Math4020 Conversion Table

Length:				
	inches	=		foot
	ft	=		yards
	ft	=		miles
	yards	=		miles
	cm	=		inch

Area:				
	sq. inches	=		sq. ft.
	sq. ft.	=		sq. yards
	sq. ft.	=		sq. miles
	sq. miles	=		acre

Mass:	ml			cm^3 (cc)
	oz.	=		lb.
	lbs.	=		ton
	liter	=		kg

Volume:	1 oz			in^3
	tsp.	=		Tbsp.
	Tbsp.	=		cup
	oz.	=		cup
	cups	=		pint
	pints	=		quart
	quarts	=		gallon
	cubic inches	=		cubic feet
	cubic feet	=		cubic yards

Time:				
	seconds	=		minute
	minutes	=		hour
	hours	=		day
	days	=		week
	weeks	=		year

Temperature:				
	degrees Celsius	=		degrees Farenheit
	degrees Farenheit	=		degrees Celsius

* Note: This should end up in your portfolio.

45

King Henry Metrics

How many???

Inches per yard		Cm /meter	
Yards per mile		Meters/ kilometer	
Pints per gallon		Dl/L	
Ounces per pound		Gram/Kg	
Ounces per quart		Cl/L	
Pounds per gallon (water)		Kg/L	
A 2 x 4 is how big			
cubic in. per pint		Cc/L	
60 mph = ? ft/sec		Km/h	

*** KING HENRY:

Long ago, there were only three major countries which had not converted to metric. Britain, Canada, USA.
King Henry did not want to be the last to convert.
He asked everyone to help him.......
Finally when it was explained,
King Henry Danced Merrily Down Center Main

K H Da M D C M

Three steps:
1. put decimal on right side of proper unit.
2. put 1 digit per column.
3. move decimal and fill in the zeros.

K Kilo	H Hecto	Da Deka	M meters	D Deci	C Centi	M Milli	
		3	2.	1			32.1 m = .0321 km
5.							5 km = 5000 m
				2	5 .		25 cm = .00025 km
						5.	5 mm = .005 m

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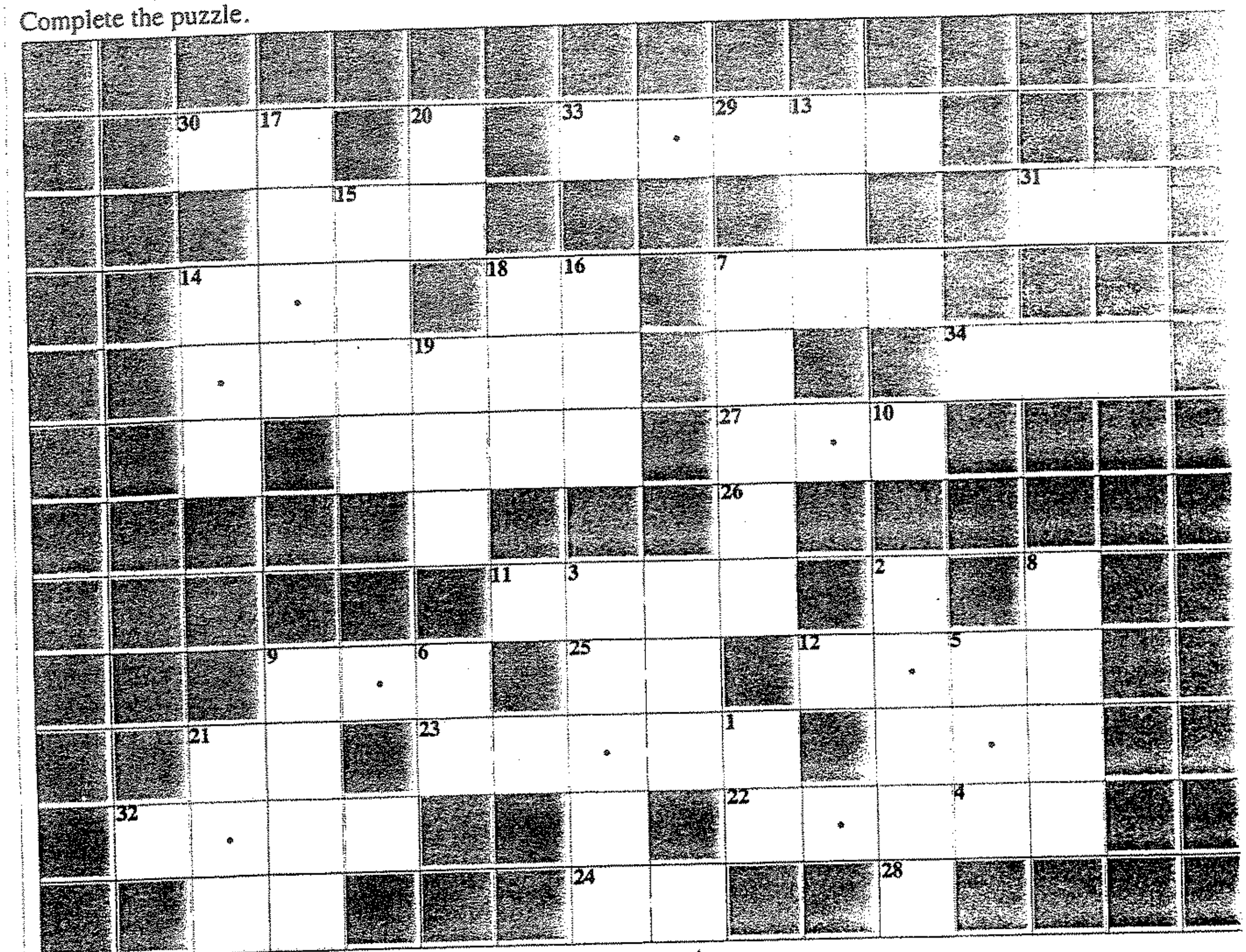
Metric Number Puzzle

(Answer ID # 0129953)

Complete the puzzle.



Down

1. 5.9 cl = ________ ml
2. 504.8 cl = ________ L
3. 6.441 cm = ________ mm
4. 3000 mm = ________ m
5. 2300 ml = ________ L
6. 8.1 cl = ________ ml
7. 11.76 g = ________ mg
8. 1.53 km = ________ m
9. 9.197 kg = ________ g
10. 1000 L = ________ kl
11. 9000 mg = ________ g
12. 4000 ml = ________ L
13. 2.7 L = ________ cl
14. 68 mg = ________ cg
15. 7.997 kg = ________ g
16. 4 m = ________ cm
17. 10600 m = ________ km
18. 4.7 L = ________ cl
19. 7.5 g = ________ cg
20. 8.3 cl = ________ ml
21. 3300 mm = ________ m

Across

1. 5000 ml = ________ L
7. 10 cl = ________ ml
9. 9800 mm = ________ m
11. 9.63 kl = ________ L
12. 4250 L = ________ kl
13. 20 mm = ________ cm
14. 690 cl = ________ L
22. 9430 g = ________ kg
23. 114.5 ml = ________ cl
24. 10000 L = ________ kl
25. 4.3 cl = ________ ml
26. 6000 mm = ________ m
27. 7100 mg = ________ g
28. 80 mg = ________ cg
29. 3.2 cl = ________ ml
30. 110 mm = ________ cm
31. 120 mm = ________ cm
32. 197 cm = ________ m
33. 832.6 cm = ________ m
34. 10.2 cg = ________ mg

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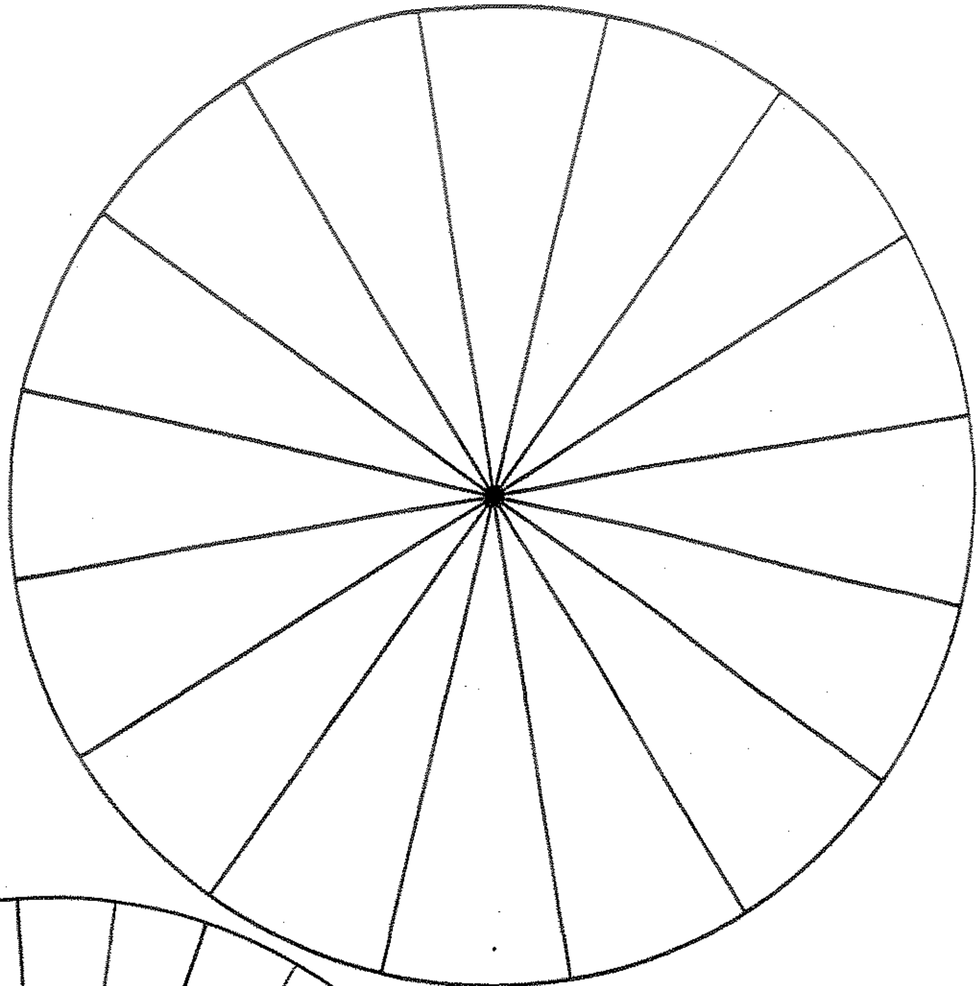

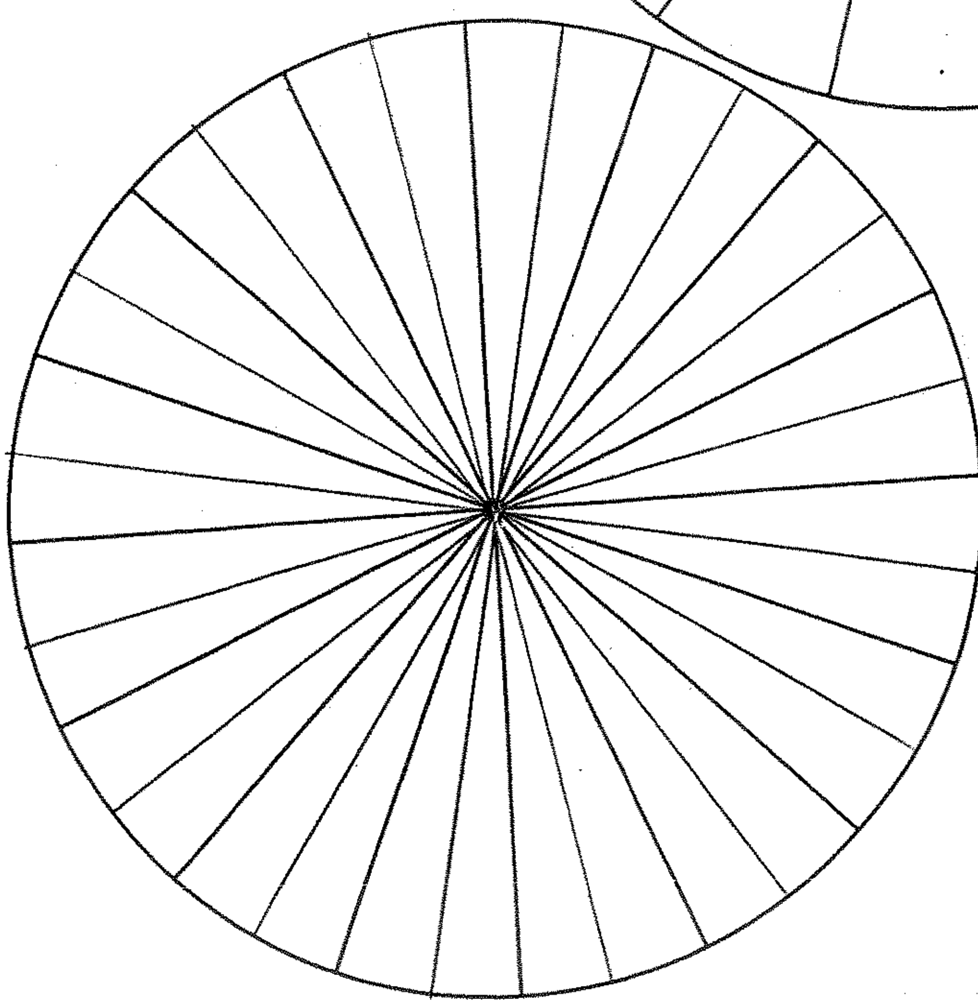

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Math4020 Geometry Notes
For 3-dimensional Objects
(Section 12.5)

Basic Terminology

Face→ Polygonal region (forms dihedral angle).

Edge→ Line segment that is common to a pair of faces.

Dihedral Angle→ The angle formed by the union of polygonal regions in space that share an edge.

Vertex→ A point of intersection between edges.

Polyhedron→ (plural is polyhedra) The union of faces, any two of which have at most one edge in common, such that a connected finite region in space is enclosed without holes (i.e. such that it will contain liquid without spilling).

Convex→ A polyhedron is convex if every line segment formed by connecting two points inside the polyhedron is wholly contained inside that polyhedron OR is on a face of the polyhedron.

Types of Polyhedra

Prism→ Has 2 opposite, parallel faces (called *bases*) that are identical polygons.

> *Right Prism*→ A prism whose *lateral faces* (those faces that are neither of the bases) are rectangles; the lateral faces meet up with the bases at a right angle.

Pyramid→ Has polygon for a base and a point not in the plane of the base (called the *apex*) that is connected with line segments to each vertex of the polygonal base.

> *Right Pyramid*→ A pyramid whose apex lies perpendicularly over the center of the base.

Regular Polyhedron→ All faces are identical regular polygons and all dihedral angles are the same.

> *Platonic Solids*→ The only 5 regular, convex polyhedra! (See handout.)

Semiregular Polyhedron→ Has several different regular polygonal faces, but it has the same arrangement of polygons at each vertex.

Other 3d Solids

Cylinder→ Has 2 opposite, parallel, identical, simple, closed curves as bases and line segments that connect corresponding points from base to base (it's like a prism, except that the bases are not polygonal).

Right Cylinder→ A cylinder whose "lateral" surface meets the bases at right angles.

Oblique Cylinder→ A cylinder that is not a right cylinder, i.e. the "lateral" surface meets the bases at acute or obtuse angles.

Cone→ Has a simple, closed curve as a base and a point not in the plane of the base that is connected with line segments to each vertex of the base (it's like a pyramid, except that the base is not polygonal).

Right Cone→ A cone whose apex lies perpendicularly over the centroid of the base.

Sphere→ The set of all points in 3d space that are the same distance from a fixed point (called the *center*).

Radius→ The length of the segment connecting the center to any point on the sphere.

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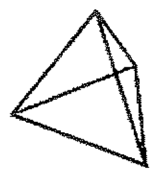
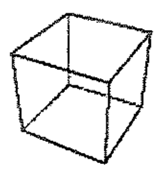
The Platonic Solids

pla submit

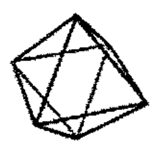
The so-called Platonic Solids are *regular polyhedra*. "Polyhedra" is a Greek word meaning "many faces." There are five of these, and they are characterized by the fact that each face is a regular polygon, that is, a straight-sided figure with equal sides and equal angles:



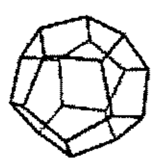
TETRAHEDRON
Four triangular faces, four vertices, and six edges.



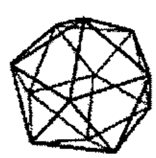
CUBE
Six square faces, eight vertices, and twelve edges.



OCTAHEDRON
Eight triangular faces, six vertices, and twelve edges.



DODECAHEDRON
Twelve pentagonal faces, twenty vertices, and thirty edges.



ICOSAHEDRON
Twenty triangular faces, twelve vertices, and thirty edges.

Johannes Kepler's Polyhedra

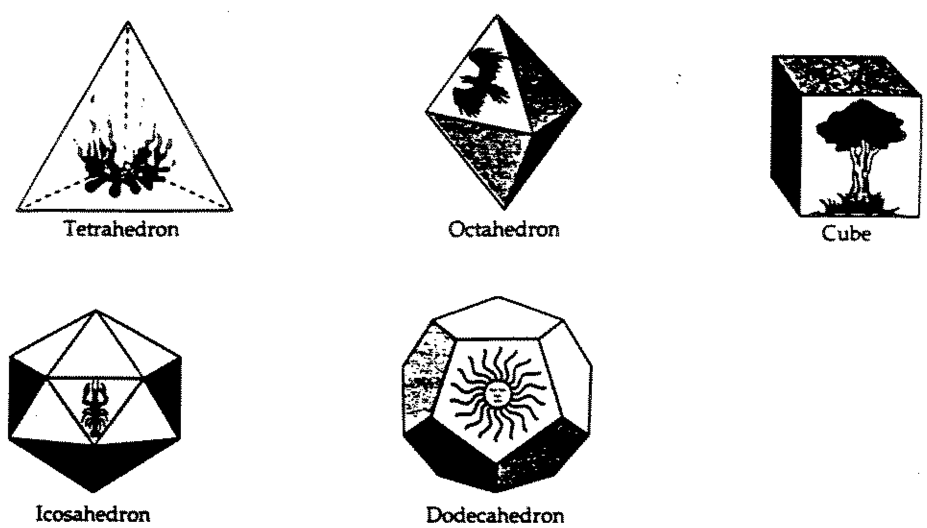
Johannes Kepler (1571-1630), best known for his three laws of planetary motion, was one of the most outstanding mathematicians of his day. In addition to his astronomical accomplishments, he systematized and extended all that was known about polyhedra in his time. While previous artist/geometers discovered particular polyhedra, he took a more mathematical approach: he defined classes of polyhedra, discovered the members of the class, and proved that his set was complete

A cube is a regular polyhedron. A triangular pyramid composed of equilateral triangles is a regular polyhedron and has a special name, [illegible]. The origin of the name is Greek: *tetra* ("four") and *hedron* ("face").

A fact that surprises many people is that there are not a large number of regular polyhedra. In fact, there are only five regular polyhedra: the tetrahedron, the cube, the octahedron (with 8 triangular faces), the [illegible] (with 12 pentagonal faces), and the icosahedron (with 20 triangular faces) (see Figure 8.70). The solids made from the regular polyhedra are called Platonic solids after the Greek philosopher Plato.

HISTORY

ver 2,000 years ago (before e knew about atoms), many eek philosopher-scientists lieved that there were four sic elements out of which all ngs arose: earth, air, fire, and ter. Some of the Greeks lieved that the smallest rticle of earth had the form of cube, the smallest particle of had the form of an ahedron, the smallest rticle of fire had the form of a ahedron, and the smallest rticle of water had the form of icosahedron. The decahedron was associated n the universe, probably cause it was the last solid covered, although it has en speculated that it is sociated with the universe cause it has twelve faces and re are twelve signs in the diac.[10]



Source: The five regular solids drawn by Johannes Kepler in *Harmonices Mundi, Book II*, 1619.
FIGURE 8.70

It is natural to wonder why there should be exactly five Platonic solids, and whether there might conceivably be one that simply hasn't been discovered yet. However, it is not difficult to show that there must be five – and that there *cannot* be more than five.

First, consider that at each vertex (point) at least three faces must come together, for if only two came together they would collapse against one another and we would not get a solid. Second, observe that the sum of the interior angles of the faces meeting at each vertex must be less than 360°, for otherwise they would not all fit together.

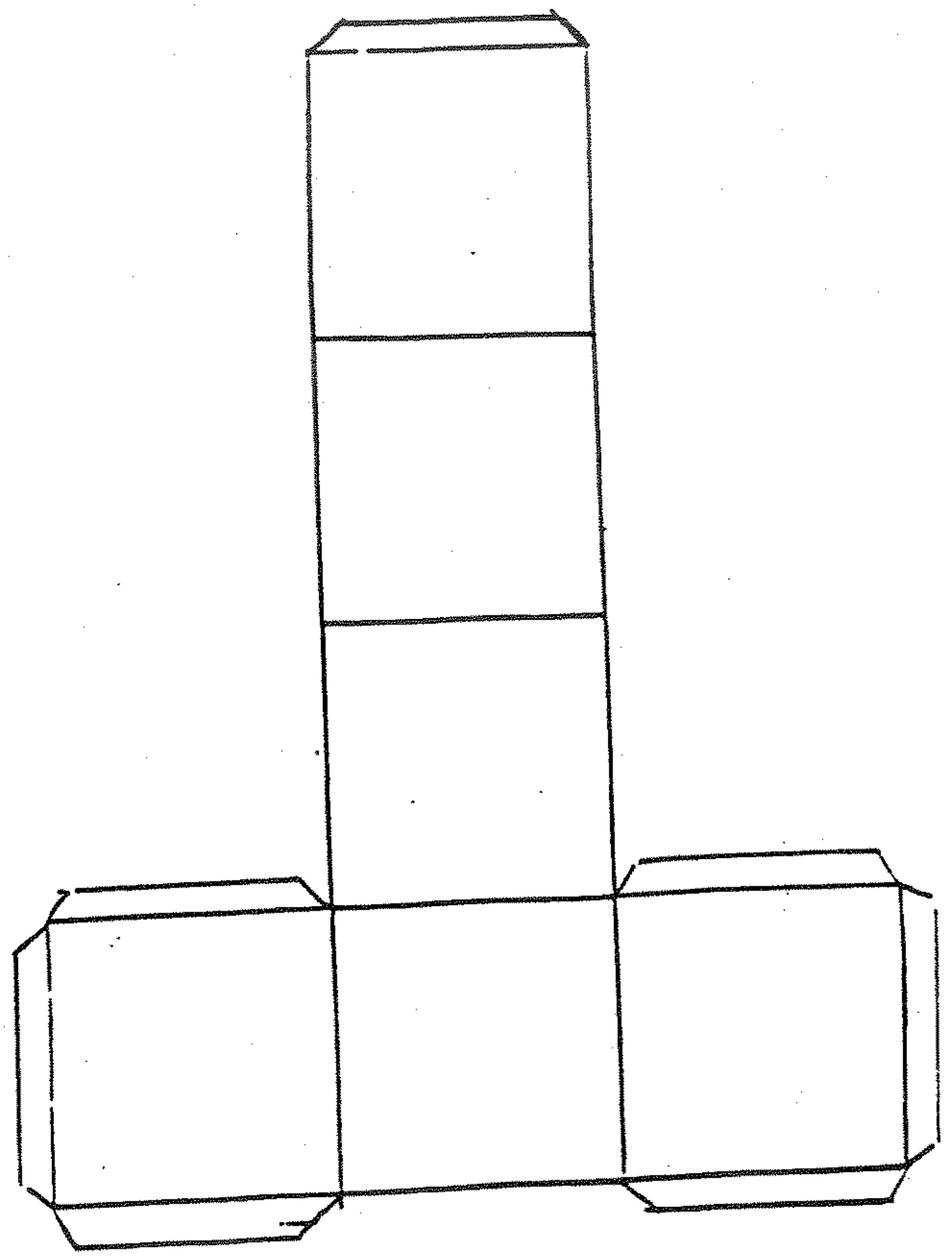
Now, each interior angle of an equilateral (i.e., regular) triangle is 60°, hence we could fit together three, four, or five of them at a vertex, and these correspond to to the tetrahedron, the octahedron, and the icosahedron. Each interior angle of a square is 90°, so we can fit only three of them together at each vertex, giving us a cube. (We could fit four squares together, but then they would lie flat, giving us a tesselation instead of a solid.) The interior angles of the regular pentagon are 108°, so again we can fit only three together at a vertex, giving us the dodecahedron.

And that makes five regular polyhedra. What about the regular hexagon, that is, the six-sided figure? Well, its interior angles are 120°, so if we fit three of them together at a vertex the angles sum to precisely 360°, and therefore they lie flat, just like four squares would do. For this reason we can use hexagons to make a tesselation of the plane, but we cannot use them to make a Platonic solid. And, obviously, no polygon with more than six sides can be used either, because the interior angles just keep getting larger.

Math4020

Solids Worksheet

Solid Type (e.g. Right Circular cone)	# Faces F	# Vertices V	# Edges E	? relationship Between F, V and E	Face shape(s)



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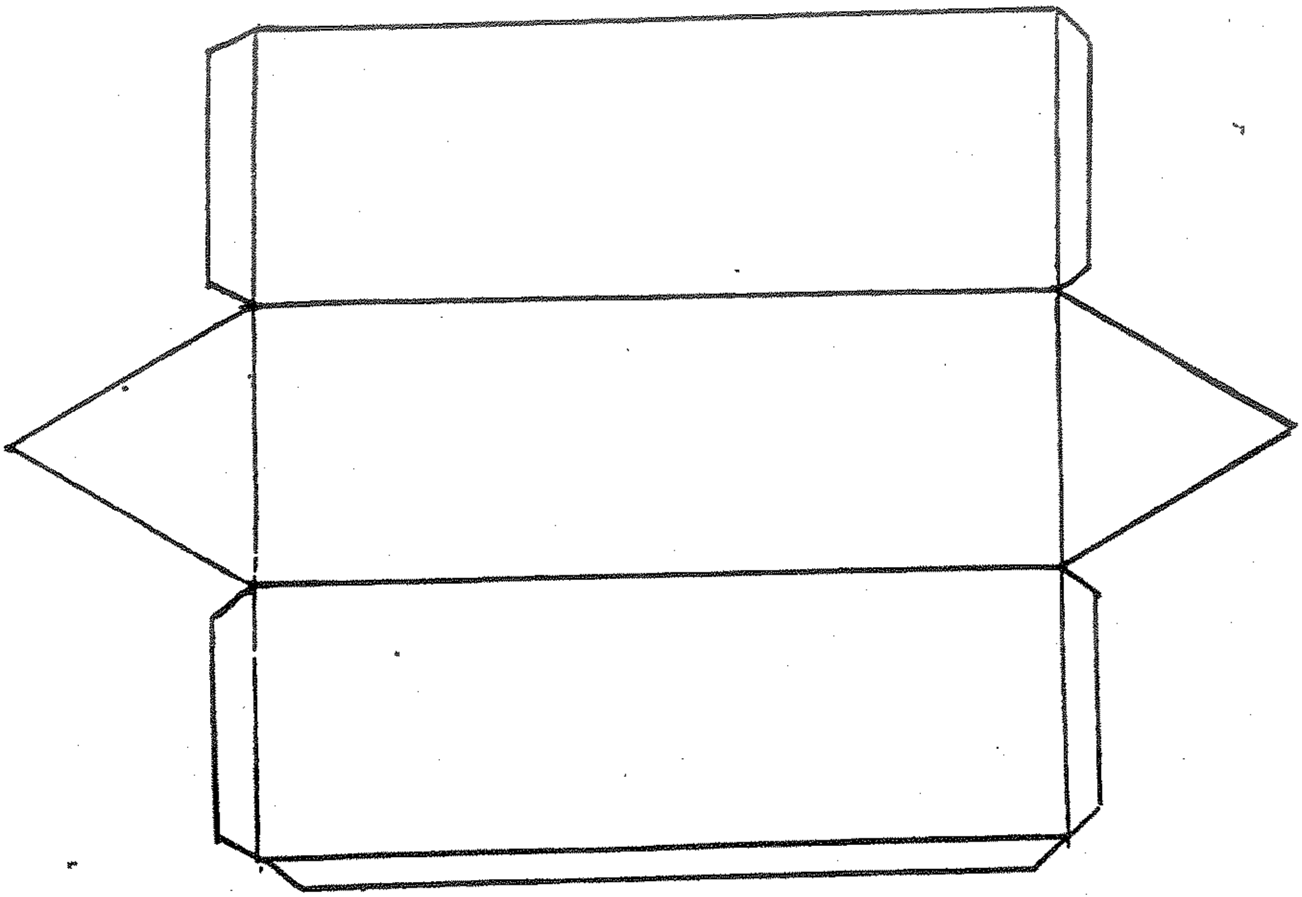

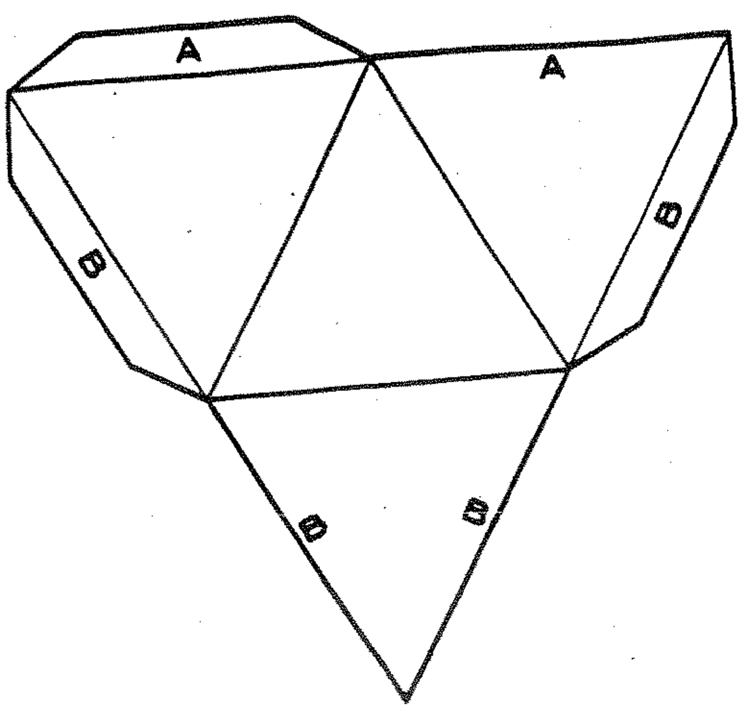

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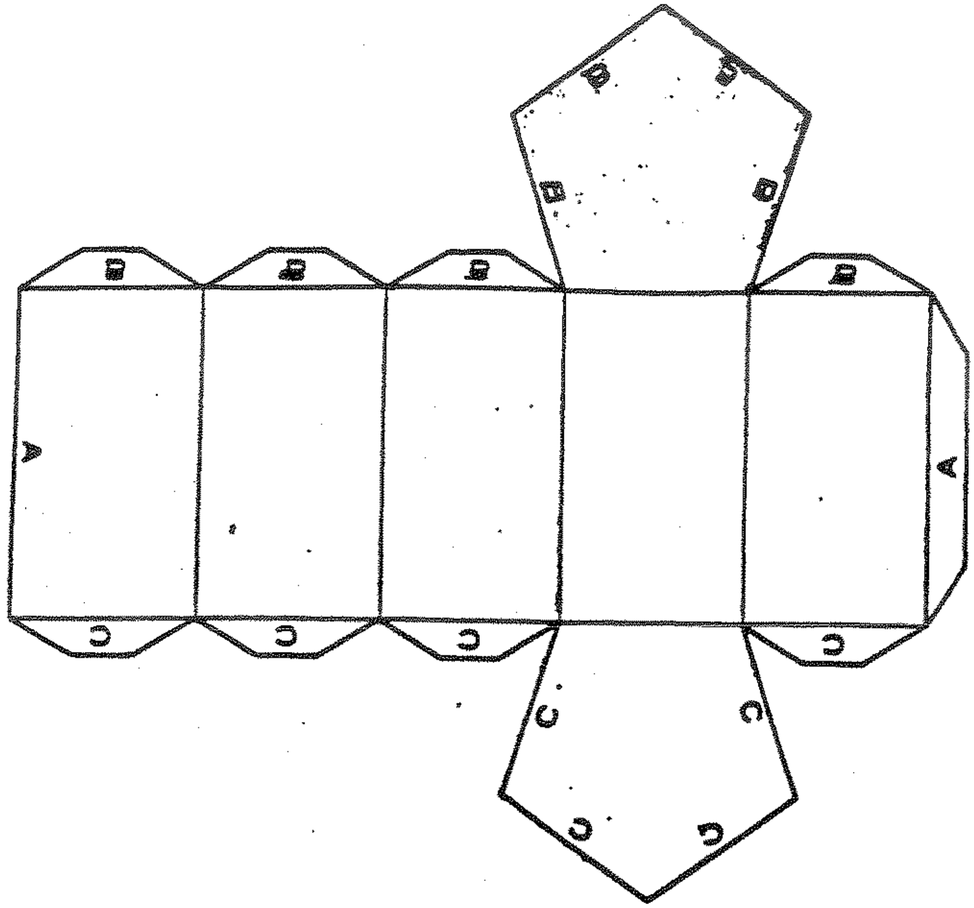

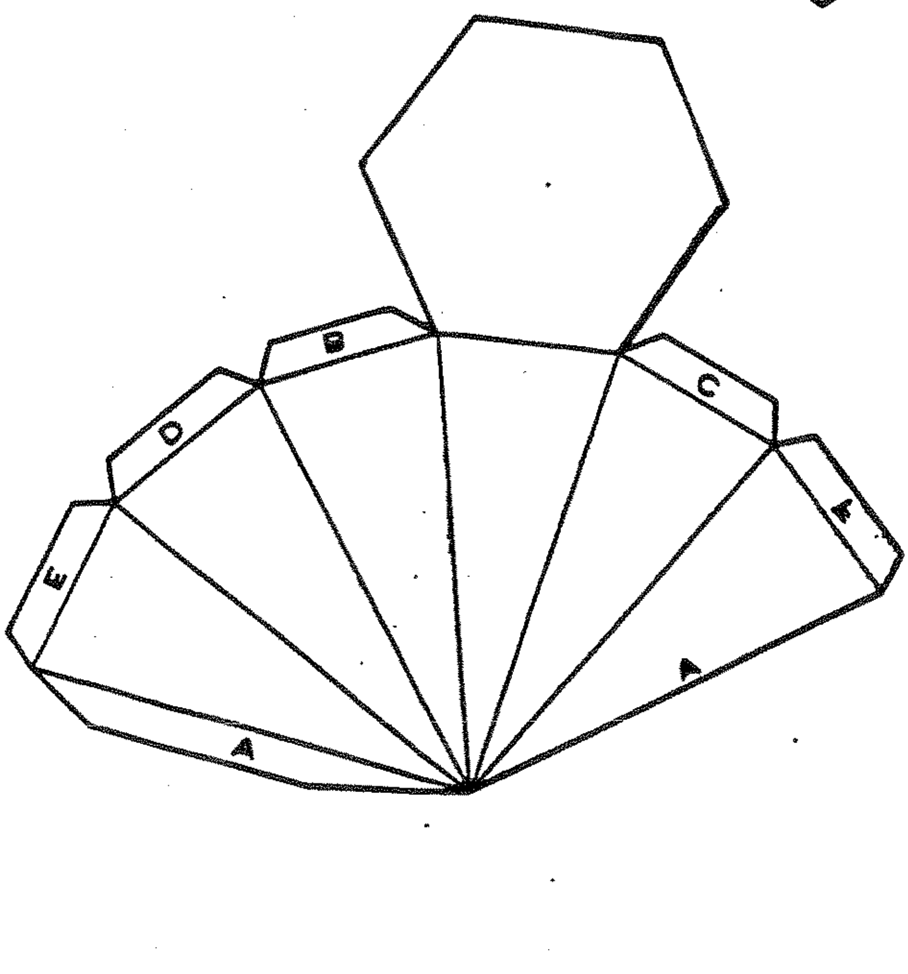

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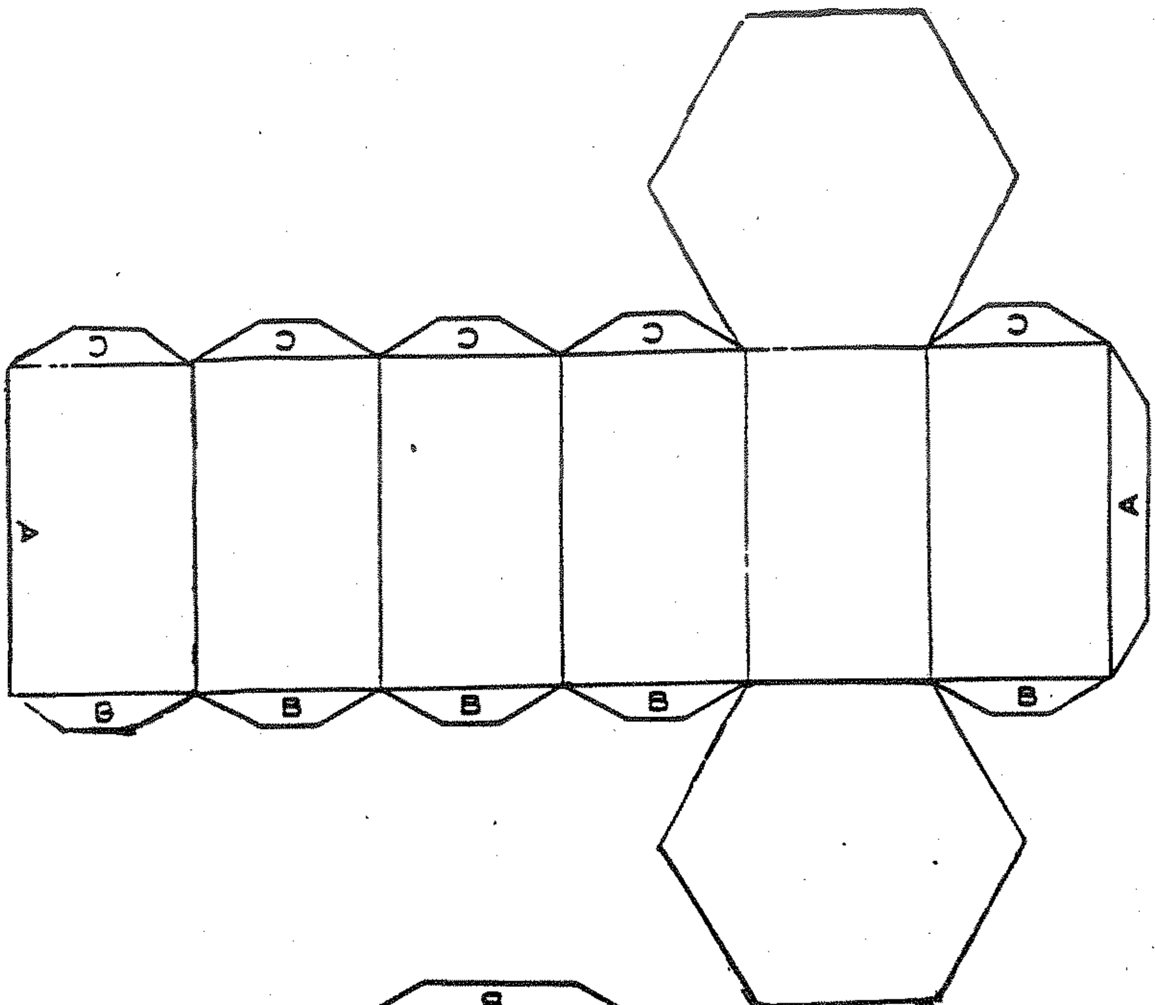

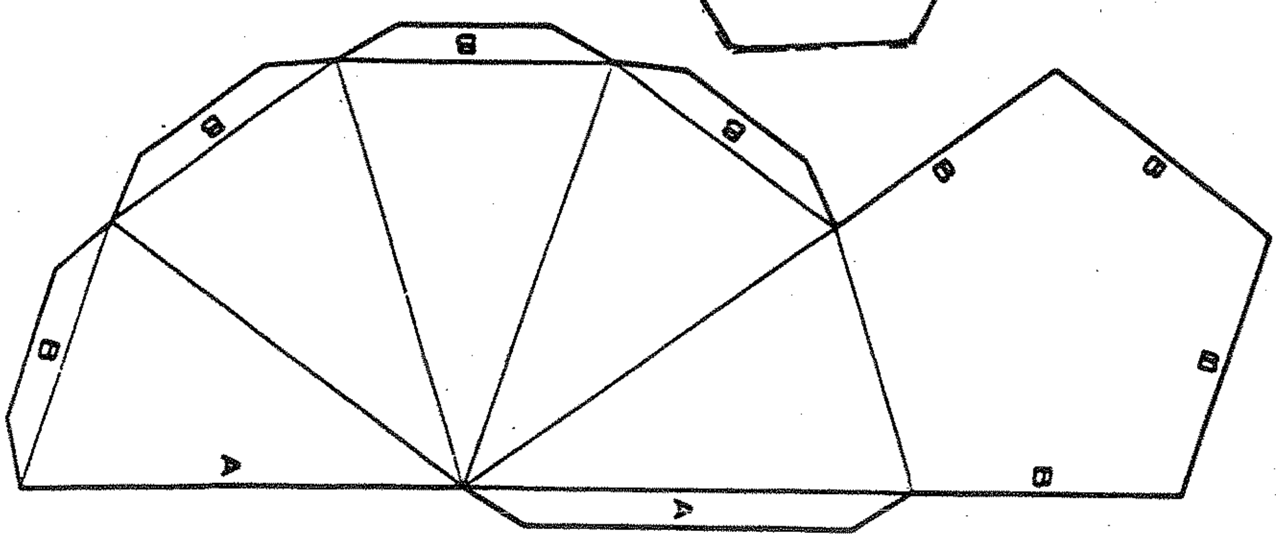

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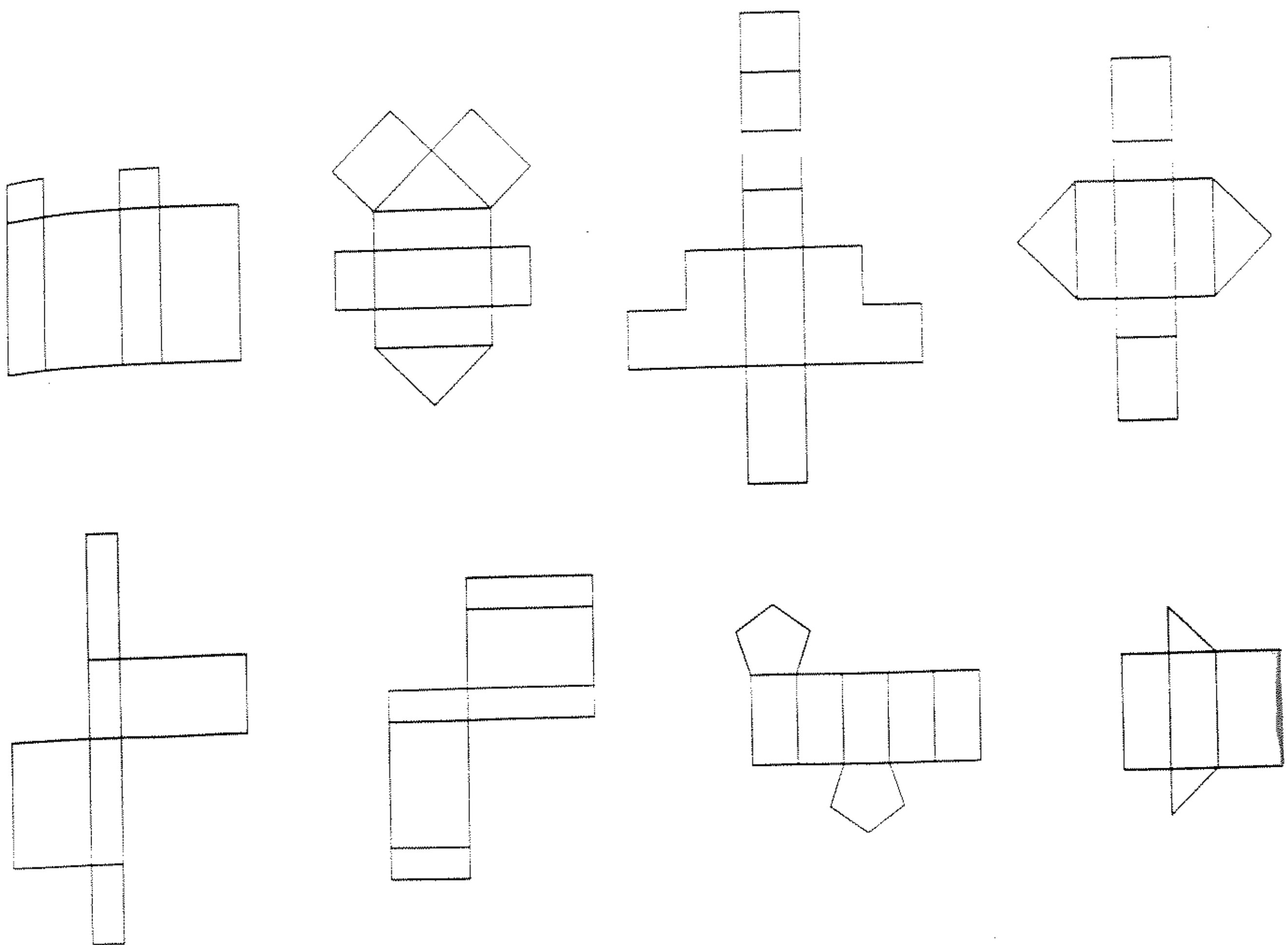
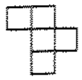
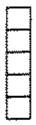
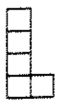
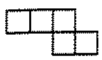
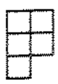
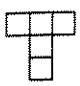
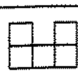
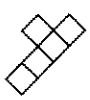
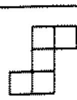
Now that you have experience making nets for shapes, let's go the other way. Following are some possible nets. Predict which will fold up into a polyhedra and which will not. Give your reasons. If you predict it will, sketch the polyhedron or describe it.



Pentominoes That Fold into Cubes

Name	Prediction and reasoning	Yes/No	Reflection
F 			
I 			
L 			
N 			
P 			
T 			
U 			
V 			
W 			
X 			
Y 			
Z 			

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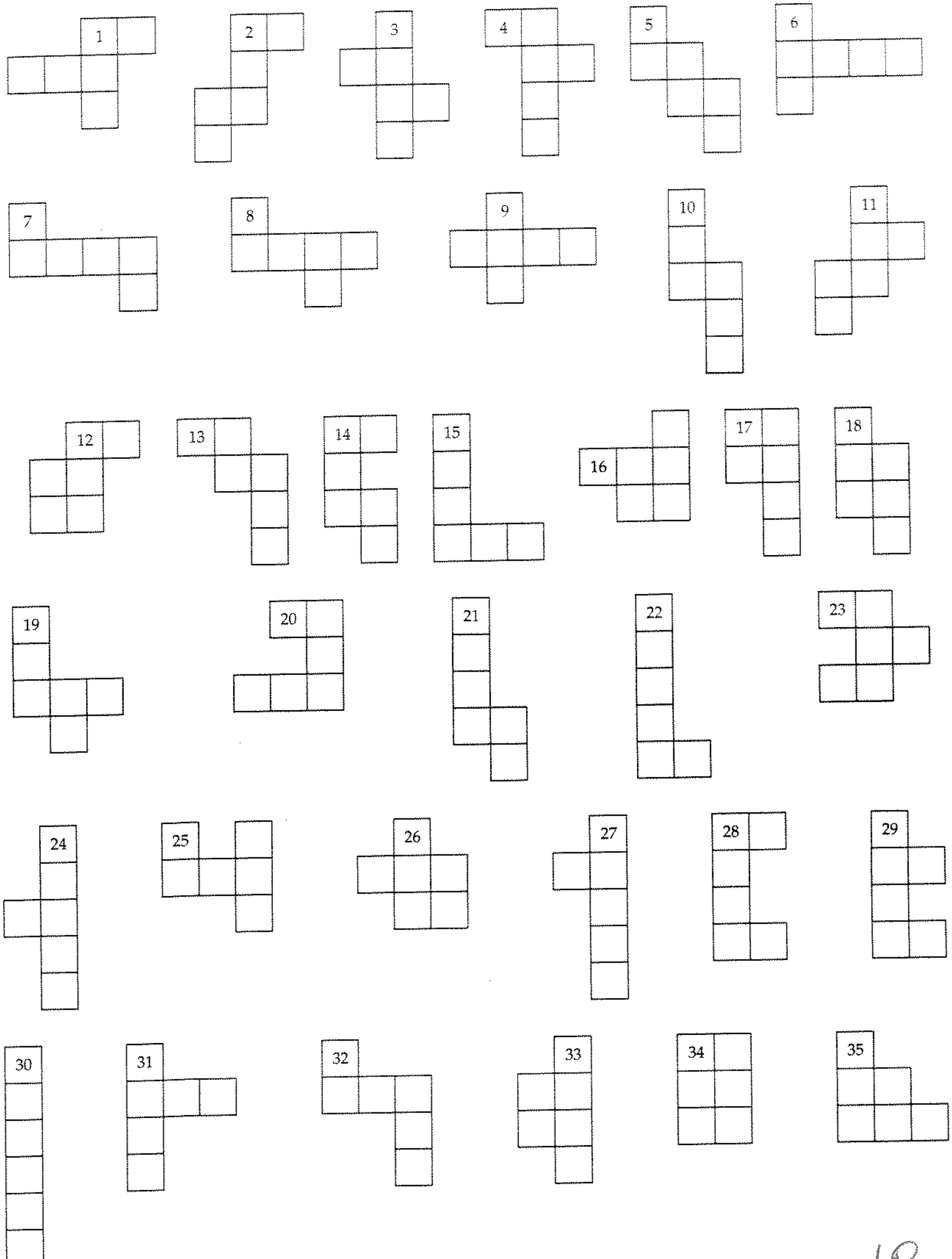
Hexomino Sheet



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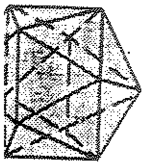
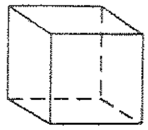
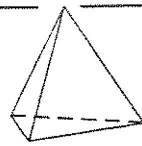
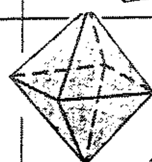
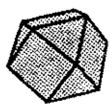
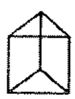
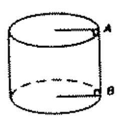
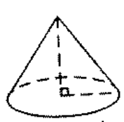
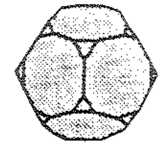


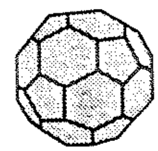
:osahedron

Planes of symmetry of the polyhedra

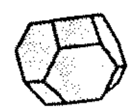
Shape	Sketch	Vertical	Horizontal	Oblique
Cube				
Tetrahedron Face down				
Octahedron Vertex down				
Dodecahedron Face down				
Cubeoctahedron Square face down				
Triangular prism				
Square pyramid				
Cylinder				
cone				



Truncated dodecahedron



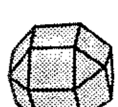
Truncated icosahedron



Truncated octahedron

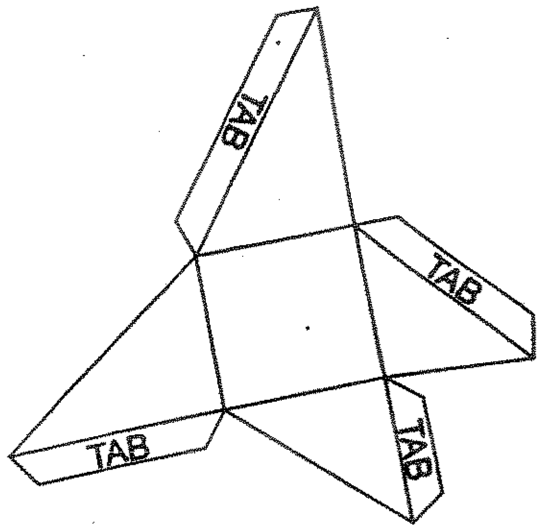
Pentagonal prism

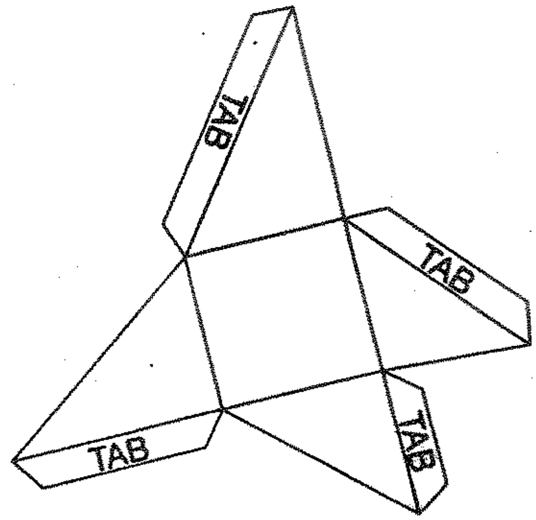
Truncated

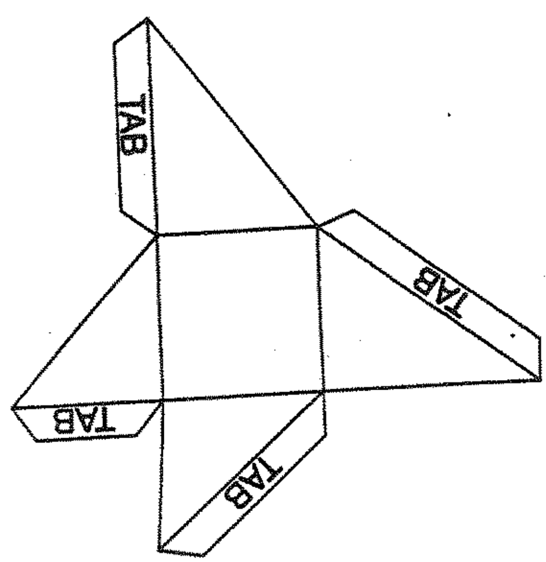


Small rhombicuboctahedron

Truncated cube







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GEOMETRIC SHAPE	SURFACE AREA	VOLUME
Right prism	$S = 2A + Ph$	$V = Ah$
Right circular cylinder	$S = 2A + Ch$	$V = Ah$
Right regular pyramid	$S = A + \frac{1}{2}Pl$	$V = \frac{1}{3}Ah$
Right circular cone	$S = A + \frac{1}{2}Cl$	$V = \frac{1}{3}Ah$
Sphere	$S = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

GEOMETRIC SHAPE	SURFACE AREA	VOLUME
Right prism	$S = 2A + Ph$	$V = Ah$
Right circular cylinder	$S = 2A + Ch$	$V = Ah$
Right regular pyramid	$S = A + \frac{1}{2}Pl$	$V = \frac{1}{3}Ah$
Right circular cone	$S = A + \frac{1}{2}Cl$	$V = \frac{1}{3}Ah$
Sphere	$S = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

Math4020

Scaling Worksheet

Cube

Side Length	Surface Area	Volume
1 m		
2 m		
3 m		
5 m		
7 m		
10 m		

Sphere

Radius	Surface Area	Volume
1 ft		
2 ft		
3 ft		
5 ft		
6 ft		
10 ft		

Rt. Circular Cylinder

Radius	Height	Surface Area	Volume
1 in	3 in		
2 in	6 in		
3 in			
5 in			
8 in			
10 in			

Rt. Circular Cone

Radius	Height	Surface Area	Volume
1 unit	2 units		
2 units	4 units		
3 units			
5 units			
9 units			
10 units			

Scaling Relationship:

If we double the lengths in a solid,
We multiply the surface area by ______
We multiply the volume by ______

If we triple the lengths in a solid,
We multiply the surface area by ______
We multiply the volume by ______

If we multiply the lengths in a solid by 5,
We multiply the surface area by ______
We multiply the volume by ______

If we multiply the lengths in a solid by n,
We multiply the surface area by ______
We multiply the volume by ______

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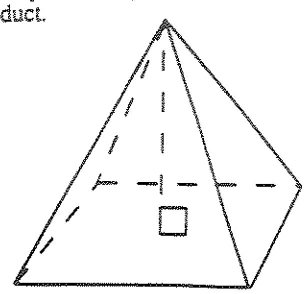
edges:
faces:
vertices:

2. (16 pts) The square pyramid drawn below is a scale model of the package for a new product.



scale model
height = 8 in
apex located over center of square base
side of square base = 5 in

a) What is the volume of the scale model? Volume:________________

b) What is the surface area of the scale model? Surface Area: _____________

c) Suppose the actual package will have a height 32 inches. What will the surface area and volume be for the actual package? Surface Area:____________

Volume:_______________

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Scaling Words

ratio a comparison of two numbers in a specific order.. We generally separate the two numbers in the ratio with a colon (:). For example, if we want to write the ratio of 8 and 12, we can write this as 8:12 or as a fraction 8/12, and we say the ratio is *eight to twelve.*

proportion an equation with a ratio on each side of the equal sign. It is a statement that says two ratios are equal. For example, 3:4 = 6:8.

scale the proportional relationship between 2 sets of measurements. For example, the distance on a map and the actual distance; 1 inch = 500 miles.

scale factor the ratio between lengths of corresponding sides of two similar figures. For example, if one similar figure is twice the size of another, then the scale factor is 2.

similar figures figures that have the same shape, but not necessarily the same size.

$$\text{scale factor} = \frac{\text{actual distance}}{\text{scaled distance}}$$

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Math4020 Midterm 2 Topics

- Classifying/recognizing 2d polygons/shapes
- Symmetry
- Corresponding angles
- Alternate interior angles
- Regular Polygons
  - Vertex angles (a.k.a. interior angles)
  - Central angles
  - Exterior angles
- Angle sum for a polygon
- Tessellations
- Converting units of measurement
- Perimeter of 2d shapes
- Area of 2d shapes
- Pi/circles
- Pythagorean theorem
- Classifying/recognizing 3d polyhedron/solids
- Platonic solids
- Surface area of 3d solids
- Volume of 3d solids
- Scaling

Math4020 Geometry Notes
Congruence and Similarity of Triangles
(Sections 14.1 & 14.2)

Definition of Congruent Triangles

Two triangles are congruent if and only if all corresponding sides are congruent and all corresponding interior angles are congruent.

Congruence Properties

(1) SSS→ If all corresponding sides of two triangles are congruent, then the two triangles are congruent.
(2) SAS→If two sides and the included angle between those two sides of a triangle are congruent to the corresponding sides and angle of another triangle, then those two triangles are congruent.
(3) ASA→If two angles and the included side between those two angles of a triangle are congruent to the corresponding angles and side of another triangle, then those two triangles are congruent.

Definition of Similar Triangles

Two triangles are similar if all corresponding interior angles are congruent and all corresponding sides are proportional.

Similarity Properties

(1) AA→ If two corresponding angles of two triangles are congruent, then the two triangles are similar.
(2) SAS→If two sides of a triangle are proportional to the corresponding sides in another triangle and the included angle between those two sides is congruent to the corresponding angle of the other triangle, then those two triangles are similar.
(3) SSS→If all corresponding sides of two triangles are proportional, then the two triangles are similar.

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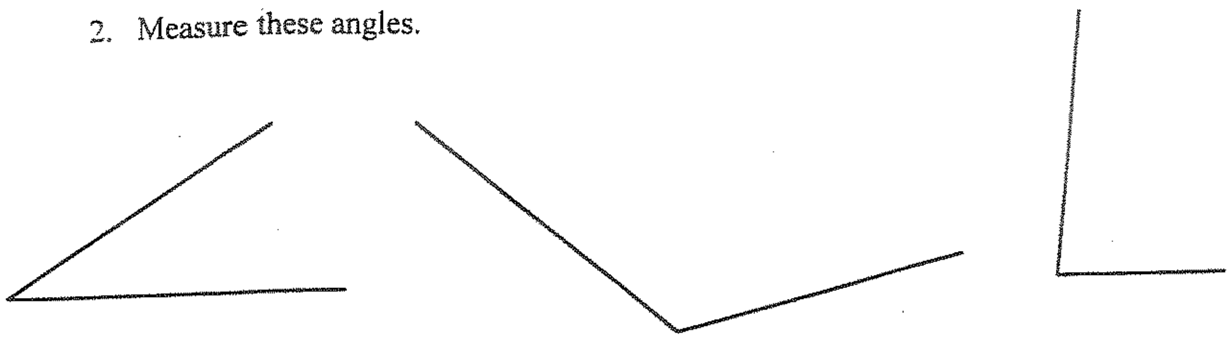
Congruence of Triangles

Math4020

1. Decide on three different length segments to use for construction, namely short, medium and long.

Write the lengths here: short__________ medium__________ long __________

2. Measure these angles.



3. Using your patty paper, do the following.
   (a) Draw a triangle with the three segments as the sides. Compare it with your partner's triangle. What do you notice?

   (b) Draw a triangle with the short and medium segments and the smallest angle between them. Can you do this another way?

   (c) Draw a triangle with the long segment and the small angle at one end and the medium angle at the other. Can you do this another way?

   (d) Draw a triangle with the smallest angle and your two shorter segments, but the angle should not be between those two segments. Can you do this another way?

   (e) Draw a random segment on your patty paper, different from your partner's segment. Put the smallest angle on the left and the medium angle on the right. Complete the triangle. Compare your triangle with your partner's triangle. What do you notice?

   (f) Draw a triangle with the two smallest angles and the shortest segment, making sure the short segment is not between the two angles. Can you do this another way?

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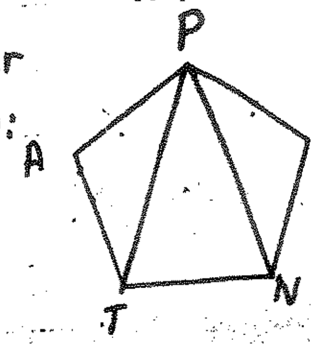
CONGRUENT TRIANGLES

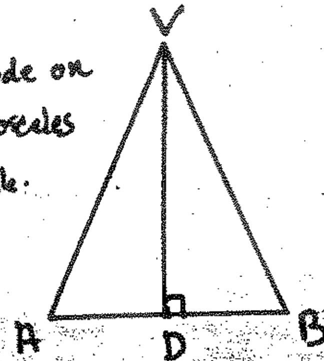
Which are congruent ? Why ?

Write a congruence statement & Reason

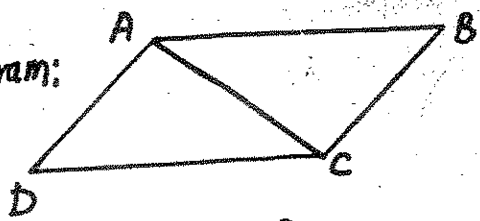
Regular Pentagon:



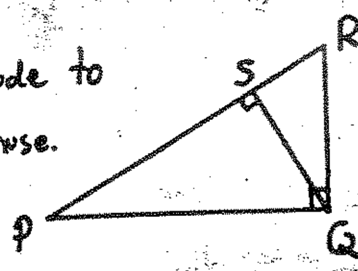
Altitude on an isosceles triangle.



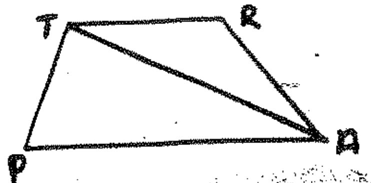
Parallelogram:



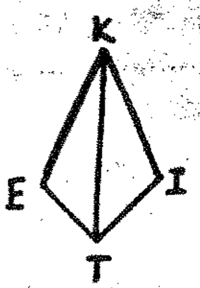
Altitude to hypotenuse.

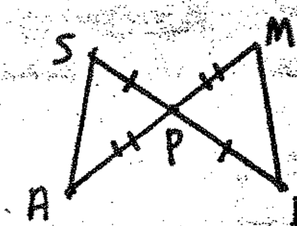


trapezoid

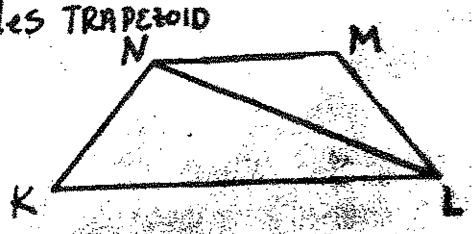


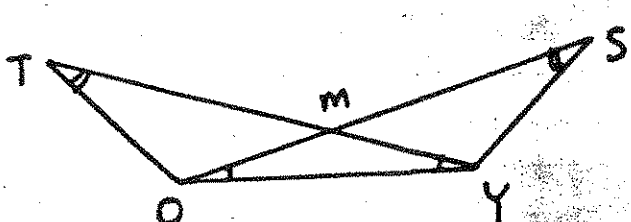
KITE:



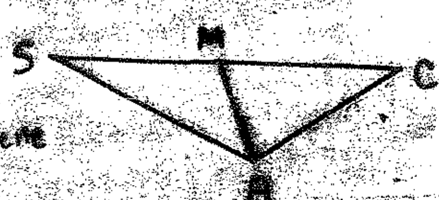


ISOSCELES TRAPEZOID





Angle Bisector a scalene triangle



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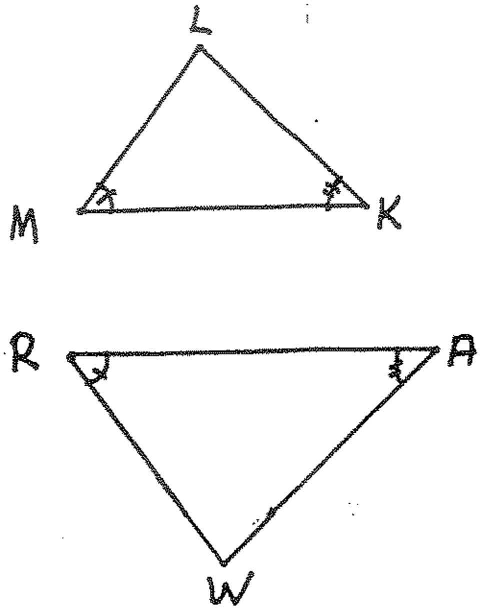
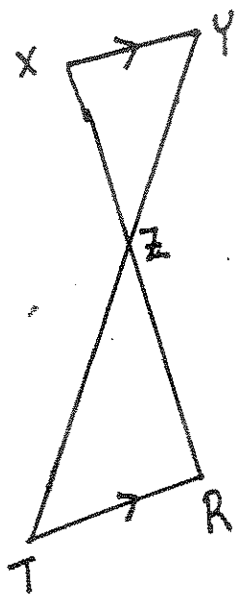
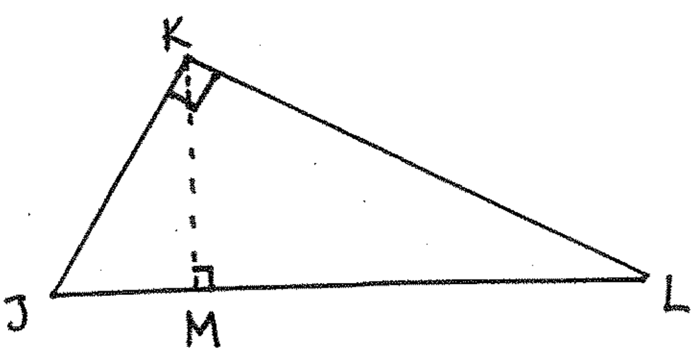
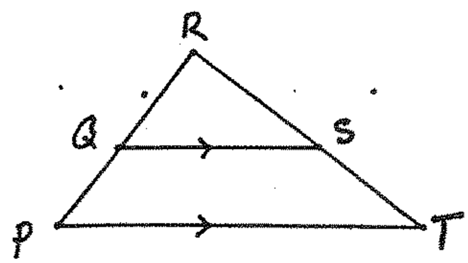
Write the similarity statement.









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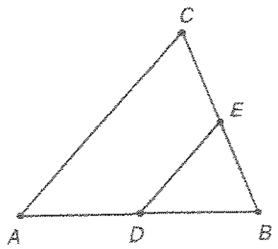
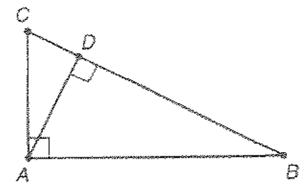
Extra Practice With Proofs 8.4

Name ____________________

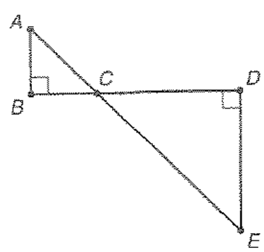
1. *Given:* $\overline{DE}$ is a midsegment of $\triangle ABC$.
Prove: $\triangle ABC \sim \triangle DBE$



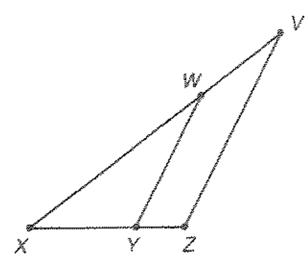
2. *Given:* $\triangle ABC$ is a right triangle.
$\overline{AD}$ is an altitude.
Prove: $\triangle ABC \sim \triangle DAC$



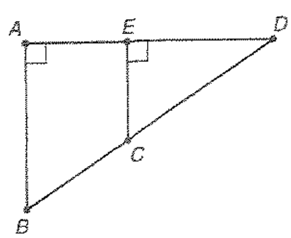
3. *Given:* $\angle ABC$ is a right angle.
$\angle EDC$ is a right angle.
Prove: $\triangle ABC \sim \triangle EDC$



4. *Given:* $\overline{WY} \parallel \overline{VZ}$
Prove: $\triangle XYW \sim \triangle XZV$



5. Complete the proof.
Given: $\overline{BA} \perp \overline{AD}$, $\overline{CE} \perp \overline{AD}$
Prove: $\triangle DEC \sim \triangle DAB$



Statements	*Reasons*
1. ?	1. Given
2. ?	2. Given
3. ?	3. $\perp$ lines intersect to form 4 rt. $\angle$s.
4. ?	4. All rt. $\angle$s are $\cong$.
5. ?	5. Reflexive Property
6. ?	6. AA Similarity Postulate

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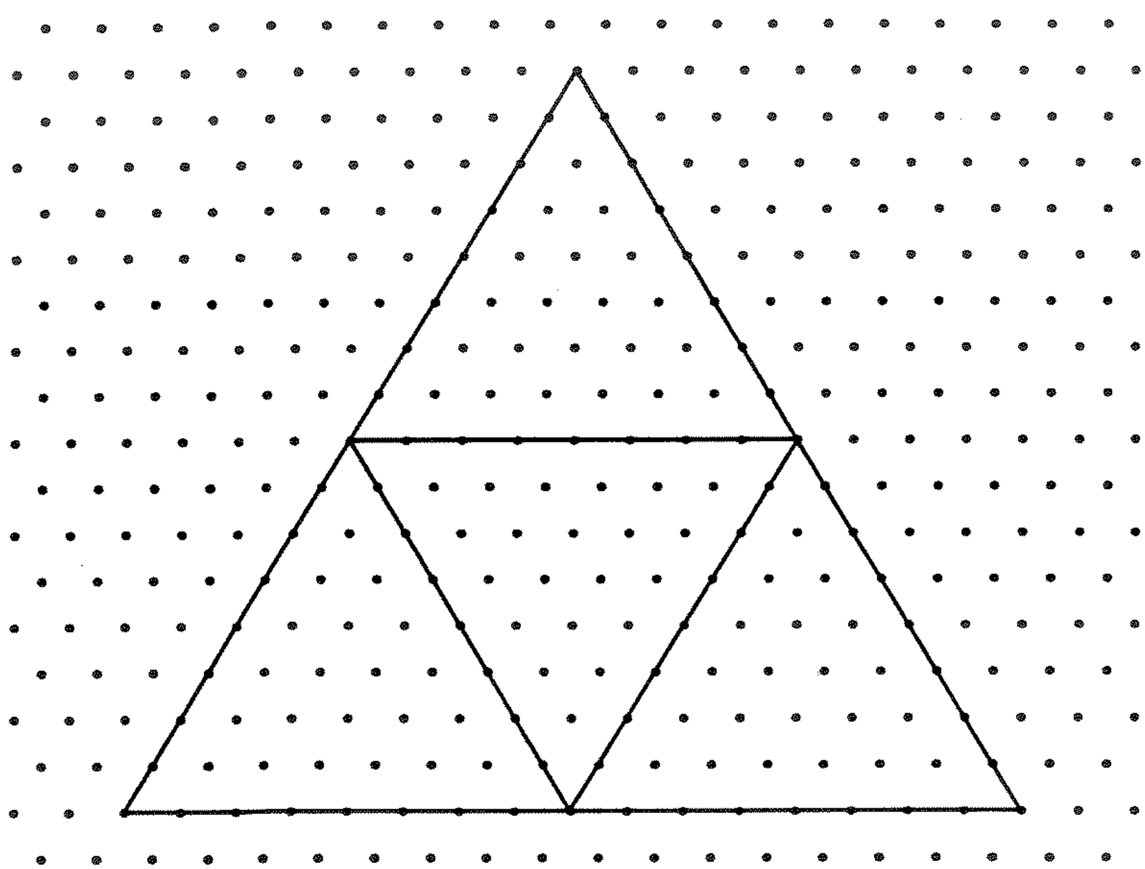
1.1 SIERPINSKI TRIANGLE AND VARIATIONS 1.1A

This construction process, when repeated over and over, generates a well known fractal image called the Sierpinski triangle (or gasket).

Construction Connect midpoints on the sides as shown, keeping only the three corner subtriangles formed.

Apply the construction process on the newly formed corner subtriangles through a second, third, and fourth stage. Remember, at every stage, each remaining triangle is transformed into three new subtriangles with sides half as long. Three times as many triangles appear at each successive stage. Count dots carefully. Every vertex of every subtriangle at each of the first four stages is on a dot on the grid paper. The resulting figure should contain 81 small triangles, representing the fourth stage in the construction of the *Sierpinski triangle.* Shade in these triangles.



1. Imagine repeating the process. Visualize and describe how the figure changes. If the process continues on without end, a Sierpinski triangle emerges.

2. What would remain of the original large triangle after four iterations if the algorithm were changed to keeping only the inner triangle?

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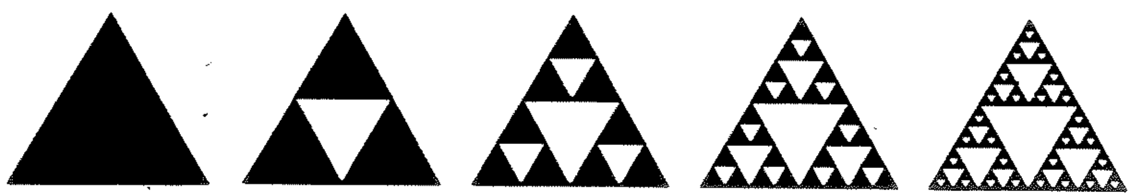
1.2 NUMBER PATTERNS WITH VARIATIONS 1.2A

This activity explores some of the number patterns found in the Sierpinski triangle.

DIRECTIONS The first four stages of the construction of the Sierpinski triangle are shown below. In subsequent stages, the subdivision continues into smaller and smaller triangles. Use these figures to explore number patterns that emerge as the Sierpinski triangle is developed through successive iterations.



NUMBER OF TRIANGLES

1. Count the number of shaded triangles at each stage 0 through 4.

STAGE	0	1	2	3	4	5	...	n
NUMBER	1							

2. Extend the pattern to predict the number of triangles at stage 5.
   What constant multiplier can be used to go from one stage to the next?

3. Generalize to find the number of triangles for level n.
   As n becomes large without bound, what happens to the number of triangles?

AREA OF TRIANGLES

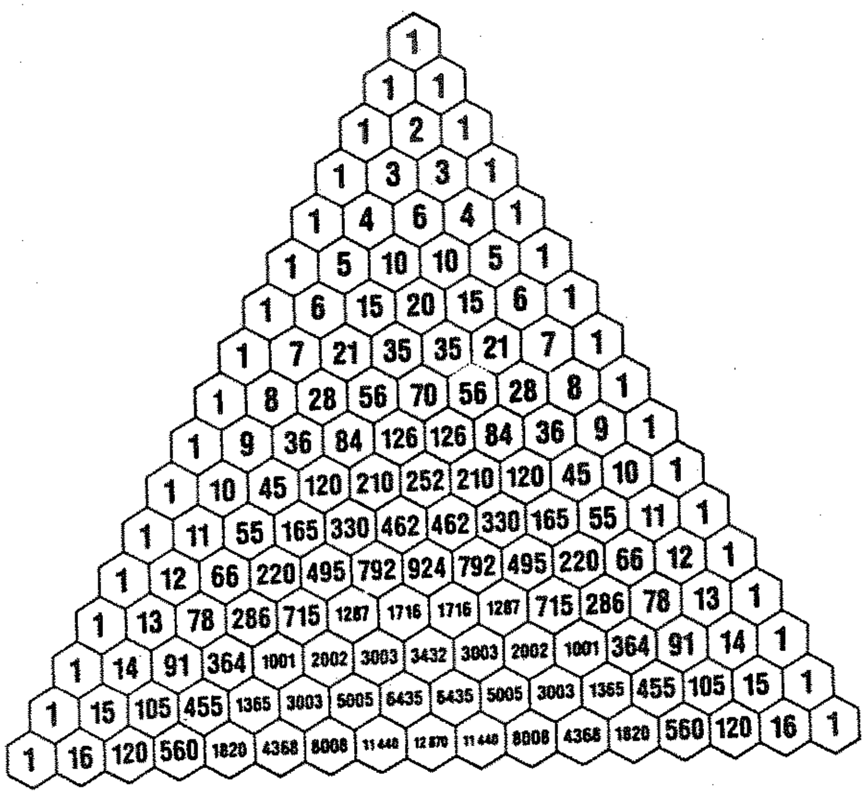
4. Let the area at stage 0 be 1. Find the total shaded areas at stages 1 through 4.

STAGE	0	1	2	3	4	5	...	n
AREA	1							

5. Extend the pattern to predict the total area at stage 5.
   What constant multiplier can be used to go from one stage to the next?

6. Generalize to find the total area at stage n.
   As n becomes large without bound, what happens to the shaded area? 74

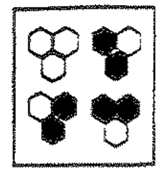
Pick a number between 2 and 9 (inclusive). Shade all multiples of that number.

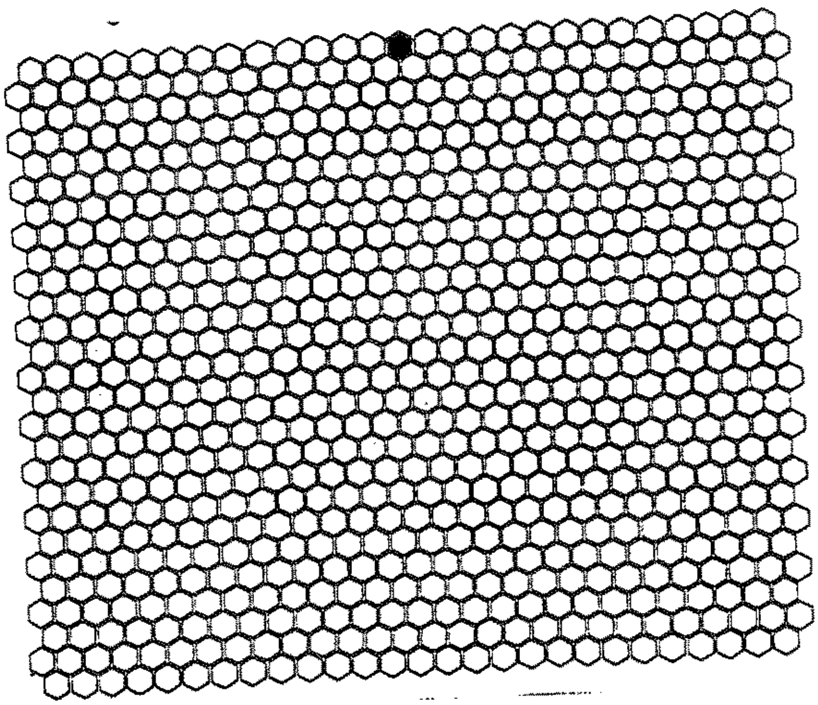


Automata

The rule for coloring the cells

If the two cells directly above are different in color, then shade in the cell so the color is black. If they are the same in color, leave the cell unshaded so the color is white





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Shade only even blocks.

In rows 13, 14, and 15, enter only the letters E for *even* or O for *odd*. Do not compute the numerical values but rather use these relationships:

E + E = E E + O = O O + E = O O + O = E

ow

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 9 36 84 126 126 84 36 9 1

0 1 10 45 120 210 252 210 120 45 10 1

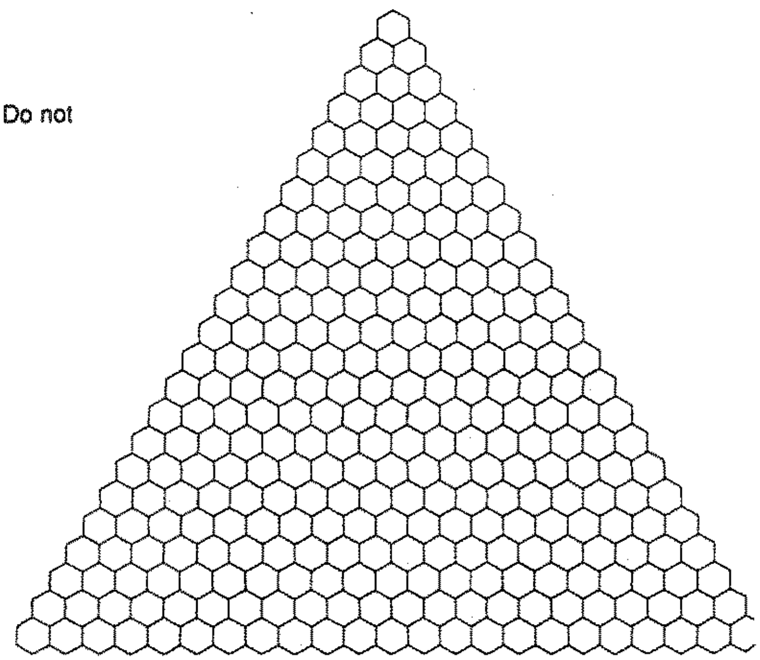
1 ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

2 ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

3 ....

4 ....

5 ....



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Math4020 Geometry Notes
Basic Euclidean Constructions
(Section 14.3 & 14.4)

Compass & Straightedge Properties

(1) For all $r > 0$ and a point C, we can construct a circle of radius r and with center C. (An arc is considered to be a connected portion of a circle.)
(2) Every two points P & Q can be connected using our straightedge to construct the line segment, ray or line between them.
(3) A line *l* can be constructed if and only if we have two points that are on line *l*. The points are located only by the intersection of lines, segments, rays or arcs.

Basic Constructions

(1) Copy a line segment→
- Draw another segment (that looks longer than the one you're copying).
- Open the compass to the length of the original segment (by putting compass point on one endpoint and compass pencil on other endpoint).
- Put the compass point on the line segment you drew (at some point you've designated), draw an arc (with the compass setting from last step) that intersects that line segment. The line segment between those two points is now your copied segment.

(2) Copy an angle (call it angle A)→
- Draw another line segment, label one endpoint as P.
- Draw an arc through angle A (with compass point on vertex A).
- With the same compass setting as last step, put compass point on P and draw an arc (making sure it goes through the line segment you've drawn and call that intersection point R).
- With compass, measure distance from arc intersection points with angle A.
- With that same compass setting, place the compass point at R and draw another arc. Call the point of intersection, with this arc and the last arc around P, point Q.
- Connect (with your straightedge) point Q and point P and you now have your copied angle.

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(3) Bisect (perpendicularly) a line segment (call endpoints A and B)➔

- Place compass at point A and open the compass to a setting that looks bigger than half the distance between A and B (but less than the whole distance). Construct arcs on both sides of line segment.
- With the same compass setting, place the compass at B and make arcs that intersect those arcs you've already drawn. Call those two new points P and Q.
- Connect P and Q with your straightedge. That new line segment is a perpendicular bisector of your original line segment.

(4) Bisect an angle (call it angle B)➔

- Place compass point at vertex B and draw an arc that goes through both legs of the angle. Label those intersection points P and Q.
- Place compass point at P and draw an arc (you can choose your compass setting).
- With the same compass setting as in last step, place your compass point at Q and draw an arc. Label the point of intersection between the two arcs as point R.
- With your straightedge, connect B and R. That line segment bisects the angle.

(5) Construct a line perpendicular to a given line through a specified point **on** the line (call that point P)➔

- Place the compass point at P and draw two arcs that intersect the line on either side of P (each with same compass setting). Label those two points S and R.
- Bisect the straight angle formed by those three points (namely P, S, and R) by above construction process.
- The constructed line segment that bisects the straight angle is perpendicular to the original line and goes through P.

(6) Construct a line perpendicular to a given line through a specified point **not** on the line (call that specified point P)➔

- Place the compass point at P and draw two arcs that intersect line on either side of P (both with same compass setting). Label those two points (on the line) as A and B.
- With the same compass setting as in last step, place compass point at A and draw an arc on the other side of the line from P.
- Still with the same compass setting, place compass point at B and draw another arc on the other side of the line from P so that this arc intersects arc from last step. Label that point Q.
- With your straightedge, draw the line segment that connects the points P and Q. This line segment is perpendicular to the original line and goes through P.

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(7) Construct a line parallel to a given line through a given point (call that point P)→

- Designate some point on the line as Q. With your straightedge, create a line segment connecting P and Q.
- Designate another point on the original line as R. Now use the technique of copying an angle to copy angle PQR to angle having its vertex at P (that will form an alternate interior angle with angle PQR).
- When you've copied that angle, the new line segment created will be parallel to the original line.

More Euclidean Constructions

(8) Construct circumscribed circle of a triangle (call triangle vertices A, B & C)→

- Construct a perpendicular bisector line of line segment AC.
- Construct a perpendicular bisector line of line segment AB.
- Label the intersection of those two created lines as point P.
- Draw the circle with center P (called *circumcenter*) and radius as length from P to A (or B or C). This circle should go through all vertices of original triangle.

(9) Construct inscribed circle of a triangle (call triangle vertices A, B & C)→

- Construct the angle bisector of angle A.
- Construct the angle bisector of angle B.
- Label the intersection of those two created lines (the angle bisector lines) as point P.
- Construct a line through P and perpendicular to line segment AB. Label the intersection point as R.
- Draw the circle with center P (called *incenter*) and radius as length of line segment PR. This circle should just touch all three sides of the original triangle.

(10) Construct an equilateral triangle (call it triangle ABC)→

- Choose points A and B at random and draw the line segment connecting them using your straightedge.
- Place the compass point at A and measure length to B.
- With that compass setting, draw an arc from A.
- With the same compass setting, draw an arc from B.
- Label intersection point, from two arcs, as point C.
- With your straightedge, make line segments connecting A to C and then B to C. You've created an equilateral triangle through points A, B and C.

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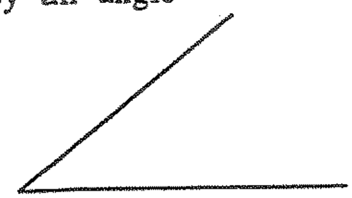
CONSTRUCTIONS WITH A COMPASS AND STRAIGHTEDGE

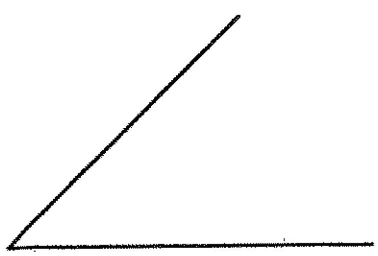
1. Copy a segment

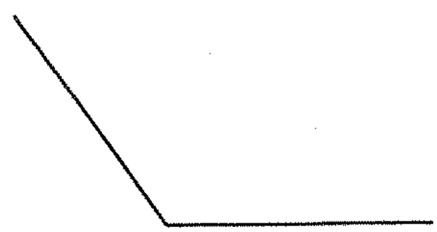
2. Copy an angle



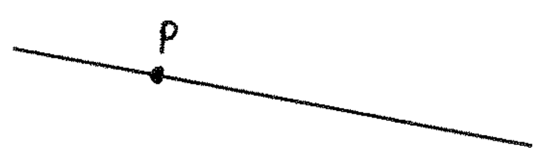
3. Bisect a segment

4. Bisect an angle





5. Draw a perpendicular through a point on a segment



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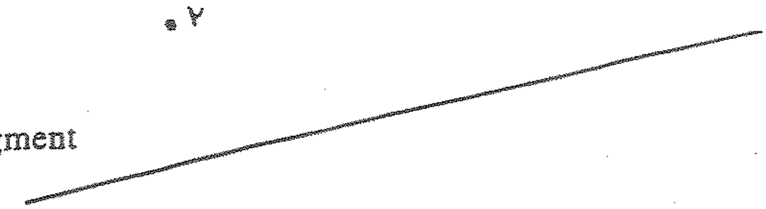
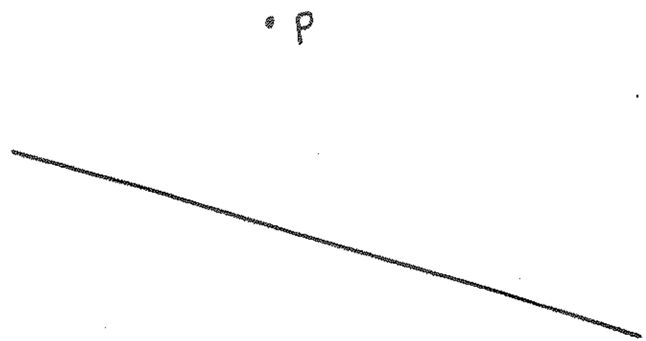
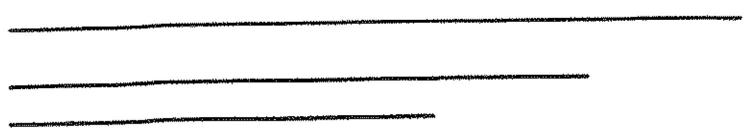
6. Draw a perpendicular through a point not on the segment





7. Draw a line parallel to a given line through a given point.

8. Create a triangle with these segments as sides:



Bisect the largest angle of your triangle.
Bisect the longest side of your triangle.
Draw an altitude to the shortest side.

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Triangles , Math 181 , Informal Geometry

Given a triangle ABC, we can construct four different types of lines with respect to the triangle.

1. The **angle bisector** bisects an angle to form two congruent angles.
2. The **perpendicular bisector**. Given a line segment, the perpendicular bisector is the unique perpendicular line passing through the midpoint of the line segment.
3. The **median** is the line passing through a vertex and the midpoint of the opposite side.
4. The **altitude** is the line passing through a vertex, perpendicular to the opposite side.

Properties of the angle bisector:

- Any point on the angle bisector is equidistant from the sides which form the angle.
- The three angle bisectors in a triangle always intersect in one point, and this intersection point always lies in the interior of the triangle.
- The intersection of the three angle bisectors forms the center of the circle inscribed in the triangle. (The circle which is tangent to all three sides.)

Properties of the perpendicular bisector:

- Any point on the perpendicular bisector of a line segment is equidistant from both endpoints.
- In a triangle the perpendicular bisectors of the three sides always meet in a single point. This point is called the circumcenter.
- If the triangle is acute, the circumcenter lies inside the triangle. If the triangle is obtuse, the circumcenter lies outside the triangle. If the triangle is a right triangle, the circumcenter will coincide with one of the sides.

- The circumcenter is the center of the circumscribed circle. (The circle which passes through all three vertices.)

Properties of the median:

- The medians of a triangle always intersect in one point (the centroid).
- The centroid always lies inside the triangle.
- The centroid divides the median into two segments. The lengths of these two segments always have a constant ratio.

Properties of the altitude:

- The altitudes of a triangle always intersect in one point.
- If the triangle is acute, the intersection point lies inside the triangle. If the triangle is obtuse, the intersection point lies outside the triangle. If the triangle is a right triangle, the intersection point will coincide with the vertex which represents the right angle.

Some other constructions using only compass and straightedge

Determine how you might:

- Construct a square
- Construct a hexagon
- Construct an octagon
- Construct an equilateral triangle
- Construct a 45° angle
- Construct a 30° angle
- Construct a 135° angle
- Construct a segment of length .5 given a unit segment.
- Construct a segment of length $\sqrt{5}$, or $\sqrt{2}$
- Divide a line into three equal segments.
- Divide an angle into three equal angles.

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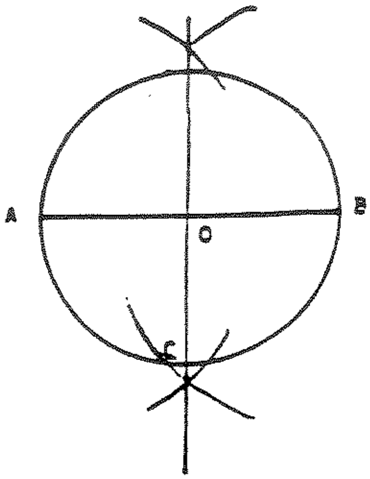
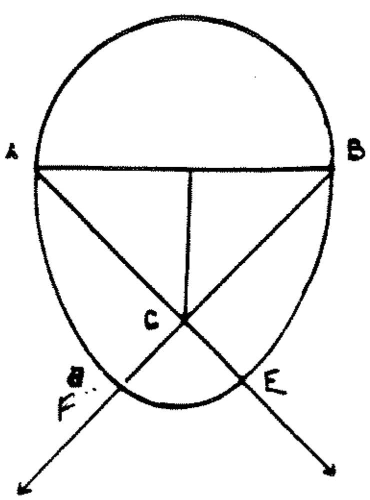
GEOMETRY LAYS AN EGG



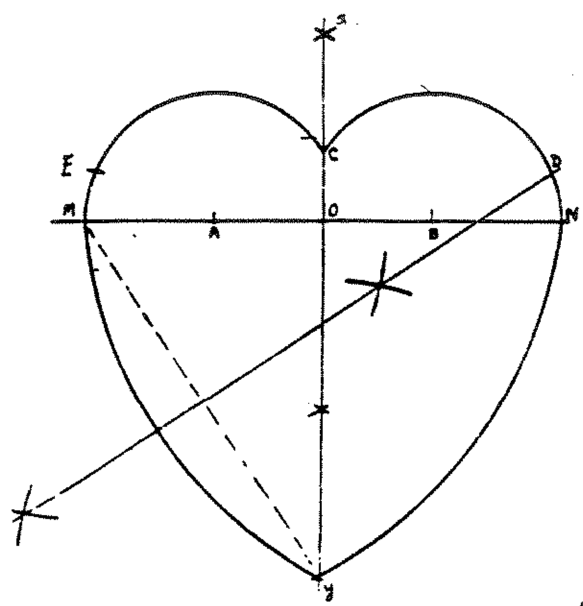
1. Draw segment AB.
2. Construct the perpendicular bisector of $\overline{AB}$.
3. Draw a circle with center O and radius $\overline{OA}$.
4. Establish C with center O at the intersection of the circle O and the perpendicular bisector.

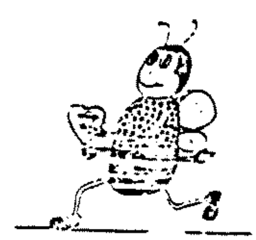


5. Draw rays $\overrightarrow{AC}$ and $\overrightarrow{BC}$.
6. Using A as center and AB as radius, draw arc $\widehat{BE}$.
7. Using B as center and AB as redius, draw arc $\widehat{AF}$.
8. Using C as center and CF as radius, draw arc $\widehat{FE}$.

CONSTRUCTION PROBLEM FOR VALENTINE'S DAY

1. Draw a segment $\overline{AB}$, 8 cm long horizontally in the center of a piece of paper
2. Construct a perpendicular bisector of $\overline{AB}$ intersecting at O. Call it $\overline{XY}$.
3. Extend the measure $\overline{OY}$ to equal 13.3 cm.
4. With A and B as centers and a radius of 5.5 cm, construct arcs of circles intersecting $\overline{XO}$ at C, and $\overline{AB}$ extended at points M and N, respectively.
5. Construct a perpendicular bisector of $\overline{MY}$ and let it intersect Arc $\widehat{CN}$ at D.
6. Using DY as a radius and D as a center construct an arc from M to Y.
7. In like manner construct an arc from N to Y using a point on $\widehat{MC}$ (E) as a center.
8. When the figure is completed, write a mathematical valentine on it





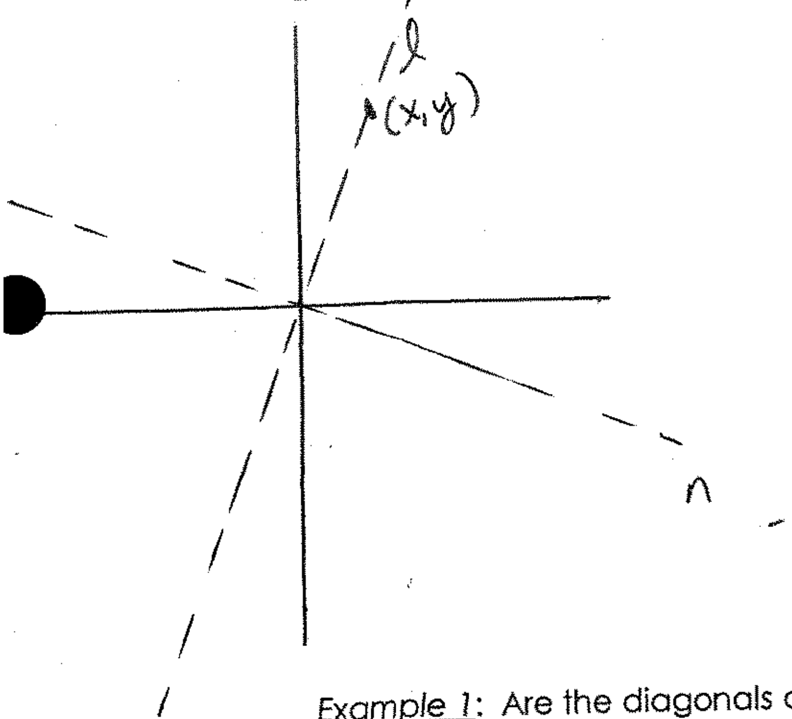
Math4020 Chapter 15 Notes
Algebra & Geometry Connections

Slope

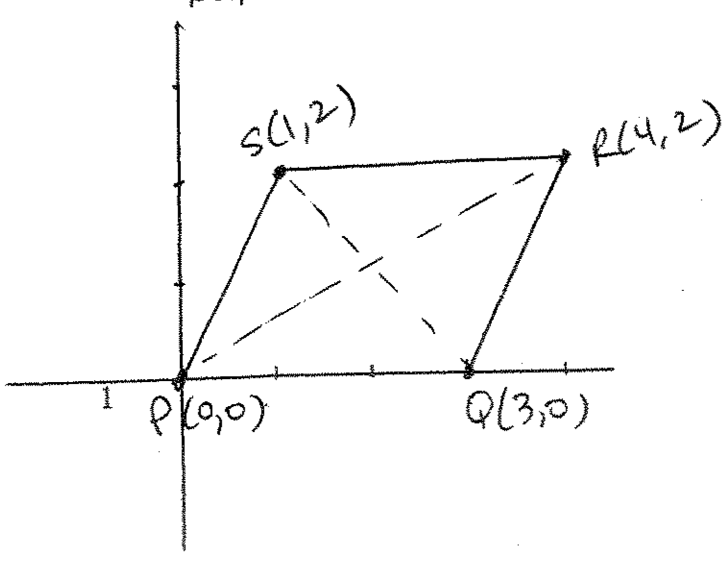
The slope of a line tells how steep it is.

Slope is given by $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\textit{vertical change}}{\textit{horizontal change}}$ for the slope of the line that goes between $P(x_1, y_1)$ and $Q(x_2, y_2)$.

- The slopes of parallel lines are equal (or both undefined).
- The slopes of perpendicular lines are negative reciprocals of one another.



Example 1: Are the diagonals of the following parallelogram perpendicular to one another?



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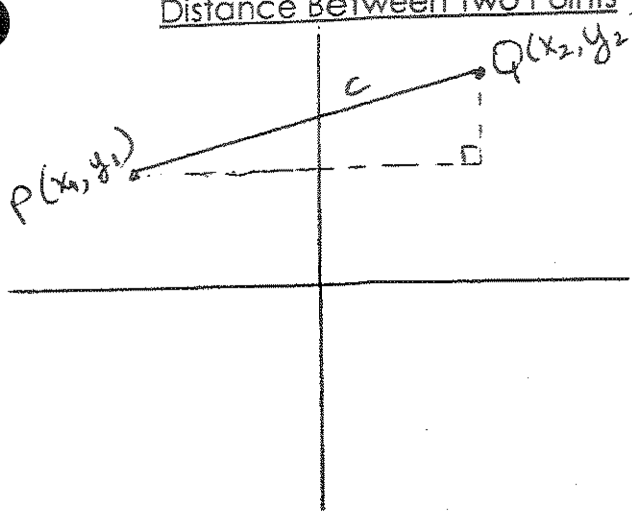
Distance Between Two Points



To find the distance between points P & Q, we can form a right triangle. By the Pythagorean Theorem, we know

$$(x_2-x_1)^2+(y_2-y_1)^2=c^2 \ .$$

=> $$c=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

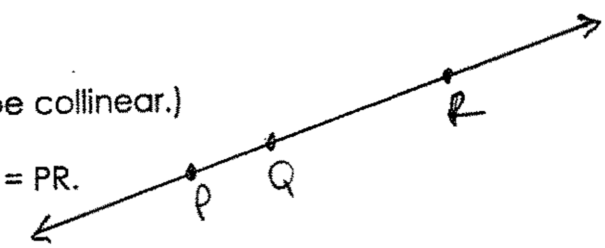
This is the distance formula (just the Pythagorean Theorem in disguise).

Example 2: Find the distance between (2,-3) and (1,5).

Collinearity

(Two points that lie on the same line are said to be collinear.)

If P, Q and R are on the same line, then PQ + QR = PR.



Example 3: Determine if P(-1,4), Q(-2,3) and R(-4,1) are collinear, using two different methods.

(1) Check distance between all the pairs of points.
PQ =

QR =

PR =

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(2) Check to see if the slope of $\overline{PQ}$ equals the slope of $\overline{QR}$.

Midpoint

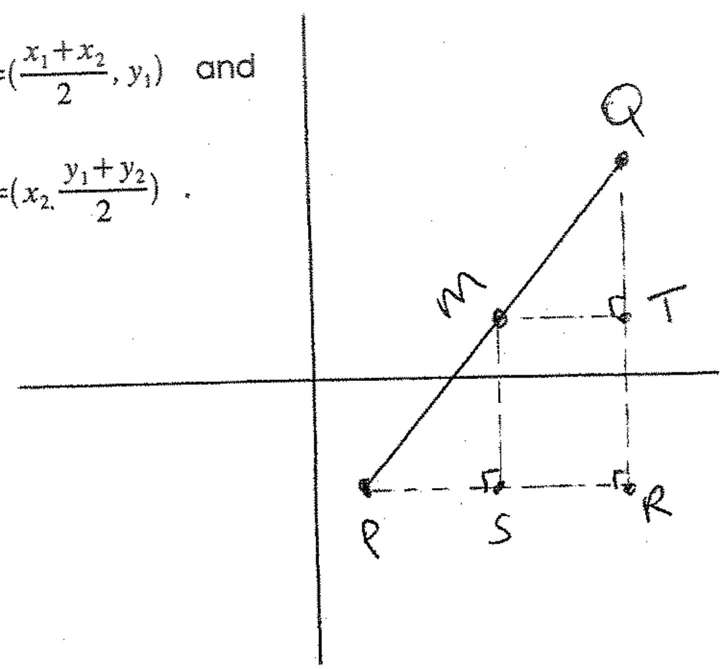
For the drawing below, let

S = midpoint of $\overline{PR} = (\frac{x_1 + x_2}{2}, y_1)$ and

T = midpoint of $\overline{QR} = (x_2, \frac{y_1 + y_2}{2})$.

Then, we claim that the midpoint, M, is

$$M = (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}).$$

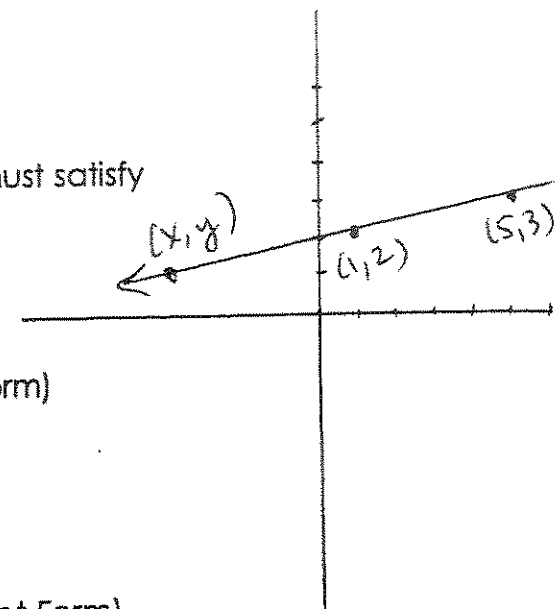


Proof:

Equation of a Line

Suppose we're given two points (1, 2) and (5, 3). The slope of the line between them is

m =



Let (x, y) be an arbitrary point on this line. Then, it must satisfy

$$\frac{y-3}{x-5}=\frac{1}{4}$$

So, if we rearrange things, we get

$$y-3=\frac{1}{4}(x-5) \qquad \text{(Point-Slope Form)}$$

$$y-3=\frac{1}{4}x-\frac{5}{4}$$

$$y=\frac{1}{4}x-\frac{5}{4}+\frac{12}{4}$$

$$y=\frac{1}{4}x+\frac{7}{4} \qquad \text{(Slope-Intercept Form)}$$

Every point on the line satisfies this equation, so it's the equation describing exactly the line we started with.

y-intercept-->The point on the line that lies on the y-axis, so x = 0 for this point.

x-intercept-->The point on the line that lies on the x-axis, so y = 0 for this point.

Point-Slope Form of a Line $y-y_1=m(x-x_1)$

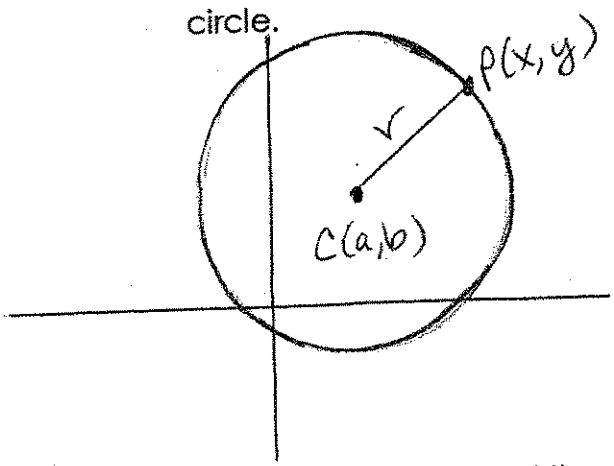
Slope-Intercept Form of a Line $y=mx+b$ where (0,b) is the y-intercept.

Example 4: Find the equation of the line perpendicular to 2x+4y=6 and goes through the point (-2,5).

Example 5: Refer to Example 3, and check for collinearity by using the line equations.

Circles

Let the point C(a,b) be the center of a circle with radius r, as drawn on a Cartesian coordinate system. Let P(x,y) be an arbitrary point lying on the circle.



Then, PC = r ==> $PC^2=r^2$
By the distance formula, we know

$r=\sqrt{(x-a)^2+(y-b)^2}$. If we squrare both sides, we get

$r^2=(x-a)^2+(y-b)^2$ which is the equation of a circle.

Example 6: Find the equation of the circle with radius r = 5 and center at (-1,3).

Example 7: P(5,4) and Q(1,2) are endpoints of a diameter of a circle. Find the equation of this circle.

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