

7.5 #33

①

$$\int \frac{(\sin^3 t - 8 \sin^2 t - 1) \cos t}{(\sin t + 3)(\sin^2 t - 4 \sin t + 5)} dt$$

$$\text{Let } u = \sin t \\ du = \cos t dt$$

$$= \int \frac{u^3 - 8u^2 - 1}{(u+3)(u^2 - 4u + 5)} du$$

$$= \int \frac{u^3 - 8u^2 - 1}{u^3 - u^2 - 7u + 15} du$$

$$(u+3)(u^2 - 4u + 5) \\ = u^3 - u^2 - 7u + 15$$

$$\begin{array}{r} u^3 - u^2 - 7u + 15 \overline{) u^3 - 8u^2 + 0u - 1} \\ \underline{-(u^3 - u^2 - 7u + 15)} \\ -7u^2 + 7u - 16 \end{array}$$

$$(\star) = \int 1 + \frac{-7u^2 + 7u - 16}{u^3 - u^2 - 7u + 15} du$$

$$\frac{-7u^2 + 7u - 16}{(u+3)(u^2 - 4u + 5)} = \frac{A}{u+3} + \frac{Bu + C}{u^2 - 4u + 5}$$

$$\Rightarrow -7u^2 + 7u - 16 = A(u^2 - 4u + 5) + (Bu + C)(u + 3)$$

$$u = -3, \quad -7(9) - 21 - 16 = A(9 + 12 + 5)$$

$$-(63 + 21 + 16) = 26A$$

$$-100 = 26A$$

$$A = \frac{-50}{13}$$

$$u = 0, \quad -16 = 5\left(\frac{-50}{13}\right) + 3C$$

$$-16 = \frac{-250}{13} + 3C$$

$$\frac{-16(13) + 250}{13} = 3C \Rightarrow \frac{42}{13(3)} = C \Rightarrow \boxed{\frac{14}{13} = C}$$

7.5 #33 (cont)

(2)

$$-7u^2 + 7u - 16 = -\frac{50}{13}(u^2 - 4u + 5) + (Bu + \frac{14}{13})(u+3)$$

$$u=1, \quad -7+7-16 = -\frac{50}{13}(1-4+5) + (B+\frac{14}{13})4$$

$$-16 = -\frac{100}{13} + 4B + \frac{56}{13}$$

$$-16 = -\frac{44}{13} + 4B$$

$$\frac{-16(13)+44}{13} = 4B \Rightarrow \frac{-164}{13} = 4B \Rightarrow \frac{-164}{13(4)} = B$$

$$\Rightarrow \boxed{\frac{-41}{13} = B}$$

$$(\star) = \int \left[1 + \frac{-50/13}{u+3} + \frac{-41/13 u + 14/13}{u^2 - 4u + 5} \right] du$$

$$= \int \left[1 - \frac{50}{13} \left(\frac{1}{u+3} \right) + \frac{1}{13} \left(\frac{-41u + 14}{u^2 - 4u + 5} \right) \right] du$$

$$(\heartsuit) = u - \frac{50}{13} \ln|u+3| + \frac{1}{13} \int \frac{-41u + 14}{u^2 - 4u + 5} du$$

$$\int \frac{-41u + 14}{u^2 - 4u + 5} du = \int \frac{-41u}{u^2 - 4u + 5} du + \int \frac{14}{u^2 - 4u + 5} du$$

$$u^2 - 4u + 5 = (u^2 - 4u + 4) + 5 - 4 = (u-2)^2 + 1$$

$$\Rightarrow -41 \int \frac{u}{(u-2)^2 + 1} du + 14 \int \frac{1}{(u-2)^2 + 1} du$$

$$= -41 \int \frac{u}{(u-2)^2 + 1} du + 14 \arctan(u-2) + C$$

$$\int \frac{u}{(u-2)^2+1} du$$

$$\text{Let } u-2 = \tan \theta$$

$$du = \sec^2 \theta d\theta$$

$$(u-2)^2+1 = \tan^2 \theta + 1 = \sec^2 \theta$$

$$\Rightarrow \int \frac{(\tan \theta + 2)}{\sec^2 \theta} (\sec^2 \theta) d\theta = \int (\tan \theta + 2) d\theta$$

$$= \int \frac{\sin \theta}{\cos \theta} d\theta + \int 2 d\theta$$

$$w = \cos \theta$$

$$-dw = \sin \theta d\theta$$

$$\Rightarrow = -\int \frac{1}{w} dw + 2\theta + C = -\ln |\cos \theta| + 2\theta + C$$

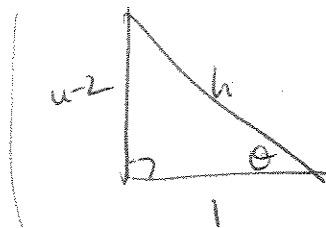
$$= -\ln \left| \frac{1}{\sqrt{(u-2)^2+1}} \right| + 2 \arctan(u-2)$$

$$= \ln(\sqrt{(u-2)^2+1}) + 2 \arctan(u-2) + C$$

$$(\heartsuit) = u - \frac{50}{13} \ln |u+3| + \frac{1}{13} \left[-41 \left(\ln(\sqrt{(u-2)^2+1}) + 2 \arctan(u-2) \right) \right. \\ \left. + 14 \arctan(u-2) \right] + C$$

$$= u - \frac{50}{13} \ln |u+3| - \frac{41}{13} \ln(\sqrt{(u-2)^2+1}) + \frac{-68}{13} \arctan(u-2) + C$$

$$= \sin t - \frac{50}{13} \ln(\sin t + 3) - \frac{41}{13} \ln(\sqrt{(\sin t - 2)^2 + 1}) \\ - \frac{68}{13} \arctan(\sin t - 2) + C$$



$$\tan \theta = \frac{u-2}{1}$$

$$(u-2)^2+1 = h^2$$

$$\sqrt{(u-2)^2+1} = h$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{(u-2)^2+1}}$$