

7.5
Ex 4

$$\int \frac{\cos x}{\sin^4 x - 16} dx$$

①

$$\begin{array}{l} u = \sin x \\ du = \cos x dx \end{array} \left| \begin{array}{l} = \int \frac{1}{u^4 - 16} du \\ = \int \frac{1}{(u-2)(u+2)(u^2+4)} du \quad (\star) \end{array} \right.$$

$$\frac{1}{(u-2)(u+2)(u^2+4)} = \frac{A}{u-2} + \frac{B}{u+2} + \frac{Cu+D}{u^2+4}$$

$$1 = A(u+2)(u^2+4) + B(u-2)(u^2+4) + (Cu+D)(u-2)(u+2)$$

$$u=2: \quad 1 = A(4)(8) \Rightarrow 32A=1 \Rightarrow \boxed{A = \frac{1}{32}}$$

$$u=-2: \quad 1 = B(-4)(8) \Rightarrow -32B=1 \Rightarrow \boxed{B = -\frac{1}{32}}$$

$$u=0: \quad 1 = \frac{1}{32}(2)(4) - \frac{1}{32}(-2)(4) + D(-2)(2)$$

$$1 = \frac{1}{4} + \frac{1}{4} - 4D$$

$$1 = \frac{1}{2} - 4D \Rightarrow \frac{1}{2} = -4D \Rightarrow \boxed{-\frac{1}{8} = D}$$

$$u=1: \quad 1 = \frac{1}{32}(1)(5) + B(-3)(5) + (-C + \frac{1}{8})(-3)(1)$$

$$1 = \frac{5}{32} - 15(\frac{-1}{32}) - 3(-C - \frac{1}{8})$$

$$1 = \frac{20}{32} + 3C + \frac{3}{8} \Rightarrow 1 = \frac{5}{8} + \frac{3}{8} + 3C \Rightarrow \boxed{C=0}$$

$$(\star) = \int \frac{\frac{1}{32}}{u-2} - \frac{\frac{1}{32}}{u+2} + \frac{-\frac{1}{8}}{u^2+4} du$$

$$= \frac{1}{32} \ln|u-2| - \frac{1}{32} \ln|u+2| - \frac{1}{8} \left(\frac{1}{2}\right) \arctan\left(\frac{u}{2}\right) + C$$

$$= \boxed{\frac{1}{32} \ln|u-2| - \frac{1}{32} \ln|u+2| - \frac{1}{16} \arctan\left(\frac{u}{2}\right) + C}$$

Ex 5 $\int \frac{x^6 - 7x^4 + 11x^3 - 13x^2 + x - 6}{x^3 - 2x^2} dx$ (improper rational fn)

$$\begin{array}{r}
 x^3 - 2x^2 \overline{) x^6 + 0x^5 - 7x^4 + 11x^3 - 13x^2 + x - 6} \\
 \underline{-(x^6 - 2x^5)} \\
 2x^5 - 7x^4 + 11x^3 - 13x^2 + x - 6 \\
 \underline{-(2x^5 - 4x^4)} \\
 -3x^4 + 11x^3 - 13x^2 + x - 6 \\
 \underline{-(-3x^4 + 6x^3)} \\
 5x^3 - 13x^2 + x - 6 \\
 \underline{-(5x^3 - 10x^2)} \\
 -3x^2 + x - 6
 \end{array}$$

$$\begin{aligned}
 & \int x^3 + 2x^2 - 3x + 5 + \frac{-3x^2 + x - 6}{x^3 - 2x^2} dx \\
 &= \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{3}{2}x^2 + 5x + \int \frac{-3x^2 + x - 6}{x^2(x-2)} dx
 \end{aligned}$$

$$\frac{-3x^2 + x - 6}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$

$$-3x^2 + x - 6 = A(x-2) + B(x-2) + Cx^2$$

$$-3x^2 + x - 6 = (A+C)x^2 + (-2A+B)x + (-2B)$$

$$A + C = -3$$

$$-2A + B = 1$$

$$-6 = -2B$$

$$-2A + 3 = 1$$

$$3 = B$$

$$1 + C = -3$$

$$-2A = -2$$

$$C = -4$$

$$A = 1$$

EX 5 (cont)

③

$$\int \frac{-3x^2 + x - 6}{x^2(x-2)} dx = \int \frac{1}{x} + \frac{3}{x^2} - \frac{4}{x-2} dx$$
$$= \ln|x| + 3\left(\frac{x^{-1}}{-1}\right) - 4 \ln|x-2| + C$$

⇒ entire answer is

$$\frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{3}{2}x^2 + 5x + \ln|x| - \frac{3}{x} - 4 \ln|x-2| + C$$