2.1 Basic Operations w/ Matrices

Defns

$A = B$

A^T

0 matrix

Square matrix

Column or row vector

Vocab

matrix ⇒

entry ⇒

scalar ⇒

order (size) ⇒

Ex 1 For $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 0 & -2 \\ 6 & 1 & 5 \end{bmatrix}$

(a) size =

(b) $a_{13} =$

(c) $A^T =$

(d) first column vector

=

2.1 (cont)

Ex 2 Given $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ -5 & 1 & 0 & 1 \\ 3 & -2 & 7 & 0 \end{bmatrix}$

(a) what size (order) is A?

(b) what is a_{24}? a_{31}?

(c) write zero matrix same size as A.

(d) Find A^T.

(e) write $-A$.

2.1 (cont)

Ex 3 Given $A = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 4 & 2 & 1 & 5 \\ -1 & 0 & -2 & 0 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 2 & 5 & 1 \\ 0 & 0 & -4 & -3 \\ 1 & 4 & -1 & 2 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -1 & 0 & 3 \\ 4 & 5 & 0 \end{bmatrix}$

(a) Find $2A + B$

(b) $A - 3C^T$

matrix Addition

$A + B$

Scalar Multiplication

cA

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2.1 (cont)

Ex 4 Given $A = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 9 & 1 \end{bmatrix}$

$C = \begin{bmatrix} -3 & 1 & 5 \end{bmatrix}$

$D = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

Find

(a) $B^T + D$

(b) $B - (A - D)^T$

(c) $(2C + A^T)^T$

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2.2 Matrix Multiplication

Defn ① matrix multiplication AB

Given A (size $m \times n$) and B (size $n \times p$), AB is an $m \times p$ matrix w/ ij entry given by

$$a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj}$$

(i.e. the product/sum of the i^{th} row of A w/ the j^{th} column of B)

② identity matrix I ⇒ always a square matrix; has 1 in each diagonal entry and zeros everywhere else

☆ closest we have to mult. identity for matrices

$$IA = AI = A \qquad I = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

Properties of Matrix Multiplication

① $(AB)C = A(BC)$ Associativity

② $A(B+C) = AB + AC$

③ $(B+C)A = BA + CA$

> Right & left Distributivity

④ $(AB)^T = B^T A^T$

WARNING $AB \neq BA$

2.2 (cont)

Ex 1 Given $A = \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \\ -1 & 0 \end{bmatrix}$

Find AB and BA, if possible.

Ex 2 Is $(AA^T)^T = A^T A$?

2.2 (cont)

Ex 3 Given $A = \begin{bmatrix} 0 & 1 & 3 \\ 2 & 0 & 1 \\ 0 & 0 & -4 \end{bmatrix}$, find A^2.

Ex 4 (#32) Solve for X.

$$\begin{bmatrix} -8 & -2 \\ -6 & 2 \end{bmatrix} X = \begin{bmatrix} -50 & 2 & -8 \\ -6 & -30 & 22 \end{bmatrix}$$

2.2 (cont)

Ex 5 Solve for x.

$$\left(\begin{bmatrix} -5 \\ 10 \\ -19 \end{bmatrix} + \begin{bmatrix} -2x \\ 13 \\ -8 \end{bmatrix} \right) + \begin{bmatrix} -8 \\ -4 \\ -7 \end{bmatrix} = \begin{bmatrix} -5 \\ 19 \\ -34 \end{bmatrix}$$

2.3 Gauss-Jordan Elimination

Vocab

Augmented matrix: A matrix that represents a system of linear eqns.

elementary row operations:

① switch two rows

② multiply any row by a nonzero constant

③ replace one row w/ the result of adding it w/ a nonzero multiple of another row.

Gauss-Jordan elimination: process for solving a system of linear eqns, using elementary row ops, until we have triangular matrix.

like this

$$\left[\begin{array}{ccc:c} 1 & 3 & 4 & 5 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & -4 \end{array}\right]$$

this is augmented matrix for

$$\begin{aligned} x + 3y + 4z &= 5 \\ y + 2z &= 7 \\ z &= -4 \end{aligned}$$

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2.3 (cont)

Ex 1 Solve.

$$\begin{aligned} 10x+y &= 6 \\ 3x+y+2z &= 3 \\ 2x-y-2z &= 2 \end{aligned}$$

Ex 2

$$\begin{aligned} -2x+y &= 1 \\ 2x-y &= 7 \end{aligned}$$

2.3 (cont)

Ex 3 Solve.

$$\begin{aligned} 3x-2y-7z&=0 \\ x-y-z&=1 \\ -x+2y-3z&=-4 \end{aligned}$$

Ex 4

$$\begin{aligned} 3x-y&=3 \\ x+z&=3 \\ 2x-y-z&=3 \end{aligned}$$

2.3 (cont)

Ex 5 Solve

$$\begin{aligned} x+y+z&=1\\ x-y-z&=1\\ -x+y-z&=1 \end{aligned}$$

2.4 Inverse Matrices

Defn A^{-1} read "A inverse" (it's not an exponent)

$$A^{-1} \cdot A = I = A \cdot A^{-1}$$

- A^{-1} can only exist for square matrix A (nxn) and is also square nxn.

Ex 1 Find A^{-1} for

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Method to find A^{-1} for A

① If A is not square, A^{-1} DNE.

② If A is square,

(a) augment A with identity matrix

(b) Perform elementary row ops on augmented matrix until the left side is I

(c) what's on the right side is A^{-1}.

2.4 (cont)

Ex 2 Find A^{-1}.

(a) $A = \begin{bmatrix} 7 & 6 \\ 2/3 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 6 & 0 & 5 \\ -3 & 5 & -3 \\ 7 & 3 & 6 \end{bmatrix}$

2.4 (cont)

Ex 3 Use A^{-1} from Ex2(b) to solve

$$\begin{aligned} 6x \quad\;\; + 5z &= 1 \\ -3x + 5y - 3z &= 0 \\ 7x + 3y + 6z &= 7 \end{aligned}$$

To solve

$$Ax = b$$

(where A is $n \times n$ matrix, x is $n \times 1$ column vector of variables and b is $n \times 1$ column vector of constants), we can left-multiply both sides by A^{-1}.

$$A^{-1}Ax = A^{-1}b$$

$$Ix = A^{-1}b$$

$$x = A^{-1}b$$

2.5 Application Problems with Matrices

Ex 1 (Encryption)

Use $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 5 & -6 \\ 3 & -2 & 2 \end{bmatrix}$ to encrypt "JOYFUL" where $A=1$, $B=2$ etc.

so $J=10$, $O=15$, $Y=25$, $F=6$, $U=21$, $L=12$

$$JOY \rightarrow \begin{bmatrix} 10 \\ 15 \\ 25 \end{bmatrix} \qquad FUL \rightarrow \begin{bmatrix} 6 \\ 21 \\ 12 \end{bmatrix}$$

2.5(cont)

Ex 2 (#14) A grocer is going to mix three kinds of nuts to make 40 pounds of a mixture that will be priced at \$5.95 /pd. The three kinds of nuts are peanuts priced at \$4.00/pd, cashews at \$6.60/pd and pistachios at \$8.20/pd. The mixture will contain twice as much in peanuts as cashews by weight. How many pounds of each nut are in the mix?

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2.5 (cont)

Ex 3 (# 22) A company needs to borrow \$150,000. For tax & related reasons, the company wants to pay 7.3% interest on this loan. There are three lenders for this money. The first charges 6%, the second charges 7% and the third charges 10%. The company is going to borrow twice as much from the first lender as from the third. How much should the company borrow from each lender?