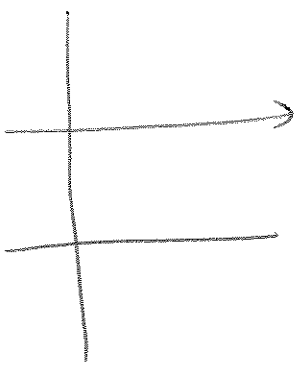
1.6 Library of Parent Functions

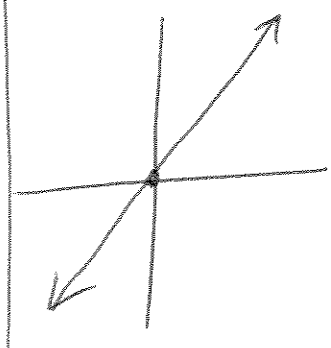
Constant Fn



$y = f(x) = c$

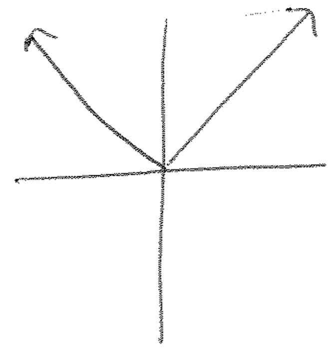
$c \in \mathbb{R}$

Identity Fn



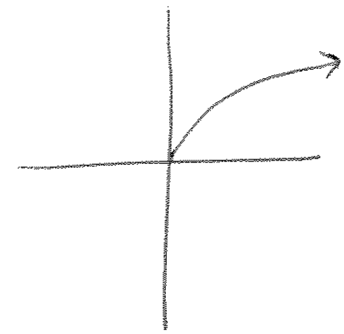
$f(x) = y = x$

Abs. Value Fn



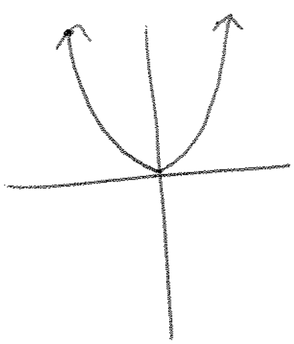
$y = f(x) = |x|$

Square Root Fn



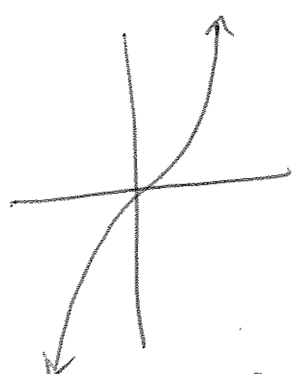
$y = f(x) = \sqrt{x}$

Quadratic Fn



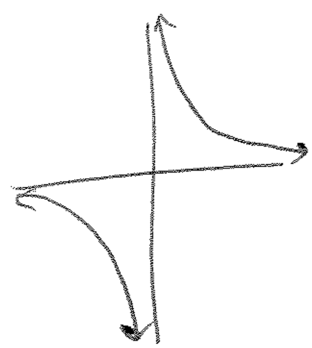
$y = f(x) = x^2$

Cubic Fn



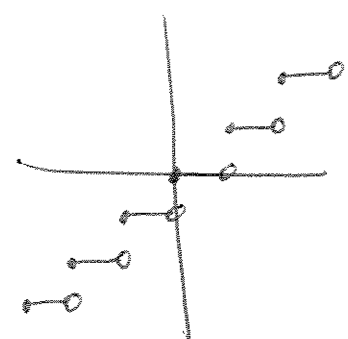
$y = f(x) = x^3$

Reciprocal Fn



$y = f(x) = \frac{1}{x}$

Greatest Integer Fn



$y = f(x) = [[x]]$

General polynomial graphs:

1.6 (cont)

Ex 1 Graph these functions.

(a) $f(x) = [[x-3]]$

(b) $y = [[x]] - 1$

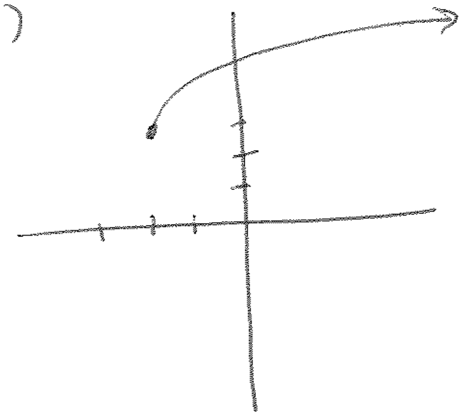
(c) $f(x) = \begin{cases} 1-(x-1)^2 & , x \leq 2 \\ \sqrt{x-2} & , x > 2 \end{cases}$

(d) $f(x) = 2(x+3)^3 + 1$

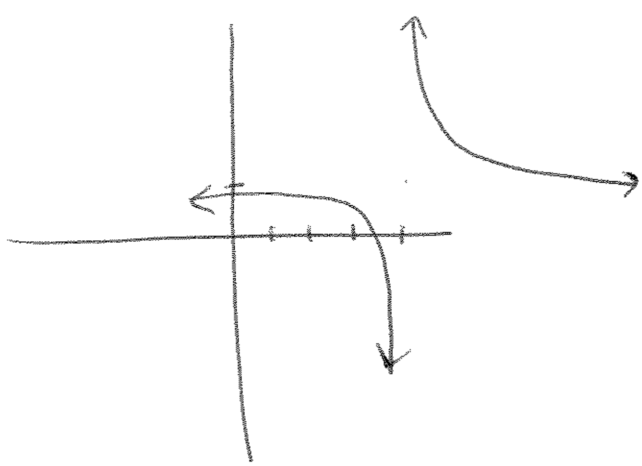
1.6 (cont)

Ex 2 Given the graph, write the equation for the function it represents.

(a)



(b)



1.7 Transformations of Functions

Types of Transformations to $y=f(x)$

		Example
① Shift :	$h(x)=f(x)+c$ $h(x)=f(x-c)$	$y=x^2+2$ $y=(x-1)^3$
② Reflect :	$g(x)=-f(x)$ $g(x)=f(-x)$	$y=-x^2$ $y=\sqrt{-x}$
③ Stretch/Shrink :	$k(x)=cf(x)$ $k(x)=f(cx)$	$y=5x^3$ $y=\sqrt{\frac{1}{2}x}$

Ex 1 Graph these fns.

(a) $y=\sqrt{-x}$

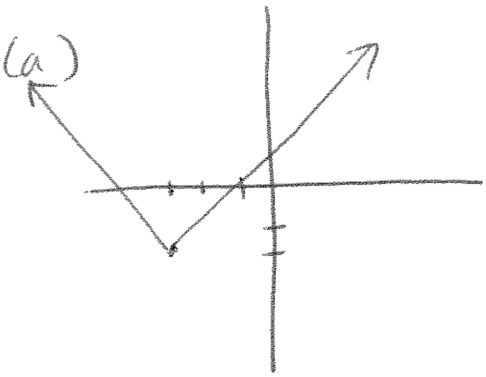
(b) $y=-x^2+3$

(c) $y=|x-2|+1$

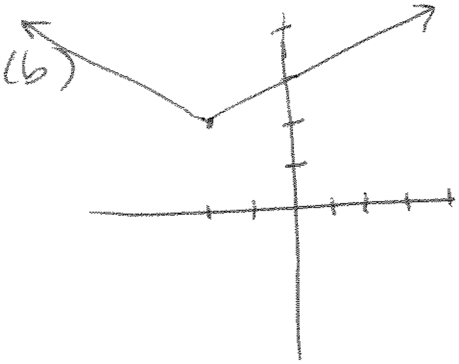
1.7 (cont)

Ex 2 Write an eqn for these graphs.

(a)



(b)



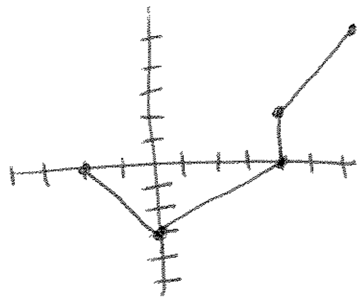
Ex 3 Describe transformations compared to base graph.

(a) $g(x) = 2(x+1)^3 - 9$

(b) $h(x) = -[[x+4]] - 3$

1.7 (cont)

Ex 4 Given this graph for $f(x)$, sketch the graphs of the transformed functions.



(a) $f(-x)$

(b) $f(x-1)+3$

(c) $-2f(x)$

1.8 Combinations of Functions: Composite Fns

Operations w/ Fns

① Sum: $(f+g)(x) = f(x) + g(x)$

② Difference: $(f-g)(x) = f(x) - g(x)$

③ Product: $(f \cdot g)(x) = f(x)\,g(x)$

④ Quotient: $(f \div g)(x) = \dfrac{f(x)}{g(x)}$, $g(x) \neq 0$

⑤ Composition: $(f \circ g)(x) = f(g(x))$

Ex 1 For $f(x) = \sqrt{x^2-4}$ and $g(x) = \dfrac{x^2}{x^2+1}$, find

(a) $(f+g)(x)$

(c) $\left(\dfrac{f}{g}\right)(x)$

(b) $(fg)(x)$

(d) $(f-g)(x)$

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1.8 (cont)

Ex 2 For $f(x)=\sqrt{x^2-4}$ and $g(x)=\frac{x^2}{x^2+1}$, find

(a) $f(g(x))$ and domain

(b) $g(f(x))$ and domain

Ex 3 For $f(x)=x^3-1$ and $g(x)=2x+5$, find

(a) $\left(\frac{f}{g}\right)(0)$

(b) $(fg)(2)+g(4)$

(c) $f(g(0))$

1.8 (cont)

Ex 4 For $f(x) = 3x+5$, find $(f \circ f)(x)$, and its domain.

Ex 5 Given $h(x) = \dfrac{4}{(5x+1)^2}$, find two fns f and g $\ni$ $(f \circ g)(x) = h(x)$.

1.9 Inverse Functions

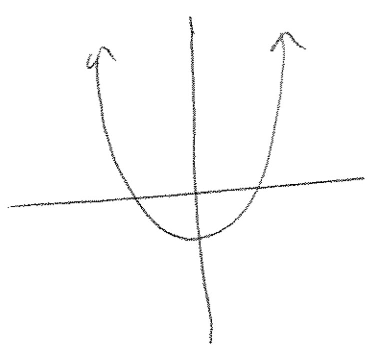
Defn If $f(f^{-1}(x)) = f^{-1}(f(x)) = x$, then $f(x)$ and $f^{-1}(x)$ are inverse functions. $\forall x$ in domains

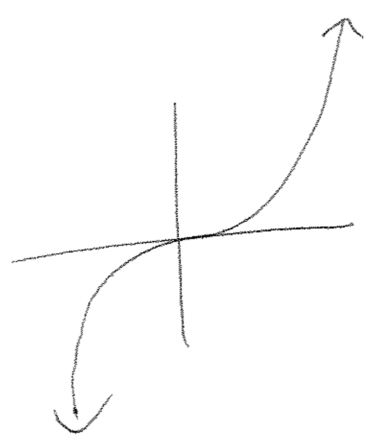
(notation: $f^{-1}(x)$ is read "f inverse of x"; that -1 indicates the inverse, it's <u>not</u> an exponent)

<u>Vocab</u>

<u>one-to-one</u>: a fn is one-to-one if each output has exactly one input.
(it passes horizontal line test)

☆ a one-to-one fn is invertible!





<u>Finding an Inverse Fn</u>

① <u>"Pants" method</u>

② <u>Algebraic Technique</u>

1.9 (cont)

Ex 1 For $f(x)$, find the inverse function $f^{-1}(x)$.

(a) $f(x) = \frac{x^5 - 1}{3}$

(b) $f(x) = \sqrt[3]{x+2} + 1$

Ex 2 Show that $f(x) = \frac{1}{1+x}$, $x \geq 0$, and $g(x) = \frac{1-x}{x}$, $0 < x \leq 1$

are inverse functions.

(a) algebraically

(b) graphically

1.9 (cont)

Ex 3 Find the inverse function of $f(x) = \frac{x-3}{x+2}$. Then, graph both $f(x)$ and $f^{-1}(x)$ on same coordinate plane. State domain and range of f and f^{-1}.

1.9 (cont)

Ex 4 Use $f(x)=3x+4$ and $g(x)=x^3$ to find $(g\circ f)^{-1}$.