5.1 Arithmetic & Geometric Sequences

<u>Vocab</u> <u>sequence</u>: $\{a_n\}$ an ordered list of numbers that ~~form~~ a pattern; it's also a function w/ domain $= \mathbb{N}$

Arithmetic sequence	Geometric Sequence
$n = 2, 3, \ldots$ $d \neq 0$ $a_n = a_{n-1} + d$, given a_1 (we add same # over & over to get next terms)	$d \neq 0$ $a_n = d a_{n-1}$ $n = 2, 3, \ldots$ given a_1 (we multiply same # over & over to get to next terms in the sequence

★ These formulas are <u>recursive</u> (they depend on previous terms)

a_1 given $a_2 = a_1 + d$ $a_3 = a_2 + d = a_1 + d + d = a_1 + 2d$ $a_4 = a_3 + d = a_1 + 2d + d = a_1 + 3d$ $a_5 = a_4 + d = a_1 + 3d + d = a_1 + 4d$ $\vdots$ $a_n = a_{n-1} + d = a_1 + (n-1)d$	a_1 given $a_2 = da_1$ $a_3 = da_2 = d(da_1) = d^2 a_1$ $a_4 = da_3 = d(d^2 a_1) = d^3 a_1$ $a_5 = da_4 = d(d^3 a_1) = d^4 a_1$ $\vdots$ $a_n = d^{n-1} a_1$

★ These formulas are <u>iterative</u> (or direct or explicit) (they <u>don't</u> depend on previous terms)

5.1 (cont)

Ex 1 Classify as arithmetic or geometric and give next 3 terms of sequence.

(a) 10, 7, 4, 1, ...

(b) 2, -6, 18, -54, ...

Ex 2 Find formula for n^{th} term of (a) arithmetic and (b) geometric sequence

(a) $a_1 = 2$, $d = -3$

(b) $a_1 = -10$, $d = 2$

5.1 (cont)

Ex 3 Given $a_1 = 2$ and $a_8 = 23$, find the 50^{th} term of this arithmetic sequence.

Ex 4 Given $a_1 = \frac{3}{2}$ and $a_6 = \frac{3}{64}$, find the 20^{th} term of the geometric sequence.

5.1 (cont)

Arithmetic Sequence Sum	Geometric Sequence Sum
$S_n = a_1 + a_2 + \cdots + a_n = ?$	$S_n = a_1 + a_2 + \cdots + a_n = ?$

Arithmetic Sequence Sum

$S_n = a_1 + a_2 + \cdots + a_n \quad = ?$

$a_1 + a_n = a_1 + (a_1 + (n-1)d)$

$\qquad = 2a_1 + (n-1)d$

$a_2 + a_{n-1} = 2a_1 + (n-1)d = a_1 + a_n$

$a_3 + a_{n-2} = 2a_1 + (n-1)d = a_1 + a_n$

$\vdots$

have $\frac{n}{2}$ groups of this

$\Rightarrow \boxed{S_n = \frac{n}{2}(a_1 + a_n)}$

Geometric Sequence Sum

$S_n = a_1 + a_2 + \cdots + a_n = ?$

$S_n = a_1 + da_1 + d^2a_1 + \cdots + d^{n-1}a_1$

$-\; dS_n = da_1 + d^2a_1 + d^3a_1 + \cdots + d^{n-1}a_1 + d^n a_1$

$S_n - dS_n = a_1 - d^n a_1$

$S_n(1-d) = a_1(1-d^n)$

$\boxed{S_n = \dfrac{a_1(1-d^n)}{1-d}}$

Ex5 (a) Find sum of first 100 terms of sequence

$1, 10, 19, 28, \ldots$

(b) Find sum of first ten terms of sequence

$3, 6, 12, 24, \ldots$

5.2 Simple & Compound Interest

Simple Interest

- add same interest to acct every period
- it's an arithmetic sequence and acct balance is the sum

P = principal = start value

Pr = principal times interest rate = interest that gets added every period. (year)

$\Rightarrow$ $S = P + Pr(t)$

$$\boxed{S = P(1+rt)}$$

P = principal, r = int. rate, t = # yrs, S = future acct value

Compound Interest

- multiply by same rate every period
- it's a geometric sequence and acct balance is the sum

P = principal = start value

$(1+r)$ = factor that's multiplied by principal every year

$\Rightarrow$ $S = P(1+r)^t$
(assuming we compound once per year)

if we compound n times per year,

$$\boxed{S = P\left(1+\frac{r}{n}\right)^{nt}}$$

n = # compoundings per year

☆ see table pgs 271-272

$\Rightarrow$ Continuous Compounding

$$\boxed{S = Pe^{rt}}$$

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5.2 (cont)

Ex 1 If \$10,000 is invested for 4 years at an annual rate of 8% (simple), how much will the acct be worth at the end of 4 years? Re-compute for compounded interest (compounded once per year).

Ex 2 You borrow \$2000 at an interest rate of 20%. How much interest is due in 39 weeks?

5.2 (cont)

Ex 3 If \$1000 is invested at 5%

(a) simple interest

(b) compounded interest w/ $n=1$

(c) " " w/ $n=12$,

how long does it take to double?

Ex 4 What is an account worth in 8 years, if we started with \$3000 + we got continuous compounding at a rate of 6%?

5.2 (cont)

Ex 5 What amt must be invested now in order to have \$1,000,000 for retirement in 45 years if money is compounded quarterly at 9%?

APY (Annual Percentage Yield)

Let $P = \$100$ be invested at 8% interest compounded (a) quarterly or (b) monthly.

(a) $S = 100\left(1 + \frac{0.08}{4}\right)^{4(1)} = 100(1.02)^4 = \$108.24$ after 1 yr

(b) $S = 100\left(1 + \frac{0.08}{12}\right)^{12(1)} = \$108.30$ after 1 yr

$\Rightarrow$ So investing at 8% compounded quarterly is basically equivalent to investing the same money at 8.24% simple interest.

That's the APY

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5.2 (cont)

$$APY = \left(1+\frac{r}{n}\right)^n - 1 \quad \text{(periodic compounding)}$$

$$APY = e^r - 1 \quad \text{(continuous compounding)}$$

Ex 6 Which is a better investment deal?

(a) 10% compounded annually

(b) 9.8% " quarterly

(c) 9.65% " continuously