2.3 Product & Quotient Rules; Higher-Order Derivatives

Product Rule

If $f(x)$ & $g(x)$ are differentiable at x, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

Quotient Rule

If $f(x)$ & $g(x)$ are differentiable,

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} \qquad (\text{if } g(x) \neq 0)$$

("low d-hi minus hi d-low over low squared")

Ex 1 Find y'.

(a) $y = -3(x+2\sqrt{x})(5x^3 - 2x + 1)$

(b) $y = \dfrac{t^2+1}{1-t^2}$

2.3 (cont)

Ex 2 Find eqn of tangent line to curve given at the point given.

$y = (x+3)(1-\sqrt{x})$ at pt where $x=1$

Ex 3 Find pts on $y = \dfrac{x^2+x-1}{x^2-x+1}$ where tangent line is horizontal.

2.3 (cont)

Ex 4 Find $\frac{dy}{dx}$ at $x=4$ for $y = (x^2+2)(x+\sqrt{x})$

Ex 5 Find $h'(0)$ if $h(x) = \frac{3x^2 - 5g(x)}{g(x)+4}$ where $g(0) = 2$ and $g'(0) = -3$.

2.3 (cont)

$y = f(x)$

Higher order Derivatives

$f'(x)$	$\frac{dy}{dx}$	$D_x(y)$	first
$f''(x)$	$\frac{d^2y}{dx^2}$	$D_x^2(y)$	second
$f'''(x)$	$\frac{d^3y}{dx^3}$	$D_x^3(y)$	third
$f^{(4)}(x)$	$\frac{d^4y}{dx^4}$	$D_x^4(y)$	fourth
$f^{(n)}(x)$	$\frac{d^ny}{dx^n}$	$D_x^n(y)$	n^{th} $(n \geq 4)$

Ex 6

Find second derivative

(a) $y = 5\sqrt{x} + \frac{3}{x^3} + \frac{3}{3\sqrt{x}} + \frac{1}{5}$

(b) $y = (x^2 - x)(2x - \frac{1}{x})$