HYPERBOLIC FORMS

78 $$\int \sinh u\, du = \cosh u + C$$

79 $$\int \cosh u\, du = \sinh u + C$$

80 $$\int \tanh u\, du = \ln(\cosh u) + C$$

81 $$\int \coth u\, du = \ln|\sinh u| + C$$

82 $$\int \operatorname{sech} u\, du = \tan^{-1}|\sinh u| + C$$

83 $$\int \operatorname{csch} u\, du = \ln\left|\tanh \frac{u}{2}\right| + C$$

84 $$\int \sinh^2 u\, du = \frac{1}{4}\sinh 2u - \frac{u}{2} + C$$

85 $$\int \cosh^2 u\, du = \frac{1}{4}\sinh 2u + \frac{u}{2} + C$$

86 $$\int \tanh^2 u\, du = u - \tanh u + C$$

87 $$\int \coth^2 u\, du = u - \coth u + C$$

88 $$\int \operatorname{sech}^2 u\, du = \tanh u + C$$

89 $$\int \operatorname{csch}^2 u\, du = -\coth u + C$$

90 $$\int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u + C$$

91 $$\int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u + C$$

MISCELLANEOUS ALGEBRAIC FORMS

92 $$\int u(au+b)^{-1}\, du = \frac{u}{a} - \frac{b}{a^2}\ln|au+b| + C$$

93 $$\int u(au+b)^{-2}\, du = \frac{1}{a^2}\left[\ln|au+b| + \frac{b}{au+b}\right] + C$$

94 $$\int u(au+b)^n\, du = \frac{u(au+b)^{n+1}}{a(n+1)} - \frac{(au+b)^{n+2}}{a^2(n+1)(n+2)} + C \quad \text{if } n \neq -1, -2$$

95 $$\int \frac{du}{(a^2 \pm u^2)^n} = \frac{1}{2a^2(n-1)}\left(\frac{u}{(a^2 \pm u^2)^{n-1}} + (2n-3)\int \frac{du}{(a^2 \pm u^2)^{n-1}}\right) \quad \text{if } n \neq 1$$

96 $$\int u\sqrt{au+b}\, du = \frac{2}{15a^2}(3au-2b)(au+b)^{3/2} + C$$

97 $$\int u^n\sqrt{au+b}\, du = \frac{2}{a(2n+3)}\left(u^n(au+b)^{3/2} - nb\int u^{n-1}\sqrt{au+b}\, du\right)$$

98 $$\int \frac{u\, du}{\sqrt{au+b}} = \frac{2}{3a^2}(au-2b)\sqrt{au+b} + C$$

99 $$\int \frac{u^n\, du}{\sqrt{au+b}} = \frac{2}{a(2n+1)}\left(u^n\sqrt{au+b} - nb\int \frac{u^{n-1}\, du}{\sqrt{au+b}}\right)$$

100a $$\int \frac{du}{u\sqrt{au+b}} = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{au+b}-\sqrt{b}}{\sqrt{au+b}+\sqrt{b}}\right| + C \quad \text{if } b > 0$$

100b $$\int \frac{du}{u\sqrt{au+b}} = \frac{2}{\sqrt{-b}}\tan^{-1}\sqrt{\frac{au+b}{-b}} + C \quad \text{if } b < 0$$

101 $$\int \frac{du}{u^n\sqrt{au+b}} = -\frac{\sqrt{au+b}}{b(n-1)u^{n-1}} - \frac{(2n-3)a}{(2n-2)b}\int \frac{du}{u^{n-1}\sqrt{au+b}} \quad \text{if } n \neq 1$$

102 $$\int \sqrt{2au-u^2}\, du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u-a}{a} + C$$

103 $$\int \frac{du}{\sqrt{2au-u^2}} = \sin^{-1}\frac{u-a}{a} + C$$

104 $$\int u^n\sqrt{2au-u^2}\, du = -\frac{u^{n-1}(2au-u^2)^{3/2}}{n+2} + \frac{(2n+1)a}{n+2}\int u^{n-1}\sqrt{2au-u^2}\, du$$

105 $$\int \frac{u^n\, du}{\sqrt{2au-u^2}} = -\frac{u^{n-1}}{n}\sqrt{2au-u^2} + \frac{(2n-1)a}{n}\int \frac{u^{n-1}\, du}{\sqrt{2au-u^2}}$$

106 $$\int \frac{\sqrt{2au-u^2}}{u}\, du = \sqrt{2au-u^2} + a\sin^{-1}\frac{u-a}{a} + C$$

107 $$\int \frac{\sqrt{2au-u^2}}{u^n}\, du = \frac{(2au-u^2)^{3/2}}{(3-2n)au^n} + \frac{n-3}{(2n-3)a}\int \frac{\sqrt{2au-u^2}}{u^{n-1}}\, du$$

108 $$\int \frac{du}{u^n\sqrt{2au-u^2}} = \frac{\sqrt{2au-u^2}}{a(1-2n)u^n} + \frac{n-1}{(2n-1)a}\int \frac{du}{u^{n-1}\sqrt{2au-u^2}}$$

109 $$\int (\sqrt{2au-u^2})^n\, du = \frac{u-a}{n+1}(2au-u^2)^{n/2} + \frac{na^2}{n+1}\int (\sqrt{2au-u^2})^{n-2}\, du$$

110 $$\int \frac{du}{(\sqrt{2au-u^2})^n} = \frac{u-a}{(n-2)a^2}(\sqrt{2au-u^2})^{2-n} + \frac{n-3}{(n-2)a^2}\int \frac{du}{(\sqrt{2au-u^2})^{n-2}}$$

DEFINITE INTEGRALS

111 $$\int_0^{\infty} u^n e^{-u}\, du = \Gamma(n+1) = n! \quad (n \geq 0)$$

112 $$\int_0^{\infty} e^{-au^2}\, du = \frac{1}{2}\sqrt{\frac{\pi}{a}} \quad (a > 0)$$

113 $$\int_0^{\pi/2} \sin^n u\, du = \int_0^{\pi/2} \cos^n u\, du = \begin{cases} \dfrac{1\cdot 3\cdot 5\cdot \cdots \cdot (n-1)}{2\cdot 4\cdot 6\cdot \cdots \cdot n}\dfrac{\pi}{2} & \text{if } n \text{ is an even integer and } n \geq 2 \\[2ex] \dfrac{2\cdot 4\cdot 6\cdot \cdots \cdot (n-1)}{3\cdot 5\cdot 7\cdot \cdots \cdot n} & \text{if } n \text{ is an odd integer and } n \geq 3 \end{cases}$$