06} = 11.55.$$

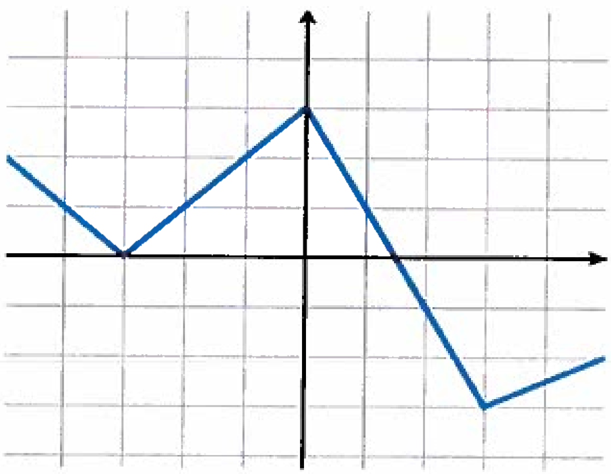
Time to double my investment: 11.55 years or

11 years and 3 quarters.

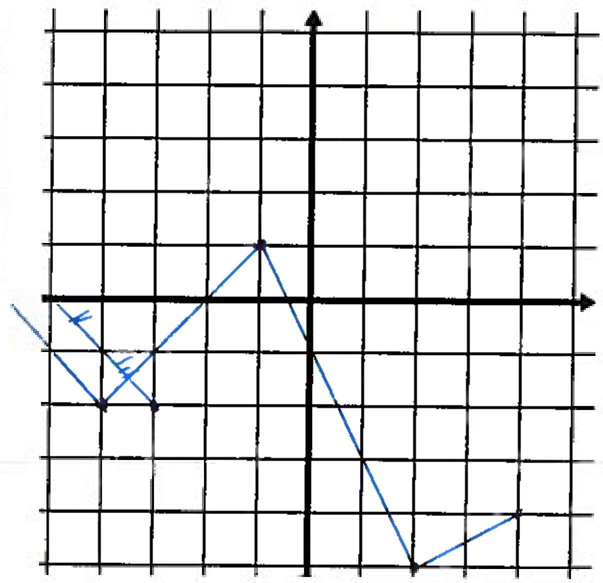
9. (points) Given this graph (on the left) for a function y = f(x), use the coordinate axes on the right to graph

$$y = f(x+1) - 2 \ .$$

y = f(x)



y = f(x+1) – 2



10. Alex purchases a home for $350,000. He takes out a loan for this, making quarterly payments with an interest rate of 4.5% compounded quarterly for 20 years.

(a) (points) What are his quarterly payments?

Formula used: $R = S \dfrac{r_c}{1-(1+r_c)^{-N}}$

$350000 \dfrac{0.01125}{1-(1.01125)^{-80}}$

$\dfrac{4.5\%}{4} = 0.01125$

$= 6658.13$

quarterly payment = $6658.13

(b) (points) Find the unpaid balance after 10 years.

Formula used: $P = R \dfrac{1-(1+r_c)^{-N}}{r_c}$ (Present value formula)

$6658.13 \dfrac{1-1.01125^{-40}}{0.01125} = \$213{,}514.59$

unpaid balance after 10 years = $213514.59

11. (points) A company is expected to replace one of their machines in 15 years. Market studies expect the cost of such a machine to be $68,000 in 15 years. The company sets up a sinking fund to pay for the new machine in the future. Payments are made to the fund twice each year. If the fund earns 7% interest compounded twice yearly, how much should each payment be?

Formula used: $R = S \dfrac{r_c}{(1+r_c)^N - 1}$

$68000 \dfrac{\frac{7\%}{2}}{\left(1+\frac{7\%}{2}\right)^{30} - 1} = 68000 \dfrac{0.035}{(1.035)^{30} - 1} = 1317.25$

sinking fund payment = $1317.25

12. Given functions $f(x)=\frac{1}{x}+5$ and $g(x)=x^2+2x-3$, compute the following.

(a) (points) Domain of $f(x)$ and $g(x)$.

Domain of $f(x)$: $x \neq 0$. $(x \in \mathbb{R} - \{0\})$

Domain of $g(x)$: $x \in \mathbb{R}$ (all real numbers)

(b) (points) $(f \circ g)(x)$

$$\frac{1}{x^2+2x-3}+5$$

$(f \circ g)(x) = \frac{1}{x^2+2x-3}+5$

(c) (points) $\left(\frac{g}{f}\right)(-1)$

$$\frac{g(-1)}{f(-1)} = \frac{1-2-3}{-1+5} = \frac{-4}{-4} = 1$$

$\left(\frac{g}{f}\right)(-1) = 1$

(d) (points) $f^{-1}(x)$

$$y = \frac{1}{x}+5$$

$$\cancel{y}x = \frac{1}{y}+5$$

$$x-5 = \frac{1}{y}$$

$$\frac{1}{x-5} = y$$

$f^{-1}(x) = \frac{1}{x-5}$