PDF $\quad f(x) = \begin{cases} \dfrac{1}{b-a} & a < x < b \\ 0 & \text{if } x \le a \text{ or } x \ge b \end{cases}$

(a) show $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = \cancel{\int_{-\infty}^{a} 0\,dx}^{\,0} + \int_a^b \frac{1}{b-a}\,dx + \cancel{\int_b^{\infty} 0\,dx}^{\,0}$$

$$= \int_a^b \frac{1}{b-a}\,dx = \frac{1}{b-a}\left(x\Big|_a^b\right) = \frac{1}{b-a}(b-a) = 1$$

(b) $$\mu = \int_{-\infty}^{\infty} x f(x)\,dx = \cancel{\int_{-\infty}^{a} x(0)\,dx}_{\,0} + \int_a^b \frac{x}{b-a}\,dx + \cancel{\int_b^{\infty} x(0)\,dx}_{\,0}$$

$$= \frac{1}{b-a}\int_a^b x\,dx = \frac{1}{2(b-a)}\,x^2\Big|_a^b = \frac{1}{2(b-a)}(b^2-a^2)$$

$$= \frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}} = \boxed{\frac{b+a}{2}}$$

(c) $$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx = \cancel{\int_{-\infty}^{a} 0\,dx}^{\,0} + \int_a^b \left(x - \frac{b+a}{2}\right)^2\left(\frac{1}{b-a}\right)dx + \cancel{\int_b^{\infty} 0\,dx}_{\,0}$$

$$= \frac{1}{b-a}\int_a^b \left(x - \frac{b+a}{2}\right)^2 dx = \frac{1}{3(b-a)}\left(x - \frac{b+a}{2}\right)^3\Big|_a^b$$

$$= \frac{1}{3(b-a)}\left[\left(b - \frac{b+a}{2}\right)^3 - \left(a - \frac{a+b}{2}\right)^3\right]$$

$$= \frac{1}{3(b-a)}\left[\left(\frac{b-a}{2}\right)^3 - \left(\frac{-(b-a)}{2}\right)^3\right] = \frac{1}{3(b-a)}\left(2\left(\frac{b-a}{2}\right)^3\right)$$

$$= \frac{2(b-a)^3}{3(8)(b-a)} = \boxed{\frac{(b-a)^2}{12}}$$

$$b - \frac{b+a}{2} = \frac{2b}{2} - \frac{b+a}{2} = \frac{b-a}{2}$$

$$a - \frac{a+b}{2} = \frac{2a}{2} - \frac{a+b}{2} = \frac{a-b}{2} = \frac{-(b-a)}{2}$$