6.6 #17

tank initially has 120 gallons of pure water. Brine w/ 1 pd salt/gallon flows into tank at 4 gallons/min. Well-stirred soln runs out at 6 gal/min. How much salt is in tank after t minutes, $0 \le t \le 60$?

Let y = amt of salt in tank at time t (pds)

$\frac{dy}{dt}$ = rate in − rate out

(but this should be in pds/min NOT gal/min)

rate in: $1 \frac{\text{pd}}{\text{gal}} \cdot 4 \frac{\text{gal}}{\text{min}} = 4 \text{ pd/min}$

rate out: $6 \frac{\text{gal}}{\text{min}} \cdot \frac{y \text{ pd}}{(120-2t)\text{gal}}$ → we're losing a total of 2 gallons per min & we started w/ exactly 120 gallons

$= \frac{6y}{120-2t}$ pd/min

$= \frac{3y}{60-t}$

$\Rightarrow \frac{dy}{dt} = 4 - \frac{3y}{60-t} \iff \frac{dy}{dt} + \left(\frac{3}{60-t}\right) y = 4$

integrating factor: $\int \frac{3}{60-t}\,dt = -3\ln|60-t| = -3\ln(60-t)$ since $t \in [0, 60]$

$\Rightarrow e^{-3\ln(60-t)} = e^{\ln(60-t)^{-3}} = \frac{1}{(60-t)^3}$

$\Rightarrow \frac{1}{(60-t)^3}\frac{dy}{dt} + \frac{3}{(60-t)^4} y = \frac{4}{(60-t)^3}$

$$\frac{d}{dt}\left(\frac{1}{(60-t)^3}y\right) = \frac{4}{(60-t)^3}$$

$$\int d\left(\frac{1}{(60-t)^3}y\right) = 4\int \frac{1}{(60-t)^3}\,dt$$

$$\frac{1}{(60-t)^3}y = 4\int (60-t)^{-3}\,dt$$

$$\frac{1}{(60-t)^3}y = \frac{-4}{-2}(60-t)^{-2} + C$$

$$\frac{1}{(60-t)^3}y = \frac{2}{(60-t)^2} + C$$

$$y = 2(60-t) + C(60-t)^3$$

we know at $t=0$, $y=0$

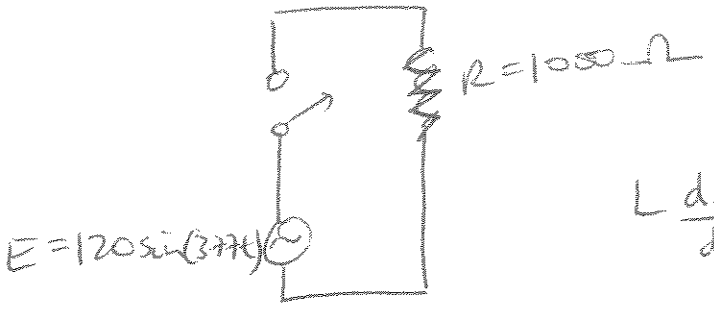
$$\Rightarrow \quad 0 = 2(60) + C(60)^3$$

$$\frac{-2(60)}{60^3} = \frac{60^3 C}{60^3}$$

$$\frac{-2}{60^2} = C \quad \Rightarrow \quad \frac{-1}{1800} = C$$

$$\Rightarrow \boxed{y = 2(60-t) - \frac{1}{1800}(60-t)^3}$$

6.6 #21



$L = 0$

$$L\frac{dI}{dt} + RI = E$$

$$\Leftrightarrow \quad 1000I = 120\sin(377t)$$

$$I = \frac{12}{100}\sin(377t)$$

$$\boxed{I = \frac{3}{25}\sin(377t)}$$