

## 2.6 Implicit Differentiation and Related Rates

Ex 1 Find  $\frac{dy}{dx}$ .

(a)  $\frac{1}{x} + \frac{1}{y} = 1$

(b)  $(3xy^2 + 1)^4 = 2x - 3y$

implicit fn

ex  $3x^2 + y^2 - xy = 4$

explicit fn

ex  $y = 3x^2 - 5x + 1$

## 2.6 (cont)

EX2 Find eqn of tangent line.

(a)  $x^2 - y^3 = 2x$  at pt  $(1, -1)$

(b)  $(x^2 + 2y)^3 = 2xy^2 + 64$  at pt  $(0, 2)$

2.6 (cont)

Ex3 when the price of a certain commodity is  $p$  (\$/unit) customers demand  $x$  hundred units, where

$$x^2 + 3px + p^2 = 79$$

How fast is demand  $x$  changing wrt time when price is \$5/unit and is decreasing at the rate of \$0.30/month?

## 2.6 (cont)

Ex 4 At a certain factory, output  $Q$  is related to inputs  $x$  and  $y$  by the eqn.  $Q = 2x^3 + 3x^2y^2 + (1+y)^3$ .

If current levels of input are  $x=30$  and  $y=20$ , use calculus to estimate change in  $y$  that should be made to offset a decrease of 0.8 unit in input  $x$  so that output will be maintained at its current level.

### 3.1 Increasing + Decreasing Fns; Relative Extrema

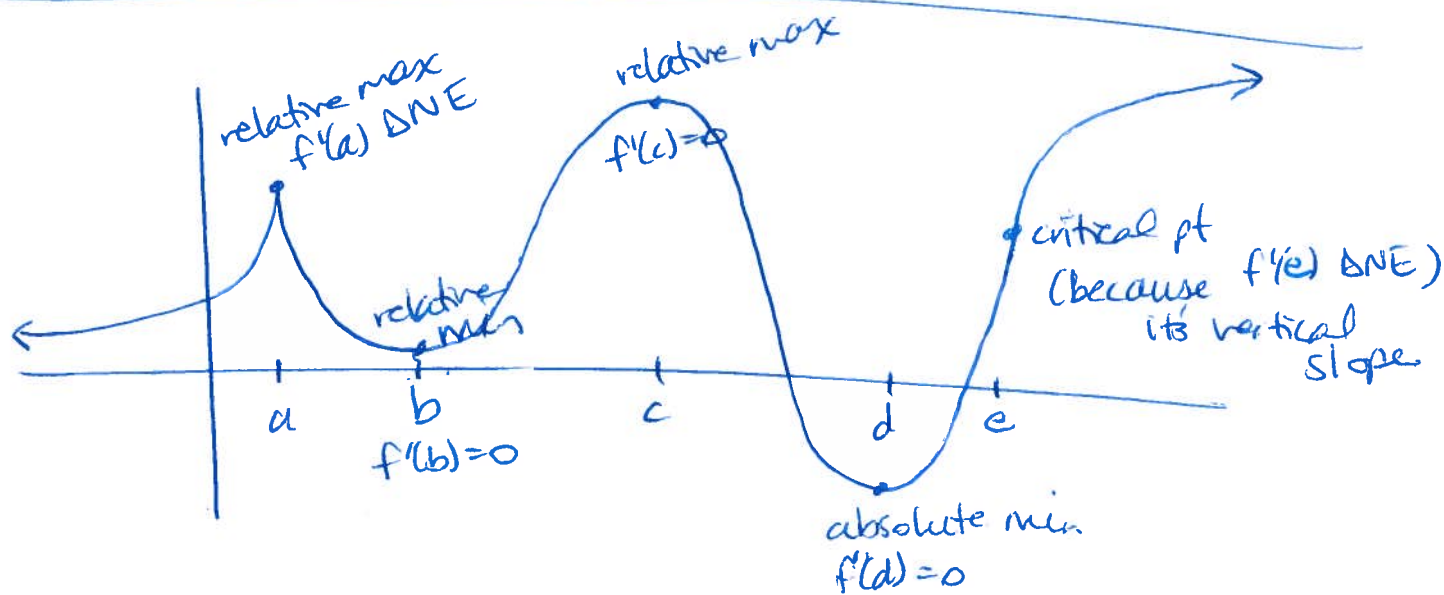
$f(x)$  is defined on  $a < x < b$ , let  $x_1, x_2$  be in that interval.

Then  $f(x)$  is increasing if  $x_2 > x_1 \Rightarrow f(x_2) > f(x_1)$

" " decreasing if  $x_2 > x_1 \Rightarrow f(x_2) < f(x_1)$

$x=c$  is critical number (or critical value) if  $f'(c) = 0$  or  $f'(c)$  is undefined

$(c, f(c))$  is critical pt



- min/max pts called critical pts
- absolute max/min means highest/lowest pt on graph
- relative max/min means graph is increasing on one side + decreasing on the other side
- pt on graph at  $x=e$  is not relative min or max

### 3.1 (cont)

Ex 1 Find where  $g(x)$  increasing/decreasing.

$$g(x) = \frac{1}{x^2+1} - \frac{1}{(x^2+1)^2}$$

Find critical  
pts +  
classify them.

### 3.1 (cont)

Ex 2 Find critical pts, classify them. Then sketch graph.

(a)  $f(x) = 3 - (x+1)^3$

(b)  $f(x) = \frac{x^2}{x-1}$

### 3.1 (cont)

Ex 3 Use calculus to sketch graph.

$$f(x) = x^3(x+5)^2$$

EX 4  $S(x) = \frac{200x + 1500}{0.02x^2 + 5}$ ,  $x \geq 0$

(a) sketch graph

$x$  = thousand dollars  
spent on advertising  
 $S$  = # units sold



3.1 (cont)

Ex 4 (cont)

(b) How many units sold if nothing spent on advertising?

(c) How much should be spent to maximize sales?  
What is max sales level?