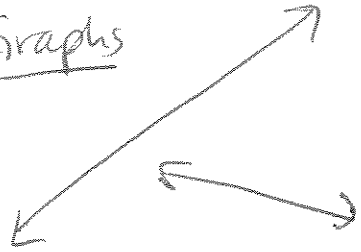
3.5 Polynomial Functions

Polynomial Fn (Defn)

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

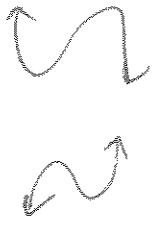
degree $= n$ $\quad (a_n \neq 0)$

Graphs



n=1

n=2



n=3

n=4

Ex 1 For these polynomials, write in standard form. What is its degree and leading coefficient? General graph shape?

(a) $4x - 12 - 2x^3 - x^2$

(b) $3x^7 - 14x + 3x^2 - 4x^4 - 5$

3.5 (cont)

Ex 2 For these polynomials, answer the following.

(a) degree
(b) zeros
(c) y-intercept
(d) x-intercept
(e) sketch graph

① $f(x) = x^4 - 8x^2 + 16$

② $g(x) = 2x^3 - 2x^2 - 4x$

3.5 (cont)

Ex 3 For these piecewise functions, fill in the points & sketch the graph.

(a) $f(x) = \begin{cases} 4 & x \geq 3 \\ |x| & -3 \leq x < 3 \\ -1 & x < -3 \end{cases}$

x	-4	-3	0	1	3	4
y						

(b) $g(x) = \begin{cases} x+5 & x \geq 1 \\ -2x+8 & x < 1 \end{cases}$

x	y
1	
0	
-1	
2	
3	

3.6 Rational Functions

Defn Rational Fn $f(x) = \frac{n(x)}{d(x)}$

where $n(x)$ and $d(x)$ are polynomials

Asymptotes

1) VA (vertical asymptote)

- lines $x=a$ where $d(a)=0$
- find VA by seeing restricted values in domain
- cannot touch or cross VA

2) HA (horizontal asymptote)

- line $y=b$ such that $b = \lim_{x \to \pm\infty} f(x)$
- graph approaches HA as x gets huge (in either direction)
- graph can touch or cross HA when x is "small"

To graph rational fn:

① find domain

(a) find VA

(b) find HA

② Find y-intercept and x-intercept(s)

③ Plot intercept pts and at least one pt on all sides of VAs.

④ Fill in graph w/ smooth curves that approach asymptotes.

Ex 1 Analyze and graph.

(a) $f(x) = \frac{2+x}{1-x}$

(b) $f(x) = \frac{10}{x^2+2}$

3.6 (cont)

(c) $g(x) = \dfrac{2x+1}{2x^2-5x-3}$