Math1220 Midterm 3
Review Problems
(Chapter 9)

Problems

Given a_n , find the first three terms of the sequence $\{a_n\}$ and find $\lim_{n\to\infty} a_n$, if it exists, or state that the sequence diverges.

1. $a_n=(-1)^{n+1}\left(\frac{1}{n}\right)$
2. $a_n=\frac{5n}{e^{2n}}$
3. $a_n=\frac{\cos^2 n}{3^n}$
4. $a_n=e^{-n}\ln(n)$
5. $a_n=\frac{n^2}{2n-1}-\frac{n^2}{2n+1}$

Determine if each series is absolutely convergent, conditionally convergent, or divergent.

6. $\sum_{n=1}^{\infty}(-1)^n\frac{n}{n^2+1}$
7. $\sum_{n=1}^{\infty}(-1)e^{-n}$
8. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{3^n}{n^2+4}$
9. $\sum_{n=1}^{\infty}\frac{2^{n-1}}{5^n(n+1)}$
10. $\sum_{n=1}^{\infty}\frac{5^n}{n(3^{n+1})}$
11. $\sum_{n=2}^{\infty}\frac{(-1)^n 3}{n^4-1}$
12. $\sum_{n=1}^{\infty}\frac{10-2^n}{n!}$
13. $\sum_{n=1}^{\infty}\frac{\sin(\sqrt{n})}{\sqrt{n^3+1}}$
14. $1+\frac{3}{2!}+\frac{3(5)}{3!}+\frac{3(5)(7)}{4!}+\ldots$
15. $\sum_{i=1}^{\infty}(-1)^i\frac{2i+1}{i^2+i^3}$
16. $\frac{2}{1+e}+\frac{2}{8+e^2}+\frac{2}{27+e^3}+\frac{2}{64+e^4}+\ldots$

17. $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^5}$
18. $\sum_{n=1}^{\infty} (-1)^n \ 3^{\frac{1}{n}}$
19. $\sum_{j=1}^{\infty} \frac{(2\mathrm{j})^j}{(5\mathrm{j}+3\mathrm{j}^{-1})^j}$
20. $\sum_{n=1}^{\infty} \frac{\cos(\frac{n\pi}{6})}{n^2}$
21. $\sum_{i=1}^{\infty} (i^2+9)(-2)^{-i}$
22. $\frac{1+\cos(1)}{1+1}+\frac{2+\cos(2)}{8+1}+\frac{3+\cos(3)}{27+1}+\frac{4+\cos(4)}{64+1}+\ldots$
23. $\sum_{n=1}^{\infty} \frac{e^n}{n^e}$
24. $\sum_{k=0}^{\infty} \frac{1}{2+\left(\frac{1}{2}\right)^k}$
25. $\sum_{n=1}^{\infty} \frac{1-\cos n}{n^2}$
26. $\sum_{n=5}^{\infty} \frac{1}{n^2-4\mathrm{n}}$
27. $\sum_{n=1}^{\infty} n^{-2} e^{\frac{1}{n}}$

Find a series representation of these functions and state the interval of convergence.

28. $f(x)=\frac{x^2+1}{x-1}$
29. $f(x)=\frac{x}{2-3\mathrm{x}}$
30. $f(x)=\frac{x^3}{4-x^3}$
31. $f(x)=x^2 e^{x^2}$
32. $f(x)=\cosh(x)$

Find the convergence set for these power series.

33. $\sum_{n=0}^{\infty} \frac{(n+1)x^n}{(-3)^n}$
34. $\sum_{n=1}^{\infty} \frac{(x+10)^n}{n(2^n)}$

35. $\sum_{n=1}^{\infty} \frac{(-1)^n (4^{2n})(x^n)}{\sqrt{n+1}}$

36. $\sum_{j=0}^{\infty} \frac{5^j x^{5j}}{(5j!)}$

37. $\sum_{n=0}^{\infty} \frac{(x+5)^n}{(n+5)!}$

Find the Taylor series for these functions through the third degree term, centered about the given a value.

38. $f(x)=\frac{1}{1+x+x^2}$, $a=0$

39. $g(x)=\frac{-\ln(1+x)}{1+x}$, $a=0$

40. $f(x)=\sin x$, $a=\frac{\pi}{6}$

41. $f(x)=2-x+3x^2-x^3$, $a=-1$

42. $h(x)=\cosh(x)$, $a=1$

Find a formula for the remainder function for the Taylor polynomial of order 6 based at a, namely $R_6(x)$. Then, obtain a good bound for $|R_6(0.2)|$.

43. $f(x)=\frac{1}{x-3}$, a = 1

44. $f(x)=\ln(1+x)$, a = 0

45. $g(x)=3e^x$, a = 0

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Answers

1. $a_1=1$, $a_2=\frac{-1}{2}$, $a_3=\frac{1}{3}$, converges to 0
2. $a_1=\frac{5}{e^2}$, $a_2=\frac{10}{e^4}$, $a_3=\frac{15}{e^6}$, converges to 0
3. $a_1=\frac{\cos^2 1}{3}$, $a_2=\frac{\cos^2 2}{9}$, $a_3=\frac{\cos^2 3}{27}$, converges to 0
4. $a_1=0$, $a_2=\frac{\ln 2}{e^2}$, $a_3=\frac{\ln 3}{e^3}$, converges to 0
5. $a_1=\frac{2}{3}$, $a_2=\frac{8}{15}$, $a_3=\frac{18}{35}$, converges to $\frac{1}{2}$
6. converges conditionally
7. converges absolutely
8. diverges
9. converges absolutely
10. diverges
11. converges absolutely
12. converges absolutely
13. converges absolutely
14. diverges
15. converges absolutely
16. converges absolutely
17. converges absolutely
18. diverges
19. diverges
20. converges absolutely
21. converges absolutely
22. converges absolutely
23. diverges
24. diverges
25. converges absolutely
26. converges absolutely
27. converges absolutely
28. $1+x+2\sum_{i=2}^{\infty} x^i$, $x\in(-1,1)$
29. $\sum_{n=1}^{\infty} \frac{3^{n-1}x^n}{2^n}$, $x\in(\frac{-2}{3},\frac{2}{3})$
30. $\sum_{n=1}^{\infty} \frac{x^{3n}}{4^n}$, $x\in(-\sqrt[3]{4},\sqrt[3]{4})$
31. $\sum_{n=0}^{\infty} \frac{x^{2n+2}}{n!}$, $x\in\mathbb{R}$

32. $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$, $x \in \mathbb{R}$

33. (-3, 3)

34. [-12, -8)

35. $(\frac{-1}{16}, \frac{1}{16})$

36. $x \in \mathbb{R}$

37. $x \in \mathbb{R}$

38. $f(x) = 1 - x + x^3 + \ldots$

39. $g(x) = -x + \frac{3}{2}x^2 - \frac{5}{3}x^3 + \ldots$

40. $f(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}(x - \frac{\pi}{6}) - \frac{1}{4}(x - \frac{\pi}{6})^2 - \frac{\sqrt{3}}{12}(x - \frac{\pi}{6})^3 + \ldots$

41. $f(x) = 7 - 10(x+1) + 6(x+1)^2 - (x+1)^3$

42. $h(x) = \frac{e}{2}\left[(e^2+1) + (e^2-1)(x-1) + \frac{1}{2}(e^2+1)(x-1)^2 + \frac{1}{6}(e^2-1)(x-1)^3 + \ldots\right]$

43. $R_6(x) = \frac{-(x-1)^7}{(c-3)^8}$, $|R_6(0.2)| \le \frac{2^6}{5^7}$

44. $R_6(x) = \frac{x^7}{7(1+c)^7}$, $|R_6(0.2)| \le \frac{0.2^7}{7}$

45. $R_6(x) = \frac{3e^c x^7}{7!}$, $|R_6(0.2)| \le \frac{e^{0.2}}{131{,}250{,}000}$