

3.1 #59

$$M(r) = \frac{1 + 0.05r}{1 + 0.004r^2}$$

$$1 \leq r \leq 8$$

(a) For what r is $M(r)$ increasing?
This is same as asking for first derivative sign line.

M = # thousand refinanced mortgages
 r = 30-yr fixed mortgage rate
($r\%$)

$$M'(r) = \frac{(1 + 0.004r^2)(0.05) - (1 + 0.05r)(0.008r)}{(1 + 0.004r^2)^2}$$

$$= \frac{0.05 + 0.0002r^2 - 0.008r - 0.0004r^2}{(1 + 0.004r^2)^2}$$

$$= \left(\frac{-0.0002r^2 - 0.008r + 0.05}{(1 + 0.004r^2)} \right) \left(\frac{10000}{10000} \right) = \frac{-2r^2 - 80r + 500}{10000(1 + 0.004r^2)}$$

$$= \frac{-2(r^2 + 40r - 250)}{10000(1 + 0.004r^2)} = 0$$

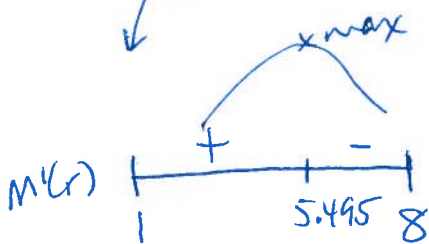
use as test

$$\Rightarrow r^2 + 40r - 250 = 0 \Rightarrow r = \frac{-40 \pm \sqrt{40^2 - 4(-250)}}{2}$$

$$r = \frac{-40 \pm \sqrt{2600}}{2} = \frac{-40 \pm 10\sqrt{26}}{2}$$

$$r = -20 \pm 5\sqrt{26} \quad (r \text{ can't be negative})$$

$$\Rightarrow r = -20 + 5\sqrt{26} \approx 5.495$$



test: $r = 2 \quad \frac{-(-)}{+} = +$

$r = 7 \quad \frac{-(+)}{+} = -$

\Rightarrow at $r = 5.495$, we have max pt.

(a) answer: $M(r)$ increasing on $[1, 5.495)$

(b) at $r = 5.495\%$, M is max + max M

$$\text{is } M = \frac{1 + 0.05(5.495)}{1 + 0.004(5.495)^2} \approx 1.137377, \text{ i.e., } \approx 1137 \text{ refinanced mortgages}$$