

Discussion of Zero

Zero stands with a very unique concept behind it, one that is difficult for most students to grasp. Zero is the symbol to mean nothing, or nothing resides in the place. The concept evolved from the concept of a placeholder, which first appeared in the Babylonian number system. Although one of the main functions of zero is a placeholder, it can also mean nothing is there. In an older number system, no symbols would be used to represent the concept of zero, which in our number system, has a symbol.

For each operation involving whole numbers, zero behaves differently. In addition, zero is the additive identity and holds high significance. The additive identity states $n + 0 = 0 + n = n$ (for this and all following examples, n represents any whole number). Basically, the additive identity allows any number added to zero to stay as that same whole number. However, subtraction is much trickier. Zero is not the identity for subtraction. Subtraction holds no identify because it is not commutative. Therefore $n - 0 \neq 0 - n$ because the first part equals n and the second part equals $-n$. The importance behind subtraction with zero is the ability to flip the positive or negative qualities of that number. Multiplication by zero is very simplistic; it always is equal to zero. There are no exceptions to this rule. $n \times 0 = 0$.

Division with zero holds many complications because there are multiple cases with different solutions for each case. Division with zero is best explained when the division problem is looked at like a fraction. It can then be broken up into three categories, when the numerator equals zero (and the denominator does not), when the denominator equals zero (and the numerator does not) or when both the

numerator and the denominator equal zero. When the numerator equals zero and the denominator does not, for instance with a problem $0 \div 6$ or $0/6$, the answer is always zero. The easiest way to prove this is to solve the division problem using the missing factor approach, what number times six equals zero. Since it has already been explained that anything multiplied by zero equals zero, then our missing factor must be zero. The answer for problems for this case is always zero. When the denominator equals zero and the numerator does not, things get more interesting. If we take the problem $6 \div 0$ or $6/0$ and again use the missing factor approach, we get the question, what number multiplied by zero gives me six, and the answer is nothing. It is already known that any number multiplied by zero gives you zero so therefore we get no answer and mark this case as undefined. The final case for division is where the problem $0 \div 0$ or $0/0$ arises. Again, we use the missing factor approach to talk about what number multiplied by zero will give us zero, and the answer is anything because any number multiplied by zero gives you zero. Since there are an infinite number of answers, we mark this case as undefined also.

Zero as an exponent (n^0) is similarly difficult, also having multiple cases. However, there are only two cases, when $n=0$ and $n \neq 0$. When $n \neq 0$, the answer is always 1. The main reasons behind this are because one is the multiplicative identity and one fits the pattern of exponents. When $n=0$, the scenario becomes more complex. If the patterns of $0^3, 0^2, 0^1, 0^0$ and $3^0, 2^0, 1^0, 0^0$ are compared, we would think of two different solutions. In the first pattern, all of the answers, besides 0^0 , are zero which leads us to the conclusion that 0^0 should also equal zero. In the second pattern, all the answers, besides 0^0 , are one which leads

us to the conclusion that 0^0 equals one. Because of our conflicting answers, we label this case as undefined.

Zero is a very complex concept. The whole concept of having nothing is extremely difficult to grasp. When adding the operations into the mix, things become much more complicated to explain. This is why as teachers, we need the most complete understanding of zero, so we can explain it to and correct our students who will make mistakes when first working with zero.