Name _____ D

Geometry

Level 4

(Answer ID # 0722866)

Fill in the space by the correct answer.

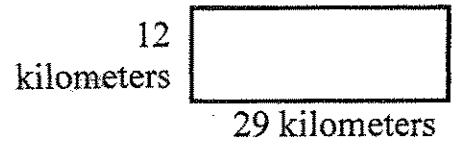
1 A diagram included seven hexagons, seven circles, twelve decagons, four squares, and eleven lines. How many polygons are in the diagram?

(A) 41
(B) 23
(C) 18
(D) 19
(E) 7
(F) 30

2 Brad's bedroom is exactly 12 feet by 18 feet. If Brad wants to put a carpet on the floor, how much carpeting is needed?

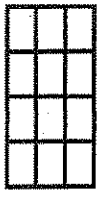
(A) 657 square feet
(B) 342 square feet
(C) 30 square feet
(D) 100 square feet
(E) 216 square feet
(F) None of the above

3 What is the area of the following figure?



(A) 348 square kilometers
(B) 386 square kilometers
(C) 2 square kilometers
(D) 52 square kilometers
(E) 415 square kilometers
(F) 932 square kilometers

4 What is the area of this figure?



(A) 7 square units
(B) 45 square units
(C) 18 square units
(D) 12 square units
(E) 4 square units
(F) 3 square units

5 Classify the following group of lines  ? Ⓐ Perpendicular Ⓑ Parallel Ⓒ Intersecting	6  Name the polygon. Ⓐ Heptagon Ⓑ Quadrilateral Ⓒ Triangle Ⓓ Pentagon Ⓔ Hexagon
7 The diameter of a circle painted by Paul is 2 feet. What is the length of its radius? Ⓐ 1 feet Ⓑ 4 feet Ⓒ 10 feet Ⓓ 8 feet Ⓔ 24.5 feet	8  Classify the angle. Ⓐ Obtuse Ⓑ Acute Ⓒ Right

9 Brad's house has a garden which is in the shape of a square. If each side of the garden is 17 feet then what is the perimeter of the garden?

(A) 79 feet
(B) 78 feet
(C) 108 feet
(D) 34 feet
(E) 68 feet
(F) 20 feet

10 

Name this figure.

(A) Ray BX
(B) Ray XB
(C) line XB
(D) line segment XB

11 When it is 8 o'clock, what type of angle is the smallest angle formed by the minute and hour hand?

(A) Right angle
(B) Acute angle
(C) Obtuse angle

12 Jane measured an area of a square to be 16 ft. Paul measured one side of the square to be 4 ft. What is the perimeter of this square?

(A) 16 ft
(B) 15 ft
(C) 11 ft
(D) 20 ft
(E) 12 ft
(F) None of the above

13 Which letter has a line of symmetry?

(A) J
(B) X
(C) R
(D) F
(E) G

14 What is the name of a polygon which has six sides?

(A) Decagon
(B) Pentagon
(C) Triangle
(D) Octagon
(E) Quadrilateral
(F) Hexagon

15 How many of the following pairs of line intersect?

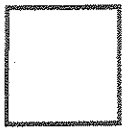
(A) three
(B) zero
(C) two
(D) one

16 A polygon has twelve vertices. How many sides does the polygon have?

(A) 11
(B) 10
(C) 13
(D) 12
(E) None of the above

17 An angle of 153° is ?

(A) Perpendicular
(B) quadrilateral
(C) right
(D) obtuse
(E) line segment
(F) acute

18 

The side d of this square is 20 m

Find the perimeter.

(A) 480 m
(B) 80 m
(C) 560 m
(D) 160 m
(E) 1 m

19 Find the perimeter of a triangle if all of the sides equal 10 mi.

(A) 100 mi
(B) 30 mi
(C) 50 mi
(D) 40 mi
(E) 60 mi
(F) None of the above

5

Name _____ D

GEOMETRY

1	A	B	C	D	E	F	11	A	B	C			
2	A	B	C	D	E	F	12	A	B	C	D	E	F
3	A	B	C	D	E	F	13	A	B	C	D	E	
4	A	B	C	D	E	F	14	A	B	C	D	E	F
5	A	B	C				15	A	B	C	D		
6	A	B	C	D	E		16	A	B	C	D	E	
7	A	B	C	D	E		17	A	B	C	D	E	F
8	A	B	C				18	A	B	C	D	E	
9	A	B	C	D	E	F	19	A	B	C	D	E	F
10	A	B	C	D									

6

Name _____ Date _____

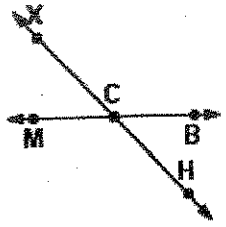
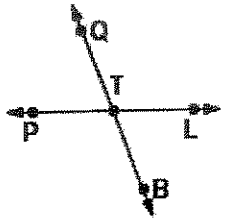
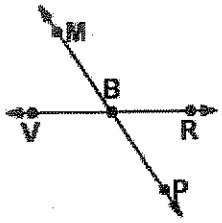
Mixed Review (Grade 6)

(Answer ID # 0949882)

Draw and label an example of each.

1. point Y	2. $\overline{KF}$	3. $\overleftrightarrow{ZP}$ intersecting $\overleftrightarrow{HN}$

Complete.

4.	5.	6.
 Name a pair of supplementary angles.	 Name a pair of vertical angles.	 Name a pair of supplementary angles.

To the nearest tenth, find the circumference. Use 3.14 for π.

7. radius = 24.64 cm	8. diameter = 76.758 ft

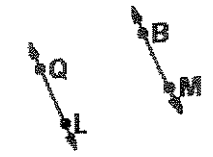
7

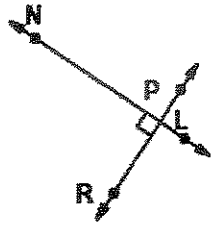
7

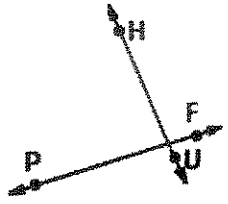
Complete the unit conversions.

9. 30,000 liters to kiloliters	10. 8 pints to quarts
11. 19 grams to milligrams	12. 20 quarts to pints
13. 54,000 secs to hours	14. 26 tons to pounds

Classify each pair of lines as *parallel*, *intersecting*, or *perpendicular*.

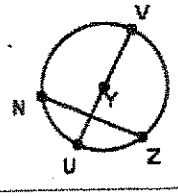
15. 

16. 

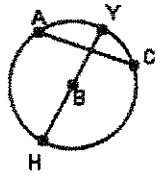
17. 

18. 

To the nearest tenth, find the circumference. Use 3.14 for π.

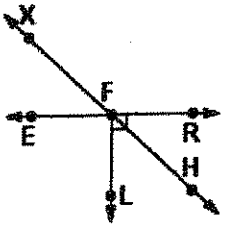
19. 

$\overline{VY}$ = 22.5 ft

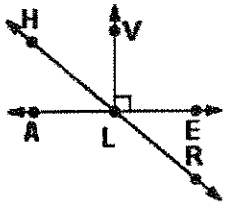
20. 

$\overline{YH}$ = 76.302 in

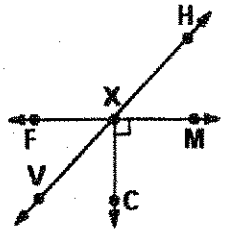
Complete.

21. 

Name a pair of complementary angles.

22. 

Name a pair of vertical angles.

23. 

Name a pair of vertical angles.

8

Complete.

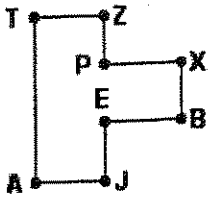
24. 222 L = _____ cL	25. 1.00003 kL = _____ cL
26. 431.008962 cL = _____ mL	27. 363,114,000 mL = _____ kL
28. 912,000 mL = _____ L	29. 698.033 kL = _____ cL

Convert each quantity to the given units.

30. 168 pt = _____ gal	31. 20 gal = _____ qt	32. 176 oz = _____ lb
33. 19 mi = _____ yd	34. 33 mi = _____ ft	35. 10 pt = _____ fl oz
36. 32 oz = _____ lb	37. 51 ft = _____ yd	38. 10,000 lb = _____ T

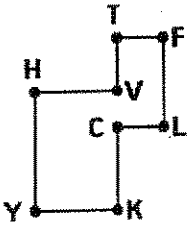
Find the perimeter of each polygon.

39.



$\overline{EJ}$=33 in
$\overline{PX}$=42 in
$\overline{XB}$=32 in
$\overline{EB}$=42 in
$\overline{TA}$=91 in
$\overline{TZ}$=39 in
$\overline{ZP}$=27 in
$\overline{AJ}$=39 in

40.



$\overline{TV}$=63 mm
$\overline{TF}$=55 mm
$\overline{FL}$=105 mm
$\overline{CL}$=55 mm
$\overline{HY}$=140 mm
$\overline{HV}$=98 mm
$\overline{CK}$=98 mm
$\overline{YK}$=98 mm

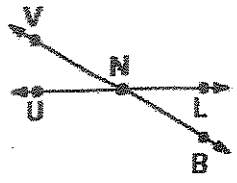
Classify each triangle as isosceles, scalene, or equilateral by the lengths of its sides.

41. 6 mm, 6 mm, 6 mm	42. 2 in, 2 in, 2 in	43. 20 cm, 23 cm, 10 mm
_____	_____	_____

9

Find the measure of the angle.

44.



m∠LNB = _____

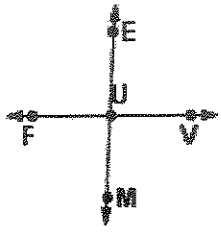
m∠UNB = 149°

45.

m∠HYG = _____

m∠GYC = 41°

46.



m∠VUE = _____

m∠FUM = 87°

Write *polygon* or *not a polygon*. For a polygon, name the type of polygon and also write *regular* or *not regular*.

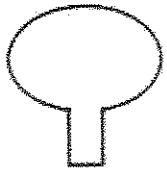
47.

_____

48.

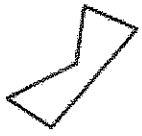
_____

49.



_____

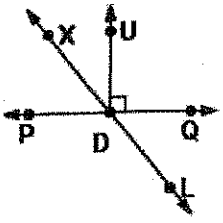
50.



_____

Find the measure of the angle.

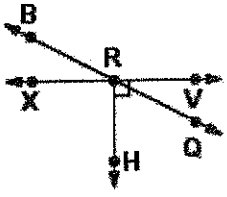
51.



m∠LDQ = _____

m∠XDP = 52°

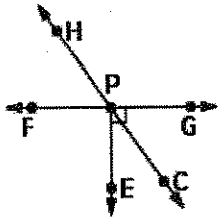
52.



m∠BRV = _____

m∠QRV = 28°

53.

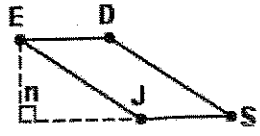


m∠CPG = _____

m∠HPF = 54°

Find the area of each parallelogram.

54.

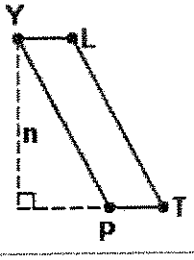


$\overline{ED}$=17 mm

n = 15 mm

$\overline{EJ}$=28 mm

55.



n = 70 in

$\overline{YP}$=79 in

$\overline{YL}$=22 in

10

10

Math4020 Geometry Notes
(Section 12.1)

Line segment--> Shortest path between two points (straight).

Angle--> Union of two rays with common endpoint called a vertex.

Right Angle--> A 90-degree angle; an angle that is equal in measure to an angle formed by perfectly horizontal and vertical line segments.

Perpendicular--> Two line segments are perpendicular when they meet at a right angle.

Triangle--> A 3-sided (straight sides), closed, two-dimensional shape.

- Isosceles Triangle--> A triangle with at least two sides that are of equal length.
- Equilateral Triangle--> A triangle with all three sides of equal length.
- Scalene Triangle--> A triangle with all three sides of different length.
- Obtuse Triangle--> A triangle with one obtuse angle.
- Right Triangle--> A triangle with one right angle.
- Acute Triangle--> A triangle with all acute angles.

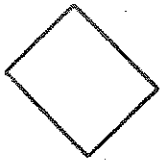
Quadrilateral--> A 4-sided (straight sides), closed, two-dimensional shape.

- Square--> A quadrilateral with four congruent sides and four right angles.
- Rectangle--> A quadrilateral with four right angles.
- Parallelogram--> A quadrilateral with two pairs of parallel sides.
- Kite--> A convex quadrilateral with two distinct pairs of adjacent congruent sides.
- Rhombus--> A quadrilateral with four congruent sides.
- Trapezoid--> A quadrilateral with exactly one pair of parallel sides.
  - Isosceles Trapezoid--> A trapezoid whose non-parallel sides are congruent.

Naming shapes:

Use these words to describe the figures below. Use as many as it takes to describe the specific figure. Give the most specific name for the figure. For example, do not call a square a rectangle for this exercise.

Segment, line, ray, angle
Isosceles, equilateral, scalene, acute, obtuse, right, convex, concave
triangle, quadrilateral, rhombus, parallelogram, rectangle, square, kite, trapezoid
Pentagon, hexagon, septagon, octagon, nonagon, decagon.
Cube, cone, sphere, cylinder, rectangular prism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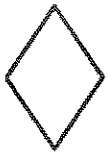
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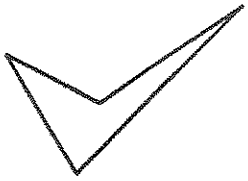
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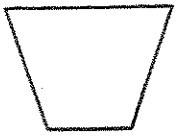
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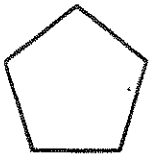
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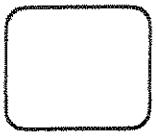
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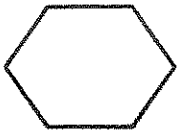
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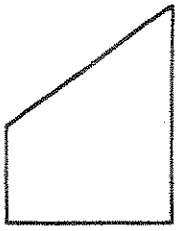
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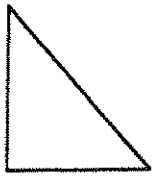
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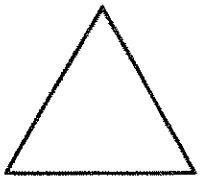
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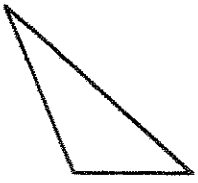
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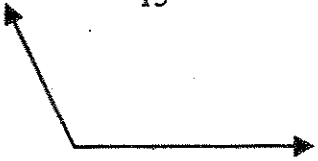
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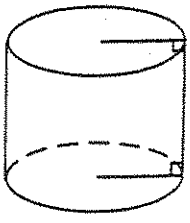
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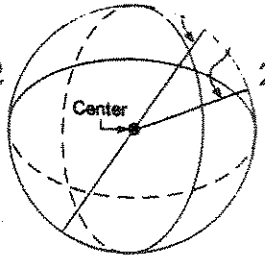

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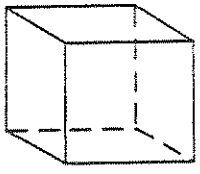
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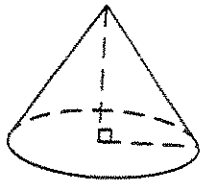
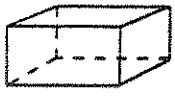

25



12

"If Possible" Worksheet

For each type of triangle listed below, decide if it's possible or impossible.
If it's possible, then write "possible" and draw an example of the triangle.
If it's not possible, then write "not" and give a reason why it won't work.

(1) An isosceles right triangle.

(2) A scalene triangle with all equal angles.

(3) A triangle with two obtuse angles.

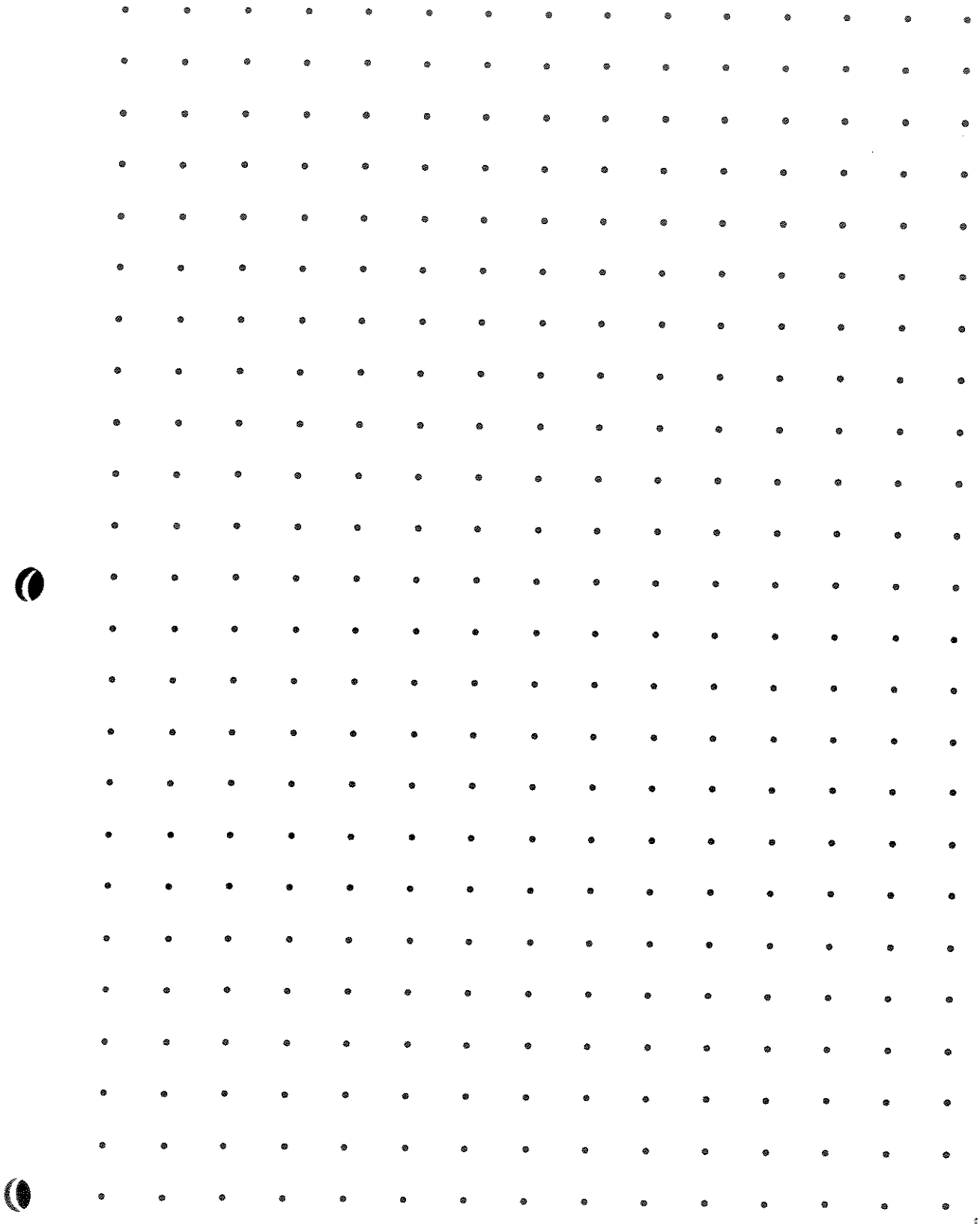
(4) An acute isosceles triangle.

(5) An acute right triangle.

(6) A triangle with two right angles.

(7) An equilateral right triangle.

Geoboard dot paper



Math4020 Geometry Notes
(Sections 12.2 & 12.3)

Point→ Is 0-dimensional.

Line→ Extends forever in two directions; has no thickness; the shortest path between two points is along a line.

Ray→ Part of a line; extends forever in only one direction and has one endpoint.

Line Segment→ Part of a line; has two endpoints; the intersection of two rays.

Angle→ A union of two rays with a common endpoint.

Straight Angle→ An angle that forms a line.

Right Angle→ A 90-degree angle.

Acute Angle→ An angle whose measure is between 0 and 90 degrees (non-inclusive).

Obtuse Angle→ An angle whose measure is between 90 and 180 degrees (non-inclusive).

Angle Relationships

Complementary angles→ Two angles whose measures add to 90 degrees.

Supplementary angles→ Two angles whose measures add to 180 degrees.

Vertical angles→ Two non-adjacent angles formed by the intersection of two lines, rays or segments.

15

Shapes (2-dimensional)

Congruent→ Two shapes are congruent if they are the same size and shape. That is, the first shape can be picked up and placed on top of the other shape and it will fit exactly (with no twisting, breaking, stretching, bending, etc.).

Similar→ Two shapes are similar if they are the same shape, but not necessarily the same size.

Polygon→ A many sided closed shape made up of line segments for sides that meet at vertices.

Convex→ A shape is convex if *every* line segment, formed by connecting *any* two points inside the shape, is wholly contained in the shape.

Concave→ A shape is concave if it is not convex.

Regular→ A polygon is regular if all of its sides and all of its interior angles are congruent.

Interior angle→ In a convex polygon, it's the inside angle formed by two adjacent sides.

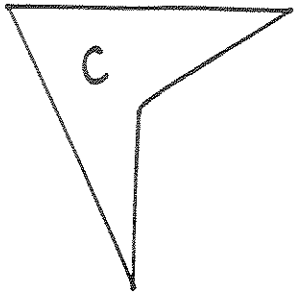
Exterior angle→ In a convex polygon, it's the angle formed by the side of the polygon and the extended line from the adjacent side (for every interior angle, there are two exterior angles).

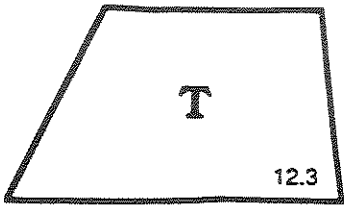
Central angle→ For a regular polygon, it's the angle formed by connecting a vertex to the center of the polygon and then to the consecutive vertex.

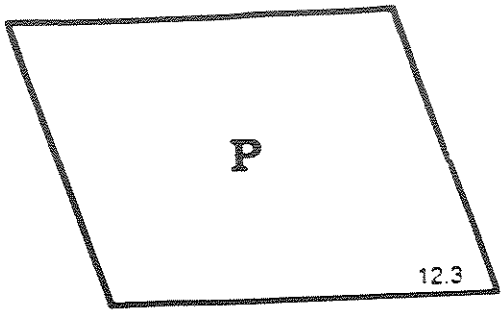
16

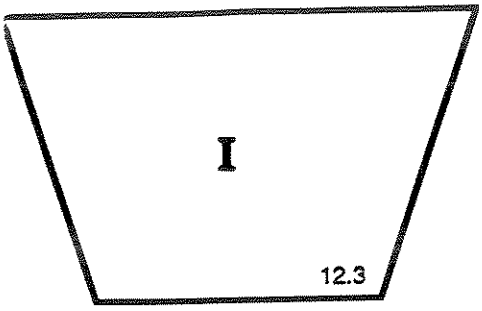
CLASSIFYING THE QUADRILATERALS

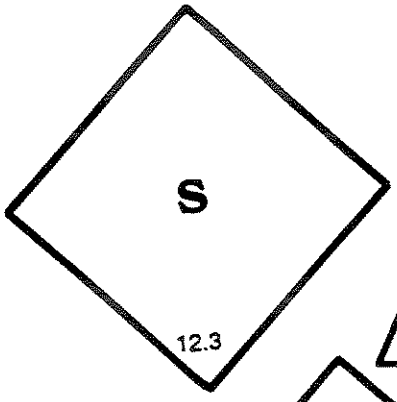
Label each quadrilateral with its most precise name. Measure the sides and angles of your figures using centimeters and degrees.

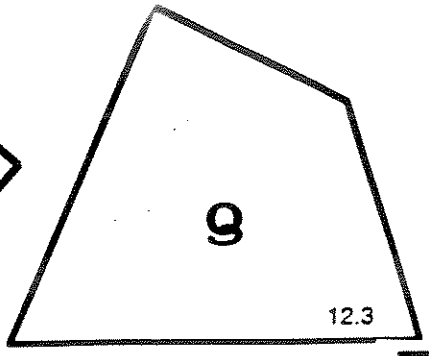


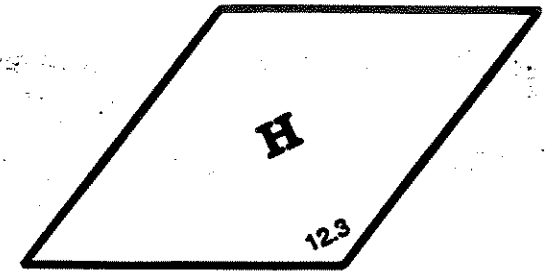


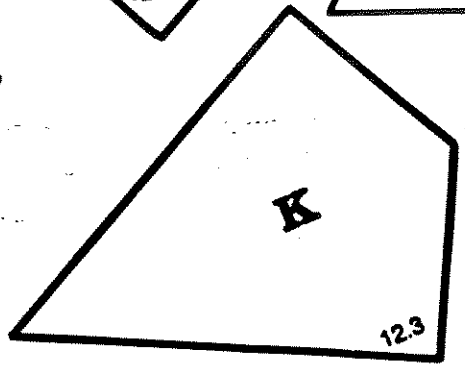


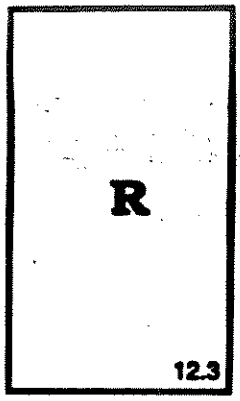












We will be discussing attributes of these shapes using this table:

		congruent sides	perp. sides	congruent diagonals	perp. diagonals	diagonals bisect angles	reflection symmetry	rotation symmetry
Q	4 sides							
T	2 sides parallel							
I	2 sides parallel; other 2 sides congruent							
K	2 pair consecutive congruent sides							
P	2 pair parallel sides							
H	4 congruent sides							
R	4 right angles; 2 pair parallel sides							
S	4 right angles; all sides congruent							
C	1 concave side; diagonal outside shape							

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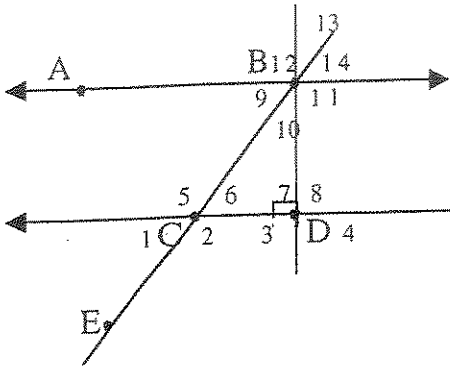
Checking up on Geometry Vocabulary:



Write a true geometric statement about perpendicular and parallel given the drawing above.

1. a ray_____ a line_____ a segment_____
2. concurrent line segments_____ collinear points_____
   coplanar segments_____
3. ∠ 1 is vertical to _____, congruent to _____ supplementary to _____
4. ∠ 6 is alternate interior to _____ complementary to _____
   corresponds to _____
5. Points inside ∠ ABD are_____ Points outside ∠ABD are_____
6. ∠ 11 is adjacent to _____ , corresponding to _____
   supplementary to _____
7. Name the point(s) between B and E. _____
8. Name all acute angles_____
9. Name all right angles_____
10. Name all obtuse angles_____
11. If m ∠13 = 30°, find the measure of all the angles in the picture.

∠1	∠2	∠3	∠4	∠5	∠6
∠7	∠8	∠9	∠10	∠11	∠12
∠13	∠14				

12. Why is the m(∠12) = ____° ?
13. Why is ∠10 complementary to ∠6?

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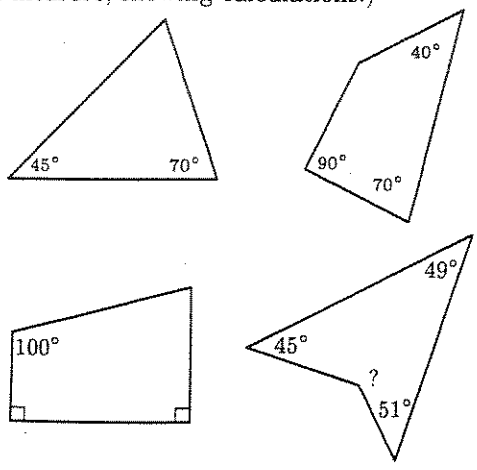
Angle Problems

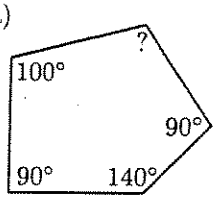
Name:_____ Group members: _____

1. What are the unmarked angles? (Write in the degree measure, showing calculations.)

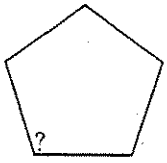


2. Figure out the measure of the unknown angles in the figures below. Show your work.

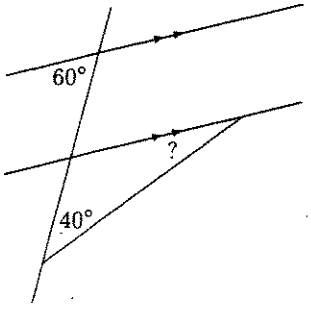
(a)



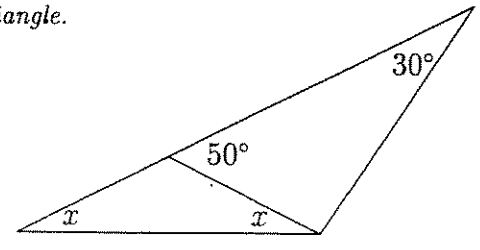
(b) Regular Pentagon



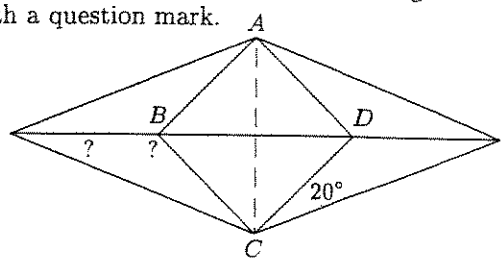
3. What is the measure of the angle marked with a question mark? *Show your reasoning.*



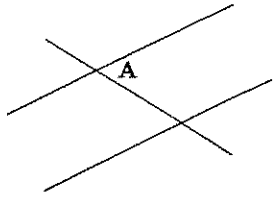
4. There are 4 unknown angles in the figure below. Figure out these angles and write in the values that you get. *Note: The lower triangle is an isosceles triangle.*



5. The quadrilateral $ABCD$ is a square and the dotted line AC is a line of symmetry for the figure below. Work out the value of the angles marked with a question mark.



6. Consider the angle marked **A** in the figure below where two lines are parallel.

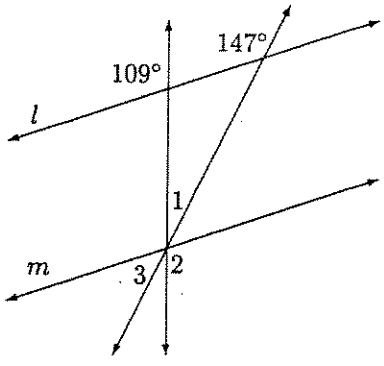


B. Mark with a **B** an angle which is an alternate interior angle to angle **A**.

C. Mark with a **C** an angle which is a corresponding angle to angle **A**.

D. Mark with a **D** an angle which is a vertical angle to angle **A**.

7. Lines l and m below are parallel. How many degrees are in the angles numbered 1, 2 and 3?



8. This figure (called a pentagram) is one of Andy's favorites. Work out the values of all of the different angles in this figure.

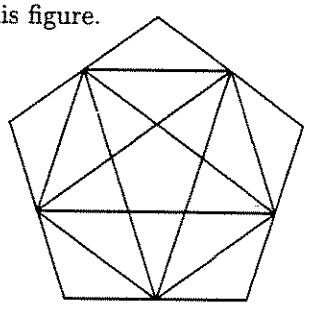


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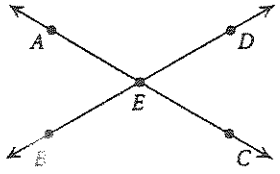
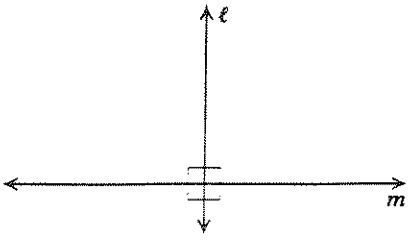
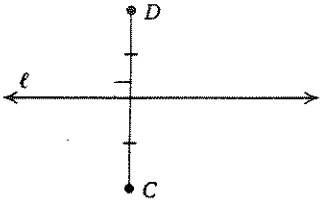
TABLE 10.2 | (continued)

Figure	Description/Symbol	Definition
	$\angle AED$ and $\angle BEC$ are a pair of **vertical angles.**	Vertical angles are a pair of angles that are formed by two intersecting lines and that are not a linear pair of angles.
	Line ℓ is **perpendicular** to line m. We write $\ell \perp m$.	Two lines are perpendicular if they intersect to form four right angles.
	Line ℓ is the **perpendicular bisector** of $\overline{CD}$.	The perpendicular bisector of a segment is a line that is perpendicular to the segment, and which divides it into two congruent segments.

The first three figures in Table 10.2 are single figures each with a special property. The remaining descriptions and definitions in the table are of *relationships* between two angles or between two lines. These figures and relationships in geometry are important and useful, and are applied in Example 10.3.

TABLE 10.2 | Defining Special Angles and Perpendicular Lines

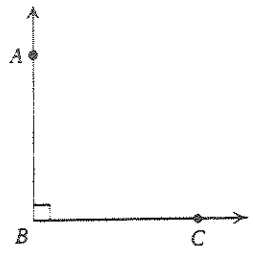
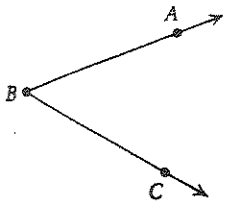
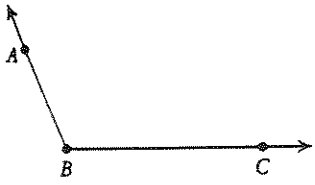
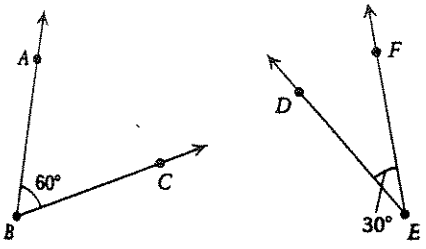
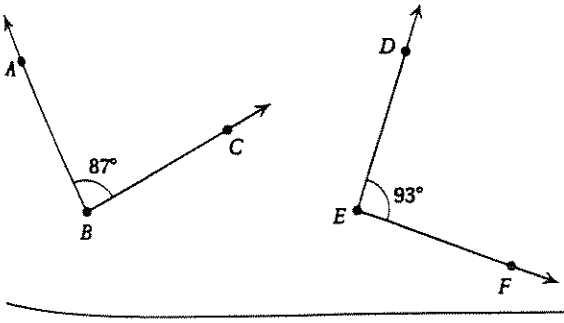
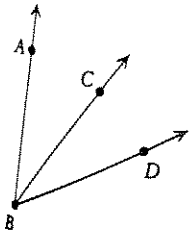
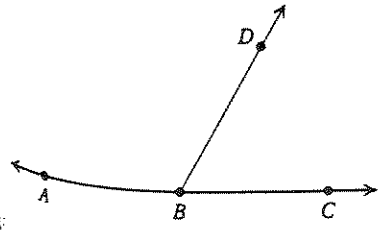
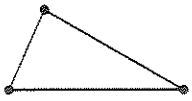
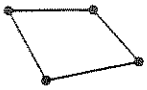
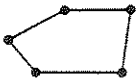
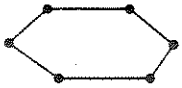
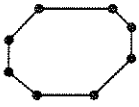
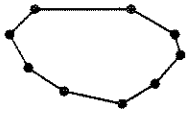
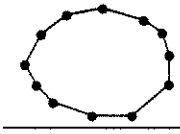
Figure	Description/Symbol	Definition
	$\angle ABC$ is a **right angle.**	Right angles are angles with a measure of 90°.
	$\angle ABC$ is an **acute angle.**	Acute angles have measures between 0° and 90°.
	$\angle ABC$ is an **obtuse angle.**	Obtuse angles have measures between 90° and 180°.
	$\angle ABC$ and $\angle DEF$ are **complementary** to each other.	Complementary angles are a pair of angles whose measures have a sum of 90°.
	$\angle ABC$ and $\angle DEF$ are **supplementary** to each other.	Supplementary angles are a pair of angles whose measures have a sum of 180°.
	$\angle ABC$ is **adjacent** to $\angle CBD$	Adjacent angles are two angles that have the same vertex and a common side, but that have no common interior points.
	$\angle ABD$ and $\angle DBC$ are a **linear pair** of angles.	A linear pair of angles is a pair of adjacent angles with two non-common sides on the same line.

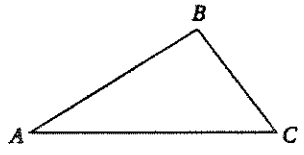
TABLE 10.3 | Classification of Polygons by Number of Sides

Polygon	Number of Sides	Name
	3	**Triangle**
	4	**Quadrilateral**
	5	**Pentagon**
	6	**Hexagon**
	7	**Heptagon**
	8	**Octagon**
	9	**Nonagon**
	10	**Decagon**
	12	**Dodecagon**

Since a triangle is a special type of polygon that we will be studying in detail, we single it out and give the following definition.

Definition of a Triangle

A **triangle,** $\triangle ABC$, is the union of three segments determined by three noncollinear points A, B, and C. A, B, and C are the **vertices** of the triangle. $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are the **sides** of the triangle.



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Two important segments associated with a triangle are defined in Figure 10.13. A **median** of a triangle is a segment from a vertex of a triangle to the midpoint of the side of the triangle opposite that vertex, as shown in Figure 10.13(a). An **altitude** of a triangle is a segment from a vertex of a triangle perpendicular to a line containing the side of the triangle opposite that vertex, as shown in Figure 10.13(b). We will explore the properties of medians and altitudes further in a later subsection.

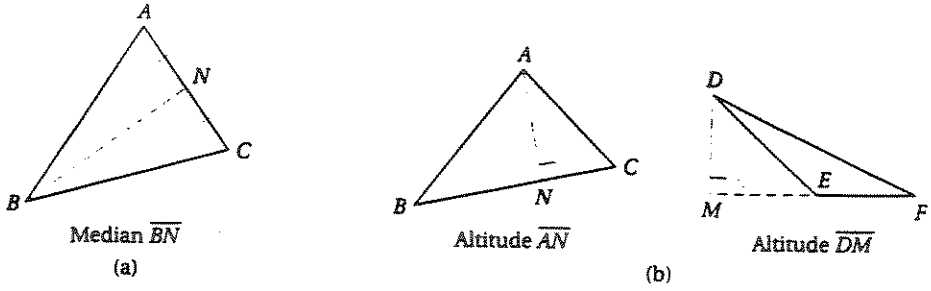
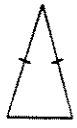
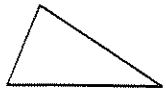
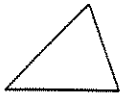
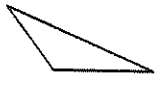


FIGURE 10.13 | A median and altitudes for some triangles.

It is useful to classify triangles according to the properties of their sides and interior angles, as in Table 10.4. Note that the definition of an isosceles triangle includes the equilateral triangle as a special case of an isosceles triangle.

TABLE 10.4 | Classification of Triangles

Triangle	Name	Properties
	**Equilateral triangle**	All sides congruent
	**Isosceles triangle**	At least one pair of congruent sides
	**Scalene triangle**	No congruent sides
	**Right triangle**	One right angle
	**Acute triangle**	All acute angles
	**Obtuse triangle**	One obtuse angle

Connection to the PreK–8 Classroom

A Sample Student Page: Properties of Quadrilaterals

In the *Principles and Standards for School Mathematics*, it is stated that "PreK–2 geometry begins with describing and naming shapes" (p. 97) and that "In grades 3–5, they should ... focus on identifying and describing a shape's properties and learning specialized vocabulary ..." (p. 165). Study the page from an elementary school mathematics textbook in Figure 10.14 and answer the questions.

- *How does this page help students distinguish a rectangle from a parallelogram?*
- *How does this page implement the Principles and Standards statement about geometry?*
- *Are there opportunities for students to reason about geometry on this page? Explain.*

Chapter 6
Lesson 4

Quadrilaterals

You Will Learn
how to classify quadrilaterals

Vocabulary
quadrilateral
a polygon with four sides

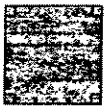
Types of Quadrilaterals
square
rectangle
parallelogram
rhombus
trapezoid

Learn

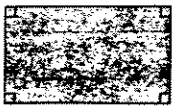
These Miao children, internationally known as Hmong, are from the Guizhou province in south central China. Their rectangular-shaped shoulder shawls are worn at one of the many festivals they celebrate each year.

Some of the shapes in Miao patterns are **quadrilaterals**, polygons with four sides.

There are about 8 million Hmong (mawng) worldwide.



Square
all sides the same length, four right angles



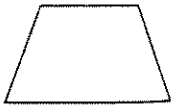
Rectangle
opposite sides parallel and the same length, four right angles

Remember
Parallel lines do not meet. Right angles form square corners.

Some quadrilaterals do not have four right angles.

Parallelogram
two pairs of parallel sides

Rhombus
two pairs of parallel sides, all sides the same length



Trapezoid
only one pair of parallel sides

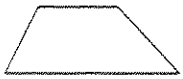
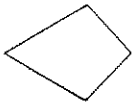
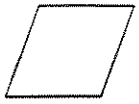
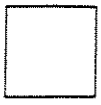
Talk About It

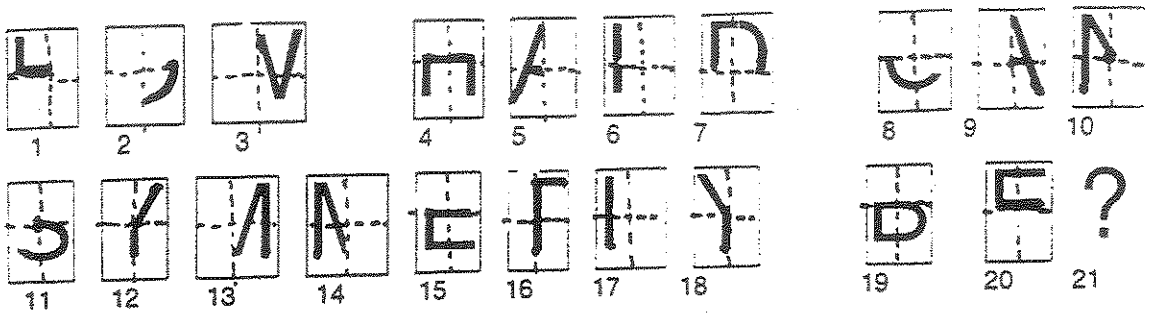
Is it true that a square is a rectangle and a rhombus? Explain.

FIGURE 10.14 | Excerpt from a grade 5 elementary school mathematics textbook.

(*Source:* Page 276 from *Scott Foresman–Addison Wesley Math Grade 5* by Randall I. Charles, et al. Copyright © 1998 by Addison Wesley Longman, Inc. Reprinted by permission of Pearson Education, Inc.)

TABLE 10.5 | Classification of Quadrilaterals

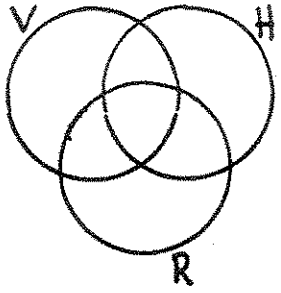
Quadrilateral	Name	Properties
	**Trapezoid**	Exactly one pair of opposite sides parallel
	**Kite**	At least two pairs of adjacent sides congruent, no side used twice in the pairs
	**Parallelogram**	Pairs of opposite sides parallel and the same length
	**Rhombus**	All sides the same length and opposite sides parallel
	**Rectangle**	Opposite sides parallel and the same length; all angles are right angles
	**Square**	All sides the same length; all angles are right angles



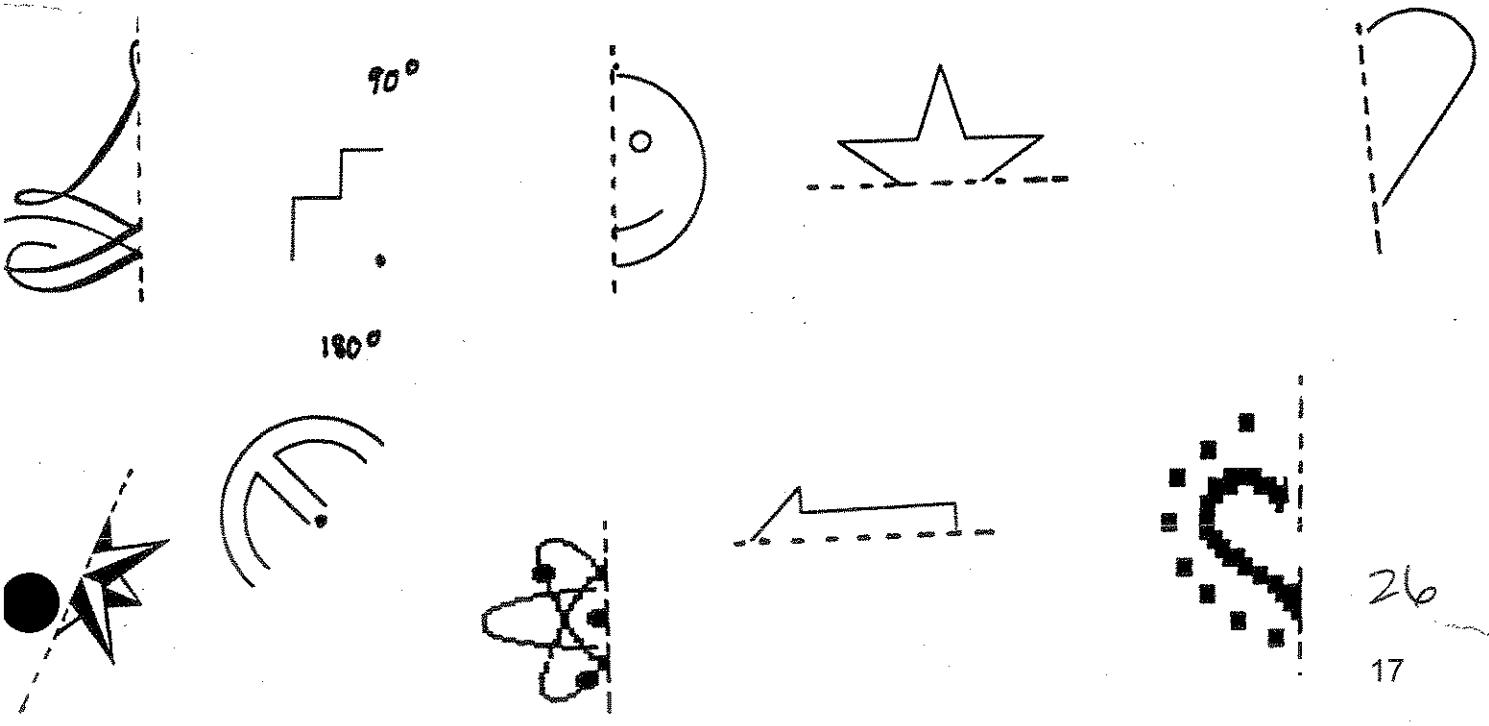
1. In the arrangement above give vertical symmetry to #'s 1, 2 3,5,9,12,13,14,16,18
   Give point (180° rotational) symmetry to # 10,11
   Give horizontal symmetry to # 4,7,8,15,19,20, 1, 2

2. Find the symmetry in these math symbols… group them by vertical, horizontal and rotational symmetry. You might put that into a Venn Diagram.

$$\Leftarrow \cup \infty \Omega \approx \sum \int \nabla \subseteq \otimes \geq \Uparrow \sqrt{\ } * \therefore$$



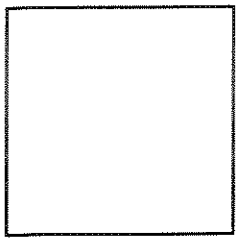
3. Complete these drawings using the indicated symmetry line or point.



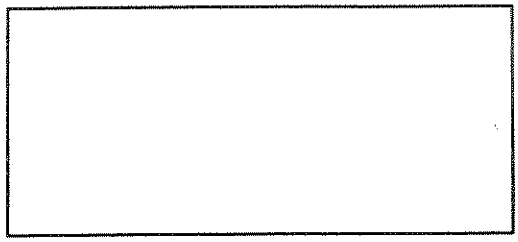
21 Lines of Symmetry

Use the Image Reflector™ to help you draw lines of symmetry. Try to find more than one for each figure.

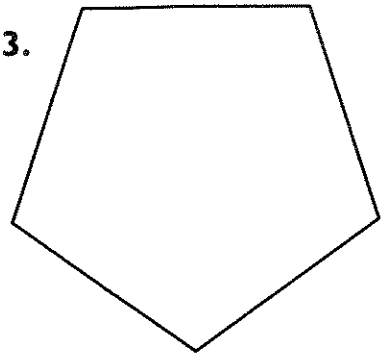
1.



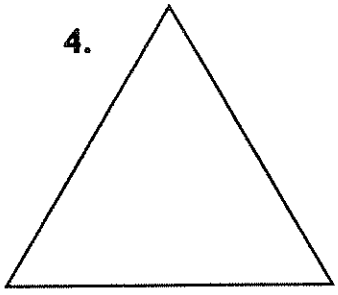
2.



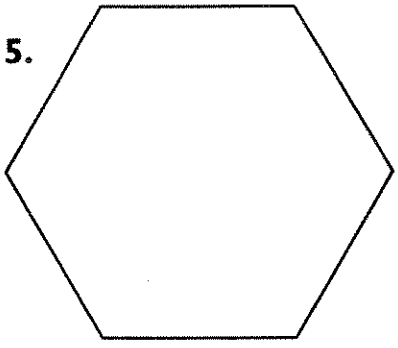
3.



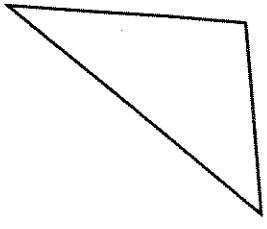
4.



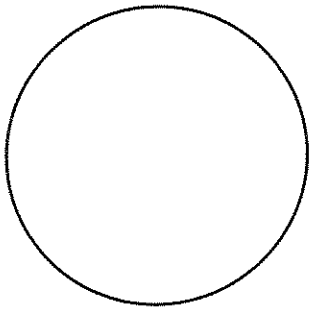
5.



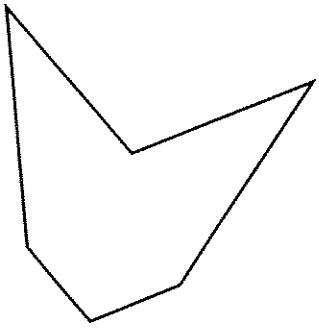
6.



7.



8.



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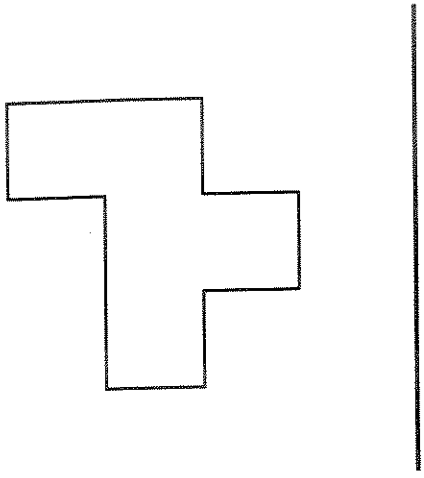
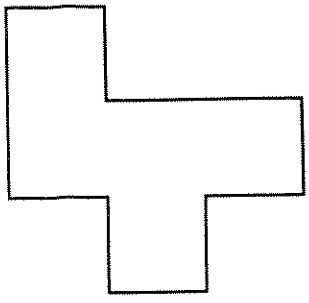
6 Using Reflection Lines

Place the Image Reflector™ on the reflection line.
Draw the image you see.

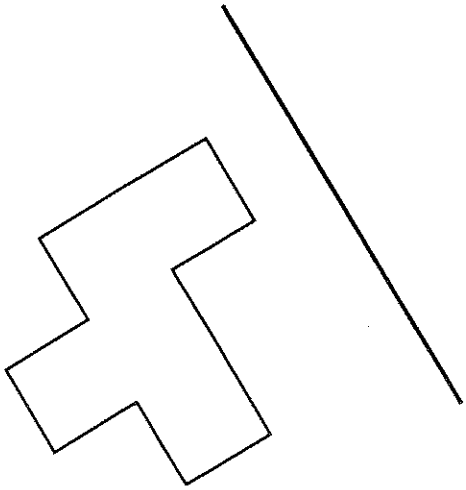
1.



3.



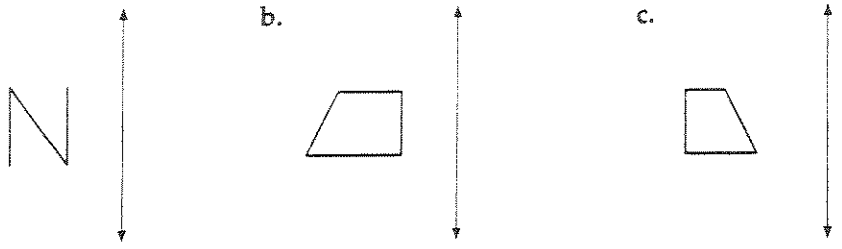
2.



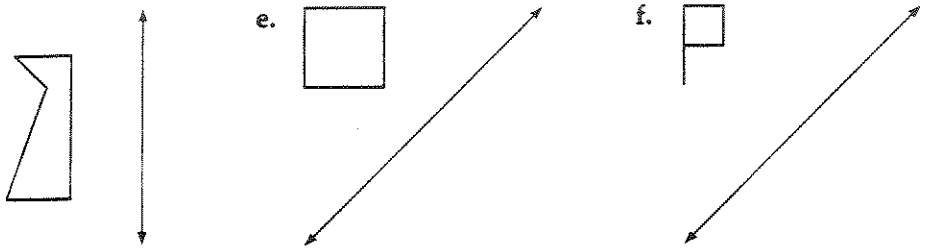
28

Figures for EXPLORATION 9.4, PART 1: Developing reflection sense

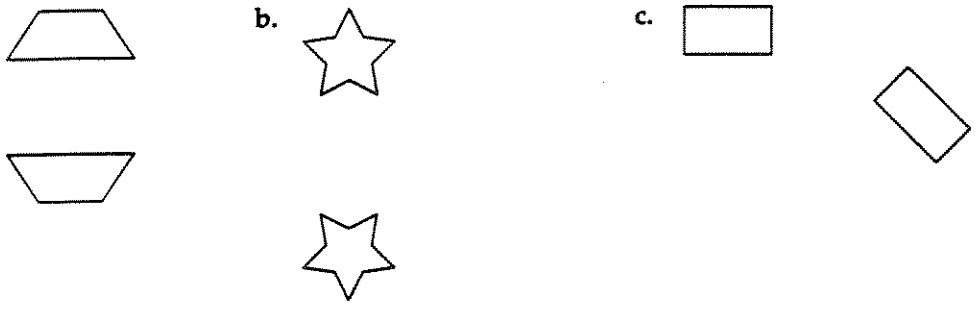
1. a. b. c.



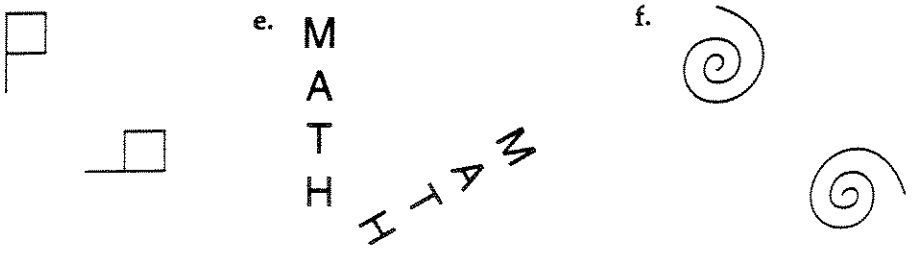
d. e. f.



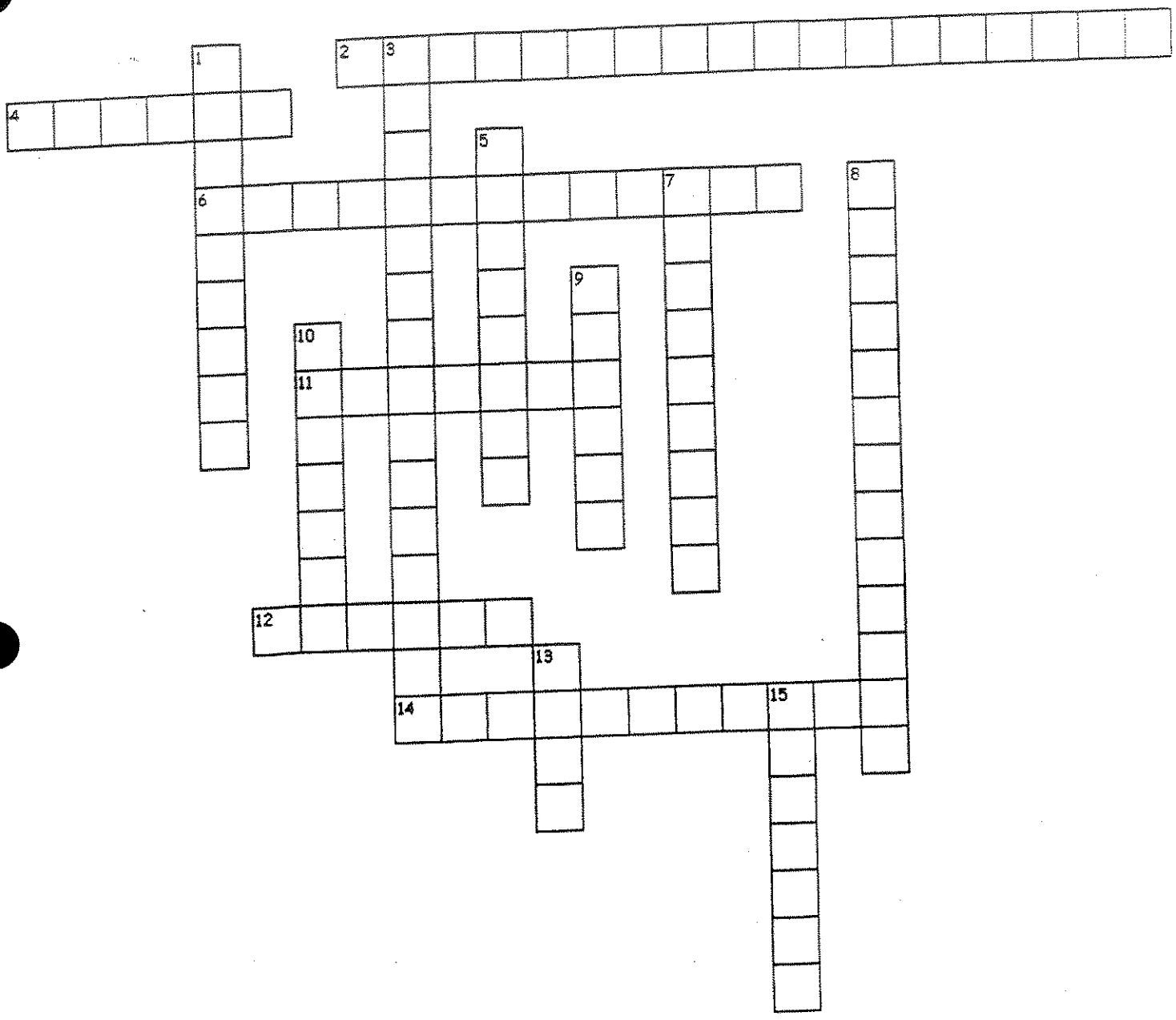
3. a. b. c.



d. e. f.



Polygons



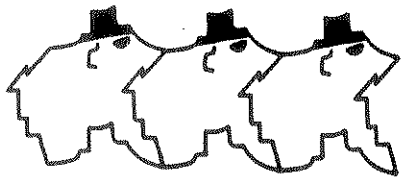
Across
2. Quadrilateral with only one line of symmetry
4. A regular quadrilateral
6. Quadrilateral with no lines of symmetry
11. The shape of a stop sign
12. Perpendicular line segment connecting base to vertex opposite it
14. Adjective for triangle that could be replaced with equiangular

Down
1. Quadrilateral with only two sides parallel
3. A polygon whose interior angles add to 180 degrees and has no sides equal in length
5. An important building in Washington D.C.
7. Most general quadrilateral with all right angles
8. Three-sided polygon with one 90-degree angle
9. Adjective describing polygon that contains every line segment connecting any two points in polygon
10. Not convex
13. Resembles a child's toy
15. A quadrilateral that has all sides congruent

30

18

d

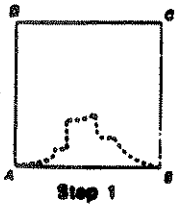


Leap Frog, Robert Caneso, Geometry student

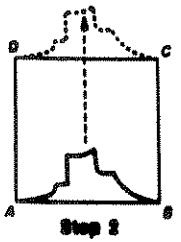
Step 1: Start with one square from a tessellation of squares (although any parallelogram will work with this method). Connect one side $\overline{AB}$ of the square with a curve, call it $\widetilde{AB}$ (curve AB).

Step 2: Place tracing paper or clear plastic over $\widetilde{AB}$ and copy it with a felt tip pen onto the tracing paper or clear plastic. Place the copy beneath the original and slide it so that the endpoints of $\widetilde{AB}$ line up with the endpoint of $\overline{CD}$. Retrace the curve on the original so that it now connects with the endpoint of $\overline{CD}$.

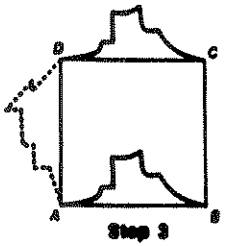
Step 3: Repeat this process with a curve connecting points A and D. That is, connect one side $\overline{AD}$ of the square with a curve, call it $\widetilde{AD}$.



Step 1



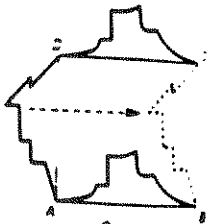
Step 2



Step 3

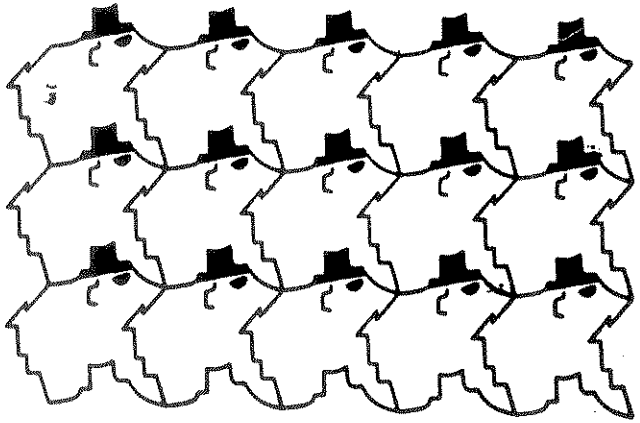
Step 4: Copy $\widetilde{AD}$ onto tracing paper or clear plastic and transfer it across to the opposite side $\overline{BC}$.

Step 5: When completed, trace the entire figure onto the tracing paper or clear plastic and move it to the next square. Trace the entire figure onto the next square. Fill the grid of squares with your figure. You have created a non-polygonal translation tessellation.



Step 4

Step 5

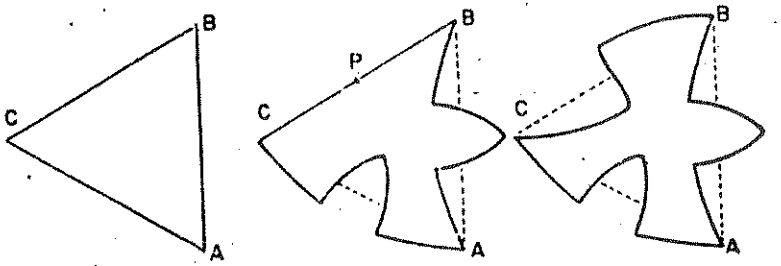


e

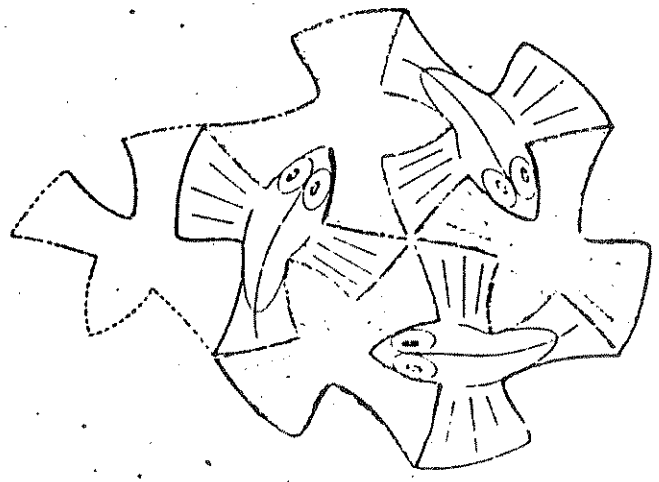
TESSELLATING WITH HEXAGONS

The artist M. C. Escher(1898–1972) is well known for his use of tessellations. By skillfully altering a basic polygon, such as a triangle or hexagon, he was able to produce intricate, artistic tessellations. The figure used here is based on one of Escher's drawings.

Step 1: Start with equilateral triangle ABC. Mark off the same curve on both sides AB and AC as shown. Mark off another curve on side BC that is symmetric about the midpoint P. If you choose the curves carefully, as did Escher, an interesting figure suitable for tessellating will be formed.



Step 2: Six of these figures accurately fit together about a point forming a hexagonal array. Trace and cut out one of the basic figures and show how it can be used to continue the tessellation over the entire sheet.



Math4020 Tangram Assignment

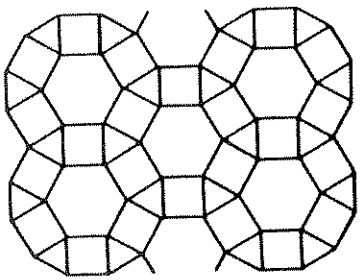
(1) How many different quadrilaterals can you make from tangram pieces where different means non-similar. Draw each quadrilateral showing the pieces you used and labeling the quadrilateral using a specific name.

(2) Make a triangle using one tangram piece, then two pieces, three pieces, etc. up to seven pieces. Sketch each of the possible triangles. If it is not possible to make a triangle with a certain number of pieces, just state that.

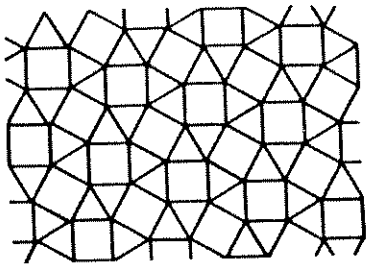
(3) Assuming that the smallest triangular piece in the tangram puzzle has an area of one square unit, how many square units of area is the tangram puzzle? Make a drawing of the tangram puzzle and show how you calculated your answer.

33

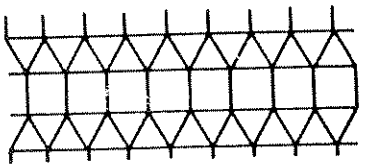
Semiregular Tessellations



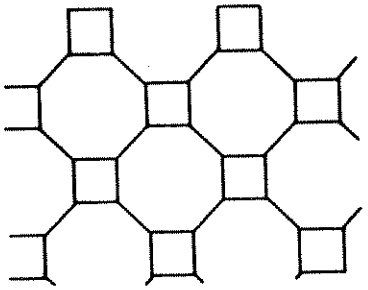
4.3.4.6



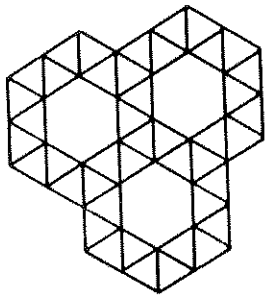
3.3.4.3.4



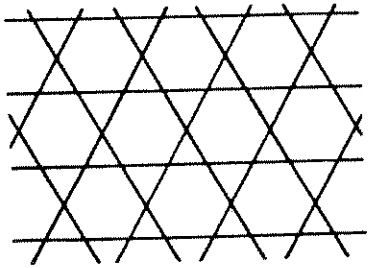
3.3.3.4.4



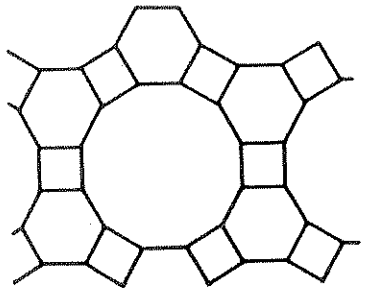
4.8.8



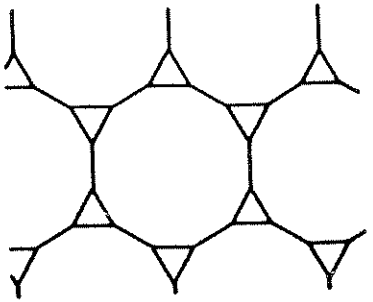
3.3.3.3.6



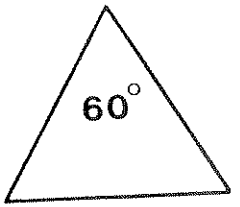
3.6.3.6

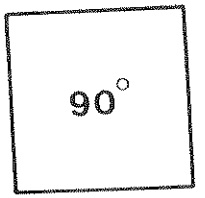


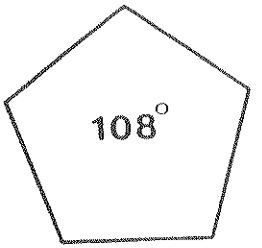
4.6.12

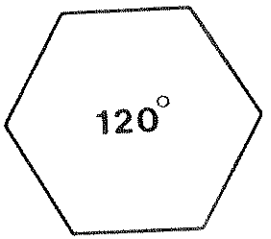


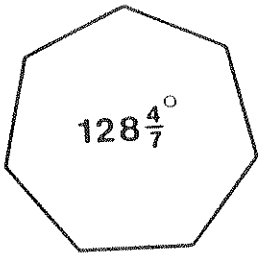
3.12.12

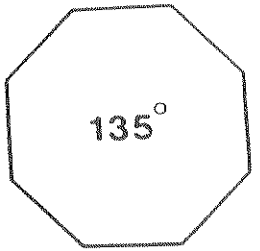


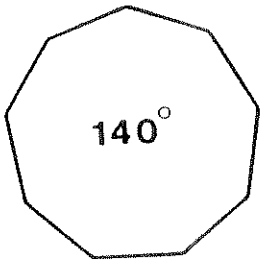


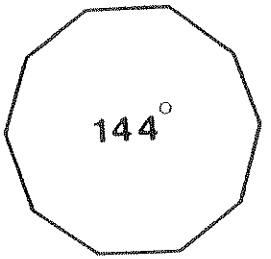












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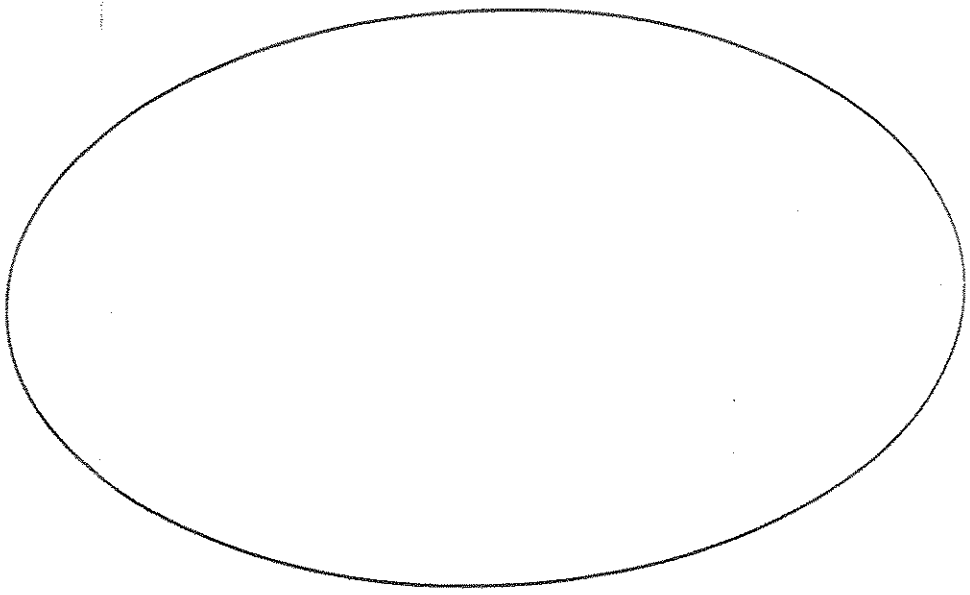

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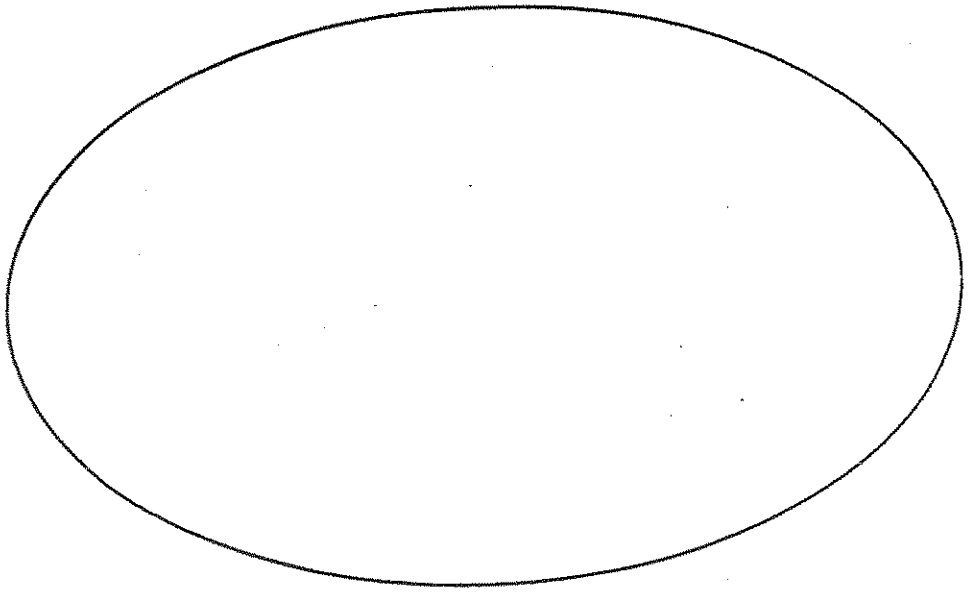

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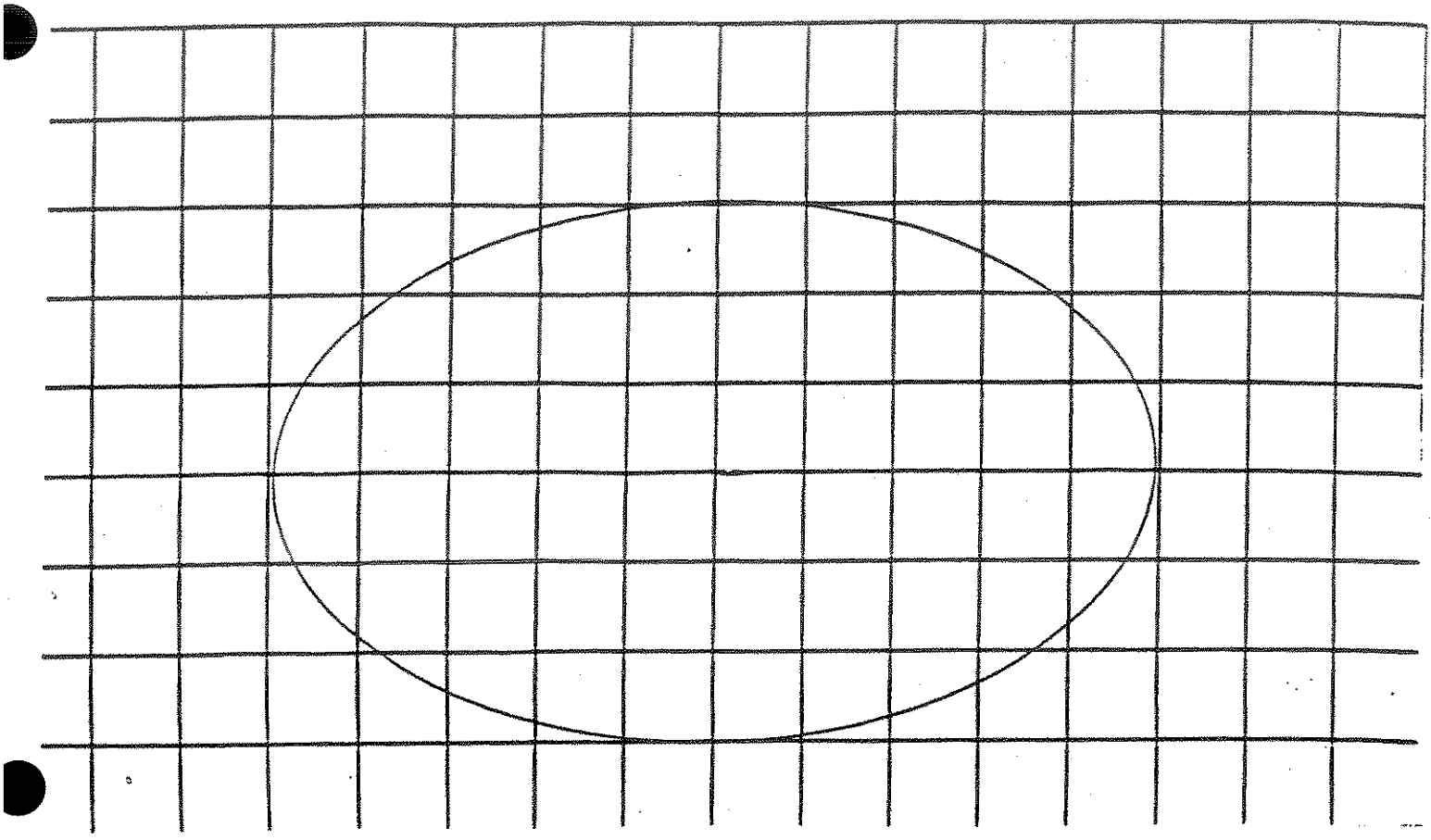

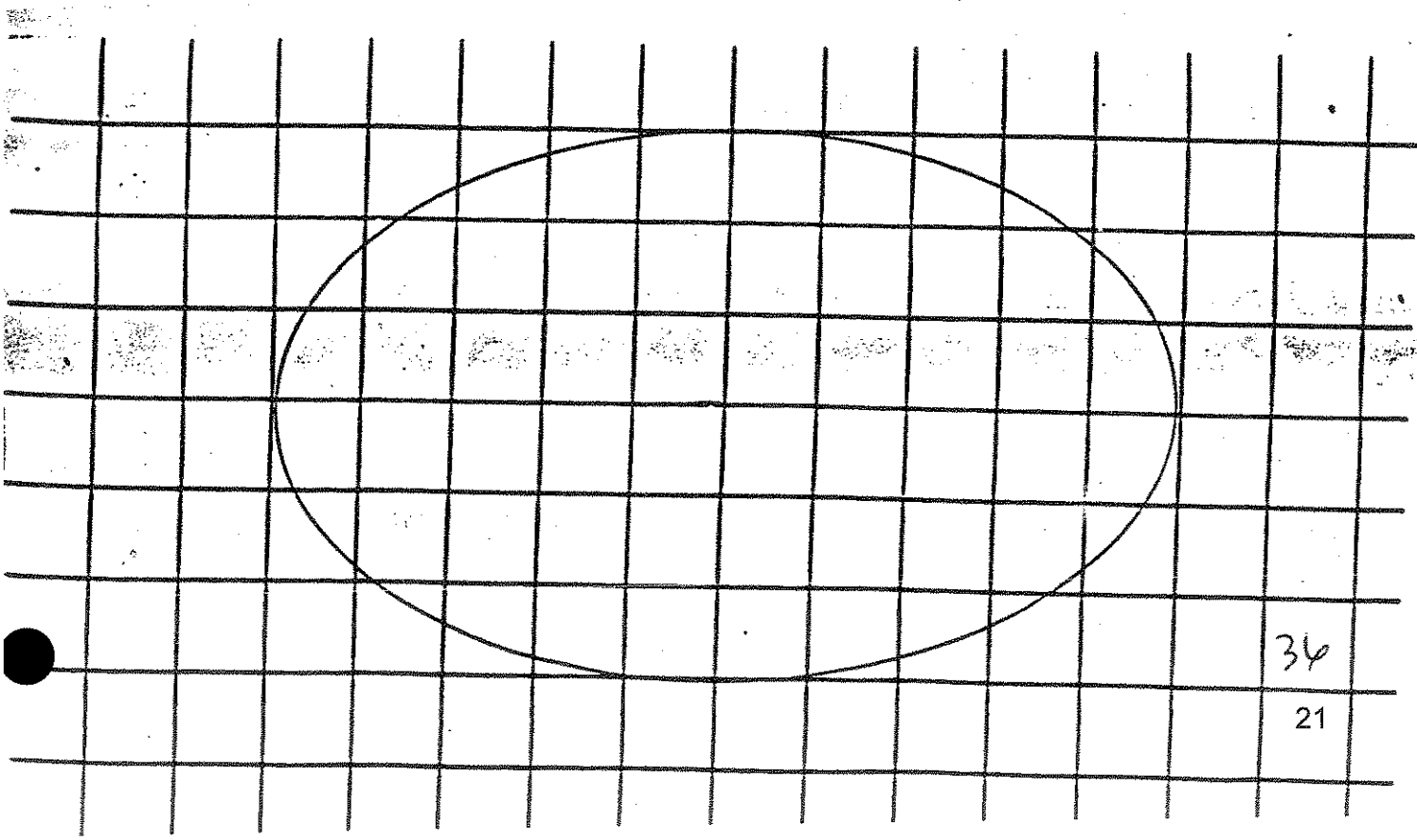

Measurement Standard

Instructional programs from prekindergarten through grade 12 should enable all students to—

Understand measurable attributes of objects and the units, systems, and processes of measurement

Pre-K–2 Expectations:

In prekindergarten through grade 2 all students should–

- recognize the attributes of length, volume, weight, area, and time;
- compare and order objects according to these attributes;
- understand how to measure using nonstandard and standard units;
- select an appropriate unit and tool for the attribute being measured.

Grades 3–5 Expectations:

In grades 3–5 all students should–

- understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute;
- understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems;
- carry out simple unit conversions, such as from centimeters to meters, within a system of measurement;
- understand that measurements are approximations and how differences in units affect precision;
- explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way.

Grades 6–8 Expectations:

In grades 6–8 all students should–

- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system;
- understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.

Grades 9–12 Expectations:

In grades 9–12 all students should–

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- make decisions about units and scales that are appropriate for

problem situations involving measurement.

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Apply appropriate techniques, tools, and formulas to determine measurements.

Pre-K–2 Expectations:

In prekindergarten through grade 2 all students should–

- measure with multiple copies of units of the same size, such as paper clips laid end to end;
- use repetition of a single unit to measure something larger than the unit, for instance, measuring the length of a room with a single meterstick;
- use tools to measure;
- develop common referents for measures to make comparisons and estimates.

Grades 3–5 Expectations:

In grades 3–5 all students should–

- develop strategies for estimating the perimeters, areas, and volumes of irregular shapes;
- select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles;
- select and use benchmarks to estimate measurements;
- develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms;
- develop strategies to determine the surface areas and volumes of rectangular solids.

Grades 6–8 Expectations:

In grades 6–8 all students should–

- use common benchmarks to select appropriate methods for estimating measurements;
- select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision;
- develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes;
- develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders;
- solve problems involving scale factors, using ratio and proportion;
- solve simple problems involving rates and derived measurements for such attributes as velocity and density.

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Grades 9–12 Expectations:

In grades 9–12 all students should–

- analyze precision, accuracy, and approximate error in measurement situations;
- understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders;
- apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations;
- use unit analysis to check measurement computations.

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Measurement

Instructional programs from prekindergarten through grade 12 should enable all students to—

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

Measurement is the assignment of a numerical value to an attribute of an object, such as the length of a pencil. At more-sophisticated levels, measurement involves assigning a number to a characteristic of a situation, as is done by the consumer price index. Understanding what a measurable attribute is and becoming familiar with the units and processes that are used in measuring attributes is a major emphasis in this Standard. Through their school experience, primarily in prekindergarten through grade 8, students should become proficient in using measurement tools, techniques, and formulas in a range of situations.

The study of measurement is important in the mathematics curriculum from prekindergarten through high school because of the practicality and pervasiveness of measurement in so many aspects of everyday life. The study of measurement also offers an opportunity for learning and applying other mathematics, including number operations, geometric ideas, statistical concepts, and notions of function. It highlights connections within mathematics and between mathematics and areas outside of mathematics, such as social studies, science, art, and physical education.

Measurement lends itself especially well to the use of concrete materials. In fact, it is unlikely that children can gain a deep understanding of measurement without handling materials, making comparisons physically, and measuring with tools. Measurement concepts should grow in sophistication and breadth across the grades, and instructional programs should not repeat the same measurement curriculum year after year. However, it should be emphasized more in the elementary and middle grades than in high school.

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Understand measurable attributes of objects and the units, systems, and processes of measurement

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A measurable attribute is a characteristic of an object that can be quantified. Line segments have length, plane regions have area, and physical objects have mass. As students progress through the curriculum from preschool through high school, the set of attributes they can measure should expand. Recognizing that objects have attributes that are measurable is the first step in the study of measurement. Children in prekindergarten through grade 2 begin by comparing and ordering objects using language such as longer and shorter. Length should be the focus in this grade band, but weight, time, area, and volume should also be explored. In grades 3–5, students should learn about area more thoroughly, as well as perimeter, volume, temperature, and angle measure. In these grades, they learn that measurements can be computed using formulas and need not always be taken directly with a measuring tool. Middle-grades students build on these earlier measurement experiences by continuing their study of perimeter, area, and volume and by beginning to explore derived measurements, such as speed. They should also become proficient in measuring angles and understanding angle relationships. In high school, students should understand how decisions about unit and scale can affect measurements. Whatever their grade level, students should have many informal experiences in understanding » attributes before using tools to measure them or relying on formulas to compute measurements.

As they progress through school, not only should students' repertoire of measurable attributes expand, but their understanding of the relationships between attributes should also develop. Students in the elementary grades can explore how changing an object's attributes affects certain measurements. For example, cutting apart and rearranging the pieces of a shape may change the perimeter but will not affect the area. In the middle grades this idea can be extended to explorations of how the surface area of a rectangular prism can vary as the volume is held constant. Such observations can offer glimpses of sophisticated mathematical concepts such as invariance under certain transformations.

The types of units that students use for measuring and the ways they use them should expand and shift as students move through the prekindergarten through grade 2 curriculum. In preschool through grade 2, students should begin their study of measurement by using nonstandard units. They should be encouraged to use a wide variety of objects, such as paper clips to measure length, square tiles to measure area, and paper cups to measure volume. Young children should also have opportunities to use standard units like centimeters, pounds, and hours. The "standardization" of units should arise later in the lower grades, as students notice that using Joey's foot to measure the length of the classroom gives a different length from that found by using Aria's foot. Such experiences help students see the convenience and consistency of using standard units. As students progress through middle school and high school, they should learn how to use standard units to

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measure new abstract attributes, such as volume and density. By secondary school, as students are measuring abstract attributes, they should use more-complex units, such as pounds per square inch and person-days.

Understanding that different units are needed to measure different attributes is sometimes difficult for young children. Learning how to choose an appropriate unit is a major part of understanding measurement. For example, students in prekindergarten through grade 2 should learn that length can be measured using linear tools but area cannot be directly measured this way. Young children should see that to measure area they will need to use a unit of area such as a square region; middle-grades students should learn that square regions do not work for measuring volume and should explore the use of three-dimensional units. Students at all levels should learn to make wise choices of units or scales, depending on the problem situation. Choosing a convenient unit of measurement is also important. For example, although the length of a soccer field can be measured in centimeters, the result may be difficult to interpret and use. Students should have a reasonable understanding of the role of units in measurement by the end of their elementary school years.

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The metric system has a simple and consistent internal organization. Each unit is always related to the previous unit by a power of 10: a centimeter is ten times larger than a millimeter, a decimeter is ten times larger than a centimeter, and so forth. Since the customary English system of measurement is still prevalent in the United States, students should learn both customary and metric systems and should know some rough equivalences between the metric and customary systems—for » example, that a two-liter bottle of soda is a little more than half a gallon. The study of these systems begins in elementary school, and students at this level should be able to carry out simple conversions within both systems. Students should develop proficiency in these conversions in the middle grades and should learn some useful benchmarks for converting between the two systems. The study of measurement systems can help students understand aspects of the base-ten system, such as place value. And in making conversions, students apply their knowledge of proportions.

Understanding that all measurements are approximations is a difficult but important concept for students. They should work with this notion in grades 3–5 through activities in which they measure certain objects, compare their measurements with those of the rest of the class, and note that many of the values do not agree. Class discussions of their observations can elicit the ideas of precision and accuracy. Middle-grades students should continue to develop an understanding of measurements as approximations. In high school, students should come to recognize the need to report an appropriate number of significant digits when computing with

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measurements.

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Apply appropriate techniques, tools, and formulas to determine measurements

Measurement techniques are strategies used to determine a measurement, such as counting, estimating, and using formulas or tools. Measurement tools are the familiar devices that most people associate with taking measurements; they include rulers, measuring tapes, vessels, scales, clocks, and stopwatches. Formulas are general relationships that produce measurements when values are specified for the variables in the formula.

Students in prekindergarten through grade 2 should learn to use a variety of techniques, including counting and estimating, and such tools as rulers, scales, and analog clocks. Elementary and middle-grades students should continue to use these techniques and develop new ones. In addition, they ought to begin to adapt their current tools and invent new techniques to find more-complicated measurements. For example, they might use transparent grid paper to approximate the area of a leaf. Middle-grades students can use formulas for the areas of triangles and rectangles to find the area of a trapezoid. An important measurement technique in high school is successive approximation, a precursor to calculus concepts.

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Students should begin to develop formulas for perimeter and area in the elementary grades. Middle-grades students should formalize these techniques, as well as develop formulas for the volume and surface area of objects like prisms and cylinders. Many elementary and middle-grades children have difficulty with understanding perimeter and area (Kenney and Kouba 1997; Lindquist and Kouba 1989). Often, these children are using formulas such as $P = 2l + 2w$ or $A = l \times w$ without understanding how these formulas relate to the attribute being measured or the unit of measurement being used. Teachers must help students see the connections between the formula and the actual object. In high school, as students use formulas in solving problems, they should recognize that the units in the measurements behave like variables under » algebraic procedures, and they can use this observation to organize their conversions and computations using unit analysis.

Estimating is another measurement technique that should be developed throughout the school years. Estimation activities in prekindergarten through grade 2 should focus on helping children better understand the process of measuring and the role of the size of the unit. Elementary school and middle-grades students should have many opportunities to estimate measures by comparing them against some benchmark. For example, a student might estimate

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the teacher's height by noting that the teacher is about one and one-half times as tall as the student. Middle-grades students should also use benchmarks to estimate angle measures and should estimate derived measurements such as speed.

Finally, students in grades 3–5 should have opportunities to use maps and make simple scale drawings. Grades 6–8 students should extend their understanding of scaling to solve problems involving scale factors. These problems can help students make sense of proportional relationships and develop an understanding of similarity. High school students should study more-sophisticated aspects of scaling, including the effects of scale changes on a problem situation. They should also come to understand nonlinear scale changes such as logarithmic scaling and how such techniques are used in analyzing data and in modeling.

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