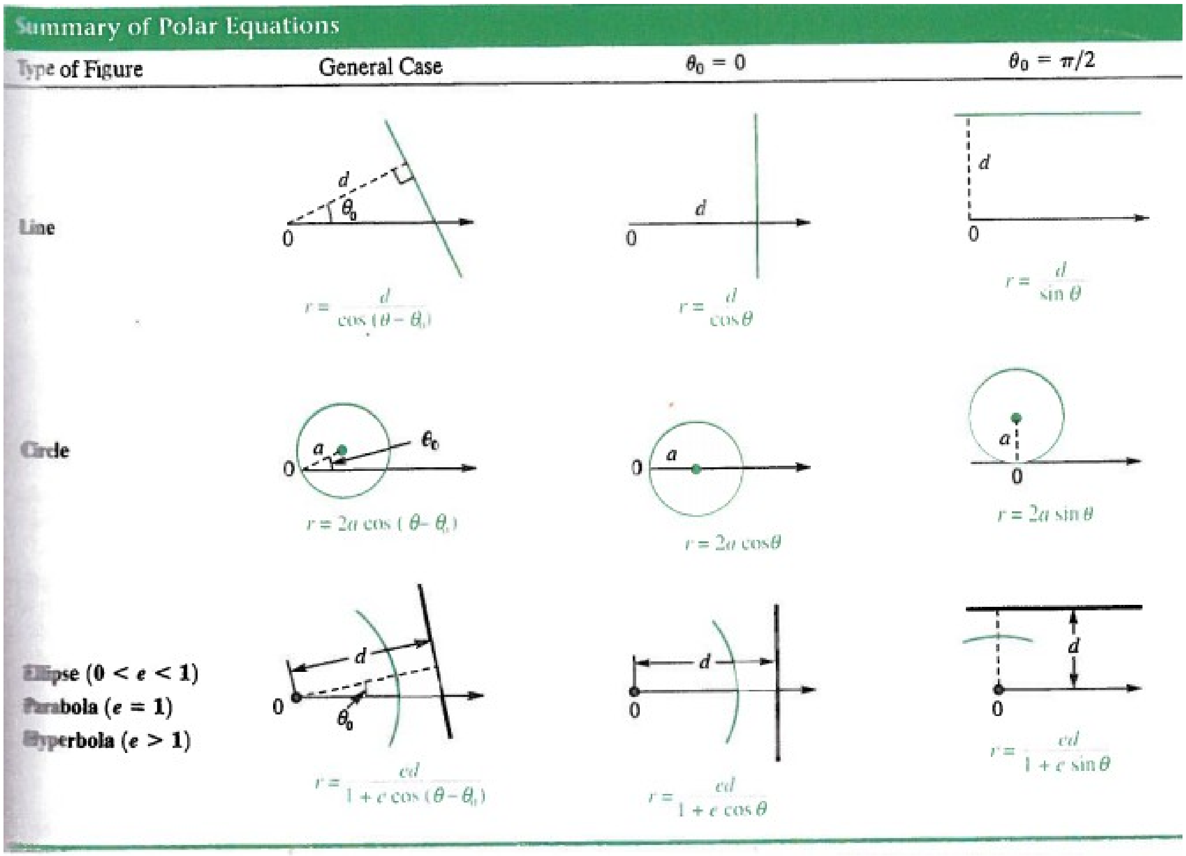
10.5 Polar Coordinates



Ex 1: Plot and give three other ways to write these points in polar coordinates.

(a) $\left(-1, \frac{15}{4}\pi\right)$

(b) $\left(-2\sqrt{2}, \frac{19}{2}\pi\right)$

Formulas

Polar to Cartesian/Rectangular Coordinates:

$$x = r\cos\theta$$
$$y = r\sin\theta$$

Cartesian/Rectangular to Polar Coordinates:

$$r^2 = x^2 + y^2$$
$$\tan\theta = \frac{y}{x}$$

Ex 2: Convert these points from polar coordinates to rectangular coordinates.

(a) $\left(-1, \frac{15}{4}\pi\right)$ (b) $\left(-2\sqrt{2}, \frac{19}{2}\pi\right)$

Ex 3: Convert these points from rectangular coordinates to polar coordinates.

(a) $(0, -2)$ (b) $(5\sqrt{2}, -5\sqrt{2})$

Ex 4: Name each curve and draw a quick sketch of the curve.

(a) $r = \frac{-9}{\cos\theta}$

(b) $r = -4\cos\theta$

(c) $r = \dfrac{4}{2 + \cos\left(\theta - \frac{\pi}{6}\right)}$

(d) $r = \dfrac{4}{2 + 2\cos\left(\theta - \frac{\pi}{6}\right)}$

(e) $r = \dfrac{4}{2 + 4\cos\left(\theta - \frac{\pi}{6}\right)}$

10.6 Graphs of Polar Equations

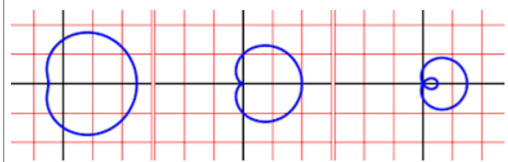
Ex 1: Name and sketch each graph. Look for symmetry.

(a) $r = 4 - 3\sin\theta$

(b) $r = 2 - 2\cos\theta$

Limacon:

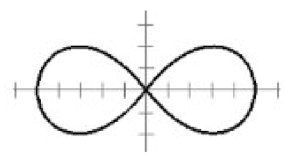
$r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$



$a > b$ $a = b$ $a < b$

Lemniscate:

$r^2 = \pm a\cos(2\theta)$ or $r^2 = \pm a\sin(2\theta)$

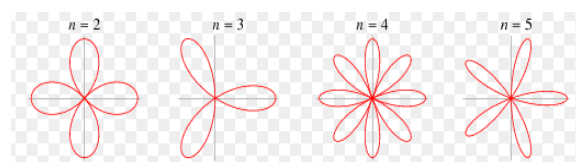


Rose:

$r = a\cos(n\theta)$ or $r = a\sin(n\theta)$

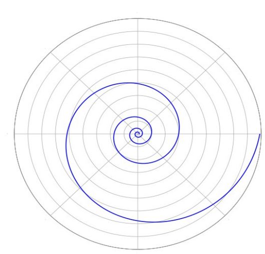
if n is odd, then there are n leaves (or petals)

if n is even, then there are 2n leaves (or petals



Spiral:

$r = a\theta$



Ex 1 (continued):

(c) $r^2 = -9\cos(2\theta)$	(d) $r = 4\cos(3\theta)$
(e) $r = 4\cos(4\theta)$	(f) $r = \frac{1}{2}\theta$

Ex 2: Sketch the curves and find the points of intersection.

$$r = 6\sin\theta \quad \text{and} \quad r = \frac{6}{1 + 2\sin\theta}$$

10.7 Calculus in Polar Coordinates

Ex 1: Find the area inside the small loop of $r=2-4\cos\theta$.

Ex 2: Setup the integral for the area in Quadrant 2 inside $r=2+2\sin\theta$ and outside $r=2+2\cos\theta$.

Area "between" two polar curves:

(1) $A=\frac{1}{2}\int_{\alpha}^{\beta} \text{radius}^2 \, d\theta$

where $\text{radius}^2=(f(\theta))^2$

if it's just one region with only one curve boundary

or possibly

(2) $A=\frac{1}{2}\int_{\alpha}^{\beta} (\text{outer radius}^2-\text{inner radius}^2)\, d\theta$

where $\text{outer radius}^2=(f(\theta))^2$ and

$\text{inner radius}^2=(g(\theta))^2$

Tangent Line Slope:

Given $r=f(\theta)$ curve, slope is

$$m=\frac{dy/d\theta}{dx/d\theta}=\frac{f(\theta)\cos\theta+f'(\theta)\sin\theta}{-f(\theta)\sin\theta+f'(\theta)\cos\theta}$$

Ex 3: Find the slope of the tangent line to

(a) $r=4-3\cos\theta$ at $\theta=\frac{\pi}{3}$.

(b) $r=\sin(2\theta)$ at $\theta=\frac{\pi}{3}$.

Ex 4: A goat is tethered to the edge of a circular pond of radius *a* by a rope of length *ka* (for some constant *k* such that $0<k\leq 2$. Find its grazing area.