$\boxed{8.4 \ \#31}$

$$\int_{2c}^{4c} \frac{dx}{\sqrt{x^2-4c^2}} = \lim_{p\to 2c^+} \int_p^{4c} \frac{1}{\sqrt{x^2-4c^2}}\,dx$$

$$\sqrt{x^2-4c^2} = \sqrt{4c^2\sec^2\theta - 4c^2} = \sqrt{4c^2\tan^2\theta} = 2c\tan\theta$$

$x = 2c\sec\theta$

$dx = 2c\sec\theta\tan\theta\,d\theta$

$x=2c,\ \theta = 0$

$x = 4c,\ \theta = \arccos(\tfrac{1}{2}) = \pi/3$

$$= \lim_{p\to 0^+} \int_p^{\pi/3} \frac{1}{2\cancel{c}\cancel{\tan\theta}} (2\cancel{c}\sec\theta\cancel{\tan\theta})\,d\theta$$

$$= \lim_{p\to 0^+} \int_p^{\pi/3} \sec\theta\,d\theta = \lim_{p\to 0^+} \ln|\sec\theta + \tan\theta| \Big]_p^{\pi/3}$$

$$= \ln\left|\sec\tfrac{\pi}{3} + \tan\tfrac{\pi}{3}\right| - \ln 1$$

$$= \ln|2+\sqrt{3}| = \ln(2+\sqrt{3})$$

#33 $\displaystyle \lim_{c\to 0^+} \int_c^1 \frac{dx}{\sqrt{x}(1+x)} = ?$

consider $c \in (0,1)$ $\displaystyle \int_c^1 \frac{dx}{\sqrt{x}(1+x)}$

$u = \frac{1}{1+x}$ $\quad v = 2\sqrt{x}$

$du = \frac{-1}{(1+x)^2}\,dx$ $\quad dv = \frac{1}{\sqrt{x}}\,dx$

$$= \frac{2\sqrt{x}}{1+x}\Big|_c^1 + 2\int_c^1 \frac{\sqrt{x}}{(1+x)^2}\,dx = 1 - \frac{2\sqrt{c}}{1+c} + 2\int_c^1 \frac{\sqrt{x}}{(1+x)^2}\,dx$$

$$\Rightarrow \lim_{c\to 0^+} \int_c^1 \frac{dx}{\sqrt{x}(1+x)} = \lim_{c\to 0^+}\left(1 - \frac{2\sqrt{c}}{1+c}\right) + 2\lim_{c\to 0^+} \int_c^1 \frac{\sqrt{x}}{(1+x)^2}\,dx$$

$$= \boxed{1 + 2\int_0^1 \frac{\sqrt{x}}{(1+x)^2}\,dx}$$

(a proper integral)