5.1 Indefinite Integral & Differential Eqns

<u>Defn</u> $F(x)$ is an <u>antiderivative</u> of $f(x)$ if
$F'(x) = f(x)$ for every x in the domain of $f(x)$.

(The process of getting an antiderivative is called antidifferentiation or indefinite integration.)

- If $F'(x) = f(x)$, then $G(x) = F(x) + C$ where C is an arbitrary constant is also an antiderivative of $f(x)$
because $G'(x) = F'(x) = f(x)$.

<u>ex</u>

$f(x) = 3x^2$

$\Rightarrow F(x) = x^3$

$G(x) = x^3 + C$

all these curves have same slope at x

<u>Indefinite Integral</u>

$$\int f(x)\,dx = F(x) + C$$

"the integral of $f(x)\,dx$ is $F(x) + C$"

$f(x)$ = "integrand"

$dx \Rightarrow$ variable of integration is x

$\int k\,dx = kx + C$, $k \in \mathbb{R}$ constant

<u>power rule</u> $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$, $n \neq -1$

<u>log</u> $\int \frac{1}{x}\,dx = \ln|x| + C$, $x \neq 0$

<u>exponential</u> $\int e^{kx}\,dx = \dfrac{e^{kx}}{k} + C$ $k \neq 0$

5.1 (cont)

Ex 1 Integrate.

(a) $\int \left(x^{1/3} - 3x^2 + 9\right) dx$

Integrals are linear Operators

①

②

(b) $\int \left(2e^u + \frac{6}{u} + \ln 2\right) du$

(c) $\int \left(\frac{1}{3y} - \frac{5}{\sqrt{y}} + e^{-y/2}\right) dy$

Ex 2 Solve the initial value problem for $y = f(x)$.

$$\frac{dy}{dx} = e^{-x} \quad \text{where } y = 3 \text{ @ } x = 0$$

Ex 3 Find $f(x)$ if $f'(x) = x^{-1/2} + x$ and $f(x)$ goes through $(1, 2)$.

5.2 Integration by Substitution

★ this is needed for functions that needed chain rule to differentiate

Ex 1 Integrate.

(a) $\int \frac{1}{3x+4}\,dx$

(b) $\int 2xe^{x^2-1}\,dx$

5.2 (cont)

Ex 2 Integrate.

(a) $\int 3t\sqrt{t^2-8}\,dt$

(b) $\int e^{\sqrt{x}}\, x^{-1/2}\,dx$

(c) $\int \frac{t-1}{t+1}\,dt$

5.2 (cont)

Ex 3 Integrate.

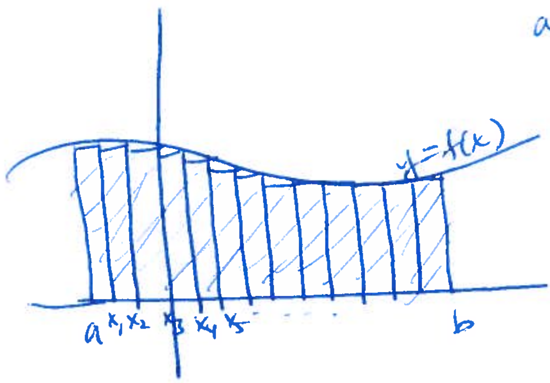
(a) $\int x\sqrt{3x+5}\,dx$

(b) $\int \frac{\ln(x^2)}{x}\,dx$

Ex 4 If $f'(x) = \frac{2x}{1+3x^2}$, find $f(x)$ if we know $f(x)$ goes through $(9,5)$.

5.3 The Definite Integral & Fundamental Thm of Calculus

Area under a curve



approximate area by adding areas of lots of rectangles

Let's say each rectangle has same width, $dx = \Delta x = \frac{b-a}{n}$ (n = # rectangles)

Then height of each rectangle is $f(x_i)$.

$\Rightarrow$ Area $\simeq \sum_{i=1}^{n} f(x_i)\Delta x$ $\Rightarrow$ Area $= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

Definite Integral (geometrically) measures area under curve from $x=a$ to $x=b$.

$$\int_a^b f(x)\,dx = \text{Area}$$

Fundamental Thm of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is antiderivative of $f(x)$

Some rules:

① $\int_a^b k f(x)\,dx = k\int_a^b f(x)\,dx$, $k \in \mathbb{R}$ constant

② $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

③ $\int_a^a f(x)\,dx = 0$

④ $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

⑤ $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

5.3 (cont)

Ex1 Evaluate.

(a) $\int_1^4 (2-3x)\,dx$

(b) $\int_0^{\ln 3} (e^x - e^{-x})\,dx$

(c) $\int_1^2 \frac{x^2}{(x^3+1)^2}\,dx$

5.3 (cont)

Ex 2 Find area under curve $y = f(x) = \dfrac{3}{\sqrt{9-2x}}$

from $x = -8$ to $x = 0$.

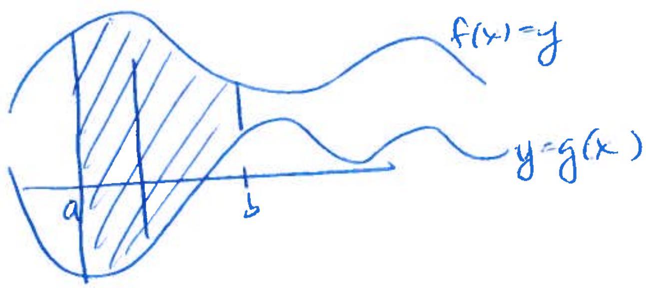
Ex 3 If $\int_{-3}^{1} f(x)\,dx = 0$, $\int_{-3}^{2} f(x)\,dx = 5$, what is

$\int_{-3}^{2} g(x)\,dx = -2$, $\int_{-3}^{1} g(x)\,dx = 4$ $\quad \int_{-3}^{1} (2f(x) + 3g(x))\,dx$?

5.4 Applying Definite Integration: Distribution of Wealth & Average Value

Area Between 2 Curves

$f(x), g(x)$ continuous with $f(x) \geq g(x)$ on $a \leq x \leq b$, then area between curves is $A = \int_a^b (f(x) - g(x))\,dx$



Ex 1 Find area between $y = \frac{1}{x^2}$ and $y = x$ and $y = \frac{x}{8}$.

5.4 (cont)

Ex2 Find area between $y = x+2$, $y = 8-x$, $x = 2$ and the y-axis.

Ex3 Find the average value of $f(x) = \frac{x+1}{x^2+2x+4}$ over $-1 \le x \le 1$.

Avg value of $f(x)$ on $[a,b]$

$$avg = \frac{1}{b-a}\int_a^b f(x)\,dx$$

assuming $f(x)$ continuous on $[a,b]$