7.2 Ex 3   (from notes)

$$\int \frac{\ln x}{\sqrt{x}}\, dx \qquad \text{(do Integration by Parts)}$$

$$u = \ln x \qquad\qquad v = 2\sqrt{x}$$

$$du = \frac{1}{x}\, dx \qquad\qquad dv = \frac{1}{\sqrt{x}}\, dx = x^{-1/2}\, dx$$

$$= 2\sqrt{x}\,\ln x - 2\int \frac{\sqrt{x}}{x}\, dx$$

$$= 2\sqrt{x}\,\ln x - 2\int x^{-1/2}\, dx$$

$$= 2\sqrt{x}\,\ln x - 2\left(2\sqrt{x}\right) + C$$

$$= 2\sqrt{x}\,\ln x - 4\sqrt{x} + C$$

Ex 6
$\boxed{n \neq 0,1}$

$$\int \cos^n x\, dx = \int \cos x\,\cos^{n-1} x\, dx$$

$$u = \cos^{n-1} x \qquad\qquad v = \sin x$$

$$du = (n-1)\cos^{n-2} x\,(-\sin x)\, dx \qquad dv = \cos x\, dx$$

$$= \sin x\,\cos^{n-1} x + (n-1)\int \sin^2 x\,\cos^{n-2} x\, dx$$

Remember $\qquad \sin^2 x + \cos^2 x = 1 \iff \sin^2 x = 1 - \cos^2 x$

$$\int \cos^n x\, dx = \sin x\,\cos^{n-1} x + (n-1)\left[\int \cos^{n-2} x\, dx - \int \cos^n x\, dx\right]$$

$$\iff \int \cos^n x\, dx = \sin x\,\cos^{n-1} x + (n-1)\int \cos^{n-2} x\, dx - (n-1)\int \cos^n x\, dx$$

$$\left(1 + (n-1)\right) \int \cos^n x \, dx = \sin x \, \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx$$

$$n \int \cos^n x \, dx = \sin x \, \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx$$

$$\boxed{\int \cos^n x \, dx = \frac{1}{n}\left[\sin x \, \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx \right]}$$

Notice that we don't get a final answer for our integral. However, we have at least reduced the integral to one of lower power of $\cos x$. Thus, if $n = 4$, we'd need this technique twice to be finished.

Likewise,

$$\boxed{\int \sin^n x \, dx = \frac{1}{n}\left[-\cos x \, \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x \, dx \right]}$$